EconPol
POLICY BRIEF

38
2021

October
Vol.5

High Public Debt in an Uncertain World: Post-Covid-19 Dangers for Public Finance

Daniel Gros (CEPS & EconPol Europe)

Key Messages

- High debt ratios represent a danger, even if interest rates are low.
- The key reason is increased uncertainty of growth prospects in a post-Covid-19 economy, coupled with an uncertainty regarding the probability of future large shocks
- Large negative shocks are more frequent than assumed in standard models.
- Another reason is that the cost of public debt might increase more than linearly as the debt ratio rises.
- Large negative shocks create much more problems when debt is already high.

EUROPE
econPOL
European Network for Economic
and Fiscal Policy Research

headed by

EconPol POLICY BRIEF
A publication of EconPol Europe
European Network of Economic and Fiscal Policy Research

Publisher and distributor: ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 89 9224-0, Telefax +49 89 9224-1462, Email Dolls@ifo.de
Editors: Mathias Dolls, Clemens Fuest

EconPol Europe: www.econpol.eu

High Public Debt in an Uncertain World
Post-Covid-19 Dangers for Public Finance

Daniel Gros

During the Covid-19 crisis, governments have had little choice but to support the economy while trying to keep the spread of the disease under control; this means accepting large deficits. Now that the health emergency is subsiding, governments have to chart a new course for public finance. The starting point is a higher level of public debt. In some countries, such as Italy or the US, public debt has increased by between 25 and 30 percentage points relative to GDP. Moreover, the levels reached by a number of countries (close to 160 percent of GDP for Italy, 130 percent of GDP for the US, 200 percent of GDP for Greece) are above the levels that would have been considered prudent a few years ago.

One reaction to these higher debt levels is: Who cares? Nominal interest rates are around zero, even for longer maturities. With nominal growth positive, even if modest, the basic debt sustainability condition that the growth rate (g) be higher than the interest rate (r) is fulfilled. It implies that the Covid-19 debt should decline over time, at least as a ratio to GDP, suggesting that today's higher debt level should be sustainable. But to paraphrase Paulo and Zhou (2021), "we cannot sleep more soundly" even if $r-g < 0$, because history shows that defaults happen even during times when this condition is fulfilled.

That debt is not free is recognized by Blanchard et al. (2020), but these authors also argue (as do many others) that its price, namely the long-term interest rate, has fallen. The corollary is that one should accept substantially higher debt ratios.

The key reason why high public debt should be considered a potential source of problems, even in an environment of low rates, is another important, but often overlooked, legacy of the Covid-19 crisis: increased uncertainty. There are two reasons why the post-Covid-19 environment should be considered more uncertain.

For one, the crisis will accelerate the shift toward a digital economy, and the demand for tourism and other personal services might remain depressed for some time. In Europe, the outlook for different countries has diverged considerably, especially among countries that specialize in tourism and have only a weak digital infrastructure.[1] This means that the medium-term growth prospects have become more uncertain.

Another reason why increased uncertainty should be an essential element of a post-Covid-19 fiscal strategy is that the realization of a "once in 100 years" crisis must lead one to reconsider the probability of other large shocks in the future. Kozlowski et al. (2020) show that the occurrence of a large event should lead agents to consider future large shocks to be more likely. If no one can know with certainty the true probability distribution of future shocks, it is rational for economic agents to update their subjective beliefs in a Bayesian approach when a large shock materializes.

This updating of the probability of large shocks should also encompass policymakers. They need to acknowledge the existence of "fat tails," i.e., the fact that extreme events occur much more often than one would expect from a "normal" distribution. This does not imply that another health crisis is around the corner, only that after the Covid-19 shock, it becomes rational to update the probability of future large shocks. Policymakers should thus assume a higher degree of overall uncertainty when making long-term plans. This paper provides evidence, based on a historical dataset spanning 200 years, showing that the distribution of growth indeed has a fat tail on the left.

[1] In addition, global value chains might also be affected by the rapidly increasing geopolitical rivalry between the US and China. This rivalry is not a consequence of the Covid-19 crisis, but has increased in intensity over the last year, adding to uncertainty about the economic outlook.

Uncertainty (about growth or future values of "r−g") becomes particularly important when the cost of public debt is not linear. There are a number of reasons to believe that the cost of public debt increases more than linearly when the debt ratio increases. One reason is that a higher debt level is usually associated with a higher risk premium, leading to a quadratic relationship between debt service cost and the debt/GDP ratio. Another reason is that the distortionary costs of raising taxes to service public debt rise as the government has to extract a larger share of national income. In public finance, it is often assumed that these costs increase with the square of the tax rate, thus implying also that the cost of debt increases with the square of debt ratio. More generally, one can describe this as a function in which the cost of debt is a convex function of debt. An immediate implication is that in the presence of uncertainty (of growth or r−g), the expected cost of public debt is higher than the cost that would result from a constant value. The usual debt sustainability calculations which assume constant values for growth or r−g are thus misleading.

Another implication of this convexity is that one should weight a large shock disproportionately more than a small one. This paper concentrates on this aspect.

The next section presents a bare-bones model of uncertainty about growth when the cost of public debt is convex. This model is then used in section 2 to determine the trade-off between the desirable level of public debt and uncertainty. Section 3 uses the historical evidence to document the existence of fat tails. Section 4 presents the conclusion.

1. An illustrative model

This two-period model describes a single country whose economy is subject to idiosyncratic growth shocks. In the first period, a certain amount of debt, indicated by *d*, is issued. The amount is taken as given from the past. The debt has to be repaid (with interest) in the second period. To illustrate the basic point, interest payments are thus incorporated into the debt service due during the second period.

An essential element of the model is a conventional social welfare loss function, which expresses the idea that increasing tax revenue leads to increasing distortions (Mankiw 1987). This implies that, at the margin, it becomes more and more costly to obtain higher tax revenue the higher the tax rate. The social loss from obtaining tax revenue, measured as a percentage of GDP, is thus assumed to be given by:

$$(1)\quad L = \beta q_{t+1}^2 \qquad \beta>0$$

where *q* indicates the ratio of tax revenue to GDP, or the overall effective tax rate. The parameter β represents the efficiency of the tax system. A higher value of β implies a lower efficiency of the tax system.

As explained above, debt, denoted by d_t, is inherited from the first period and repaid during the second period. This implies that $q_{t+1} = d_{t+1}$. Tax revenue is needed in the second period only to service debt. This implies that the social cost of debt is simply given by

$$(2)\quad L = \beta d_{t+1}^2 \qquad \beta>0$$

Other factors that can increase debt are interest payments on past debt and any primary deficit during the current period. They are all rolled into the debt that has to be repaid in the future. Although the results are interpreted in terms of debt, they could also be interpreted in terms of the primary deficit, or the interest rate.

A trivial implication of the quadratic cost function is that the marginal cost of debt increases with the debt ratio: the difference between the marginal and the average cost of debt. The average cost is given by:

$$(3)\quad average\ cost = \beta d_{t+1}$$

The marginal cost is given by:

$$(4)\quad marginal\ cost\ = 2\beta d_{t+1}$$

It follows that in this simple setup, the marginal cost is twice as high as the average cost. This key issue of the marginal cost of public debt exceeding the average was already emphasized by Alcidi and Gros (2019) in the context of a standard risk premium model of the interest rate on public debt.

A second key element of the setup is uncertainty about growth. To keep the analytics as simple as possible, it is assumed here that growth oscillated between two values, high and low. One way to formalize this in a simple way is to assume that output in the second period will be equal to 1+θ with probability 0.5 and with equal probability 1-θ (of course 0<θ<1). The parameter θ thus describes the uncertainty about future growth, so an increase in θ represents an increase in the mean preserving spread. The parameter θ could also be considered as “GDP at risk” (Adrian et al. 2019).[2]

For any given amount of debt, a shock to GDP affects the (average) tax rate that is necessary to service the debt. For one unit of debt, the tax burden falls to 1/(1+θ) in the event of a positive shock and rises to 1/ (1-θ) in the event of a negative shock. For the purposes of this illustrative setup, it does not matter whether the debt is fixed in nominal or real amounts. If debt is fixed in nominal amounts, what matters is uncertainty about future nominal GDP; similarly, if the debt is fixed in real terms, what matters is uncertainty about future output. In the following, it is implicitly assumed that all units are in real terms, i.e., units of some numeraire good.

With this form of uncertainty, the expected loss arising from debt service is given by:

$$(5)\quad E(L) = d^2\beta\frac{1}{2}\left\{\left[\frac{1}{(1+\theta)}\right]^2 + \left[\frac{(1)}{(1-\theta)}\right]^2\right\}$$

Given the quadratic loss function, it is not surprising that the expected loss is proportional to the square of the debt (relative to GDP). This can be simplified to:

$$(6)\quad E(L) = \beta d^2\left\{\frac{(1+\theta^2)}{(1-\theta^2)^2}\right\} > \beta d^2$$

[2] In a “plucking” model (Dupraz et al. 2019), the distribution would be asymmetric, with a low probability of a large negative value and a higher probability of smaller positive values.

The two variables of interest that determine the expected social loss are thus the debt to be serviced and the size of "GDP at risk." Notice that the variance of output is equal to $\theta^2/2$. The expected loss is thus a function of the variance of output.

It is apparent from equation (6) that the social cost of debt is higher when there is uncertainty about growth ($\theta > 0$) and that the social cost of debt is a convex function of the GDP at risk. Moreover, the marginal cost of additional debt is also an increasing function of uncertainty. This implies that the (marginal) social cost of adding to existing public debt not only increases with the debt level, but also increases with the degree of uncertainty. This is a first general corollary of the combination of a convex cost of debt and uncertainty.

An increase in either debt or GDP at risk increases the (expected) social loss. This implies that an increase in uncertainty requires a lower debt level if one wants to keep the expected cost of debt service constant. This is a second important general corollary if the cost of debt is convex in the debt ratio.

2. The trade-off between debt and uncertainty

The trade-off between volatility (represented by theta θ) and the debt level, which keeps the expected debt service cost constant, can be calculated by differentiating equation (3) above

$$(7)\quad \Delta[E(L)] = 2\beta\left\{d\left[\frac{(1+\sigma)}{(1-\sigma)^2}\right]\partial d + 2\theta\beta d^2\left[\frac{(3+\sigma)}{(1-\sigma)^3}\right]\partial\theta\right\}$$

For a constant expected loss (i.e., $\Delta[E(L)] = 0$), the trade-off between d and σ is given by:

$$(8)\quad \frac{\partial d}{\partial\theta_{\Delta[E(L)=0]}} = -\frac{\theta d[(3+\theta^2)]}{(1-\theta^4)}$$

This equation indicates the amount by which the debt ratio would have to decline if growth uncertainty, as measured by the parameter θ, increases. One way to interpret this equation is to say that by what amount post-Covid-19 debt levels would have to be lower than pre-Covid if the main impact of Covid-19 is an increase in uncertainty.

Equation (8) implies that higher uncertainty requires only a small reduction in debt if the initial uncertainty is very small (i.e., if θ is small). But at a given value of GDP at risk the required reduction in debt increases proportionally with the initial level of debt.

As mentioned above, a change in the interest rates is equivalent to a change in the initial debt level because a higher interest rate also contributes to higher debt service cost in the future. Another way to interpret equation (8) is thus that it describes the fall in the interest rate which can offset an increase in uncertainty. Formally, this can be obtained by writing d as $d = d_{t-1}(1+r)$ plus primary deficit. If only r changes one can rewrite equation (8) in the steady state of a constant debt ratio as:

$$(9)\quad \frac{\partial r}{\partial\theta_{\Delta[E(L)=0]}} = -\frac{\theta[(3+\theta^2)]}{(1-\theta^4)}$$

This relationship illustrates the trade-off between higher volatility and lower interest rates that one needs to take into account when discussing post-Covid fiscal policy.

3. Fat tails

In the illustrative model presented above, the impact of an increase in the GDP at risk on welfare and the required adjustment is convex. This implies that a key issue is the likelihood of large shocks. The low probability of one large shock would constitute a much stronger argument for prudence regarding debt than the high likelihood of many small shocks. Fagiolo et al. (2008) showed that the distribution of growth rates is fat-tailed, even for OECD economies, which should be more stable. The data used by these authors does not include the Great Financial Crisis (nor, naturally, the Covid-19 crisis) and it covers only the post-WWII period, which arguably constitutes an unusually favorable growth period in the broader sweep of history.

It might thus be useful to consider the evidence from longer time series. Mauro et al. (2013) provide public finance and growth data since 1800, covering most of (today's) advanced economies as well as a number of other countries.[3]

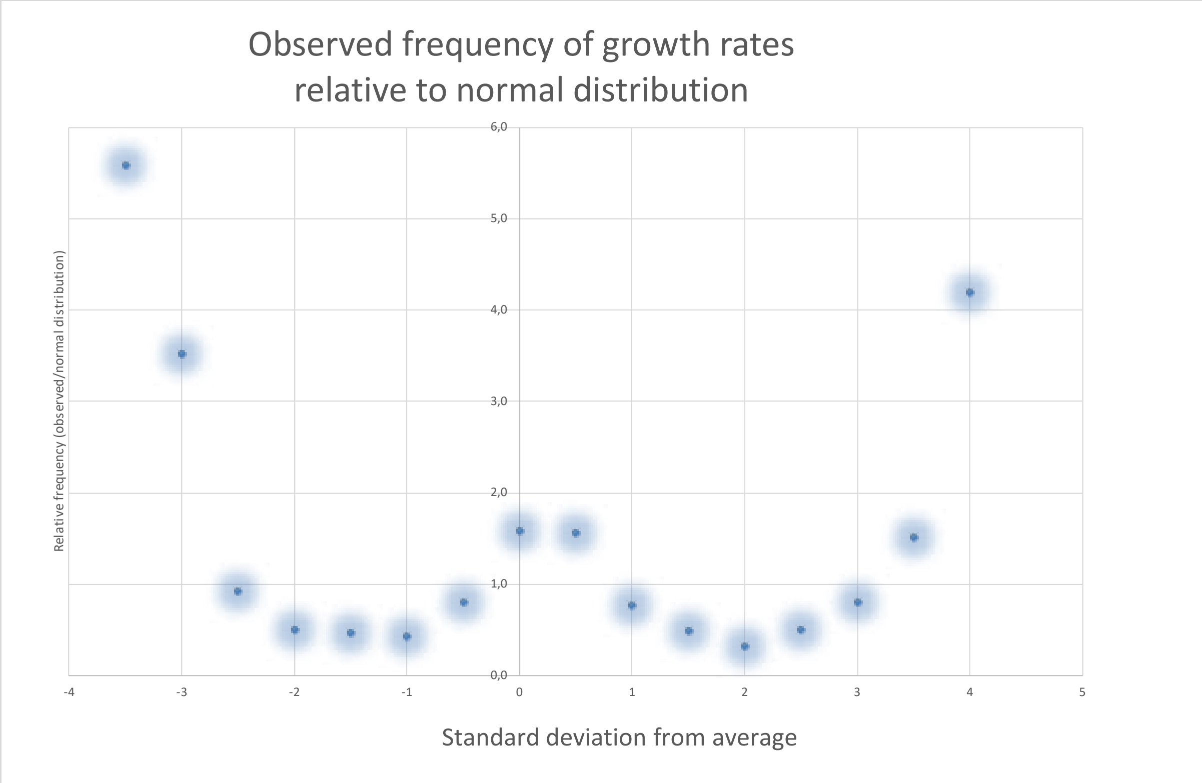
In the following, we concentrate on 24 advanced economies covered in this dataset, which yields about 3,300 observations of (annual) growth rates. The overall average growth rate of real GDP across all 215 years, and all countries, is close to 3 percent, with a sample standard deviation very close to 6 percent. If the distribution of growth rates were normal, one would expect to find at most two observations of growth below three standard deviations from the mean (i.e., a growth rate of less than 21 percent). But in reality, one finds many more. The discrepancy between the observed frequency of tail events and the theoretical one under a normal (Gaussian) distribution is illustrated in figure 1 below. This figure shows the ratio between the observed frequency in different buckets calculated in how many standard deviations they are above or below the mean (here 3 percent).

The figure shows two deviations from a normal distribution: in the center and in the tails. In the center, i.e., close to the mean, one finds over 50 percent more observations than one would expect under a normal distribution. But the real difference is in the tails from 2.5 to 3 times the standard deviations from the mean, where the actual observations are several times more frequent than one would expect under a Gaussian distribution.

The figure shows the ratio only up to four times the standard deviation because "five sigma" would be off the chart. Five-sigma growth declines are almost 20 times more frequent than they should be if the distribution were normal.

Figure 1

[3] Cotarelli et al. (2010) show the danger of relying on a short sample. Just before Greece de facto defaulted, they argued that "default in today's advanced economies [is] unnecessary, undesirable, and unlikely."

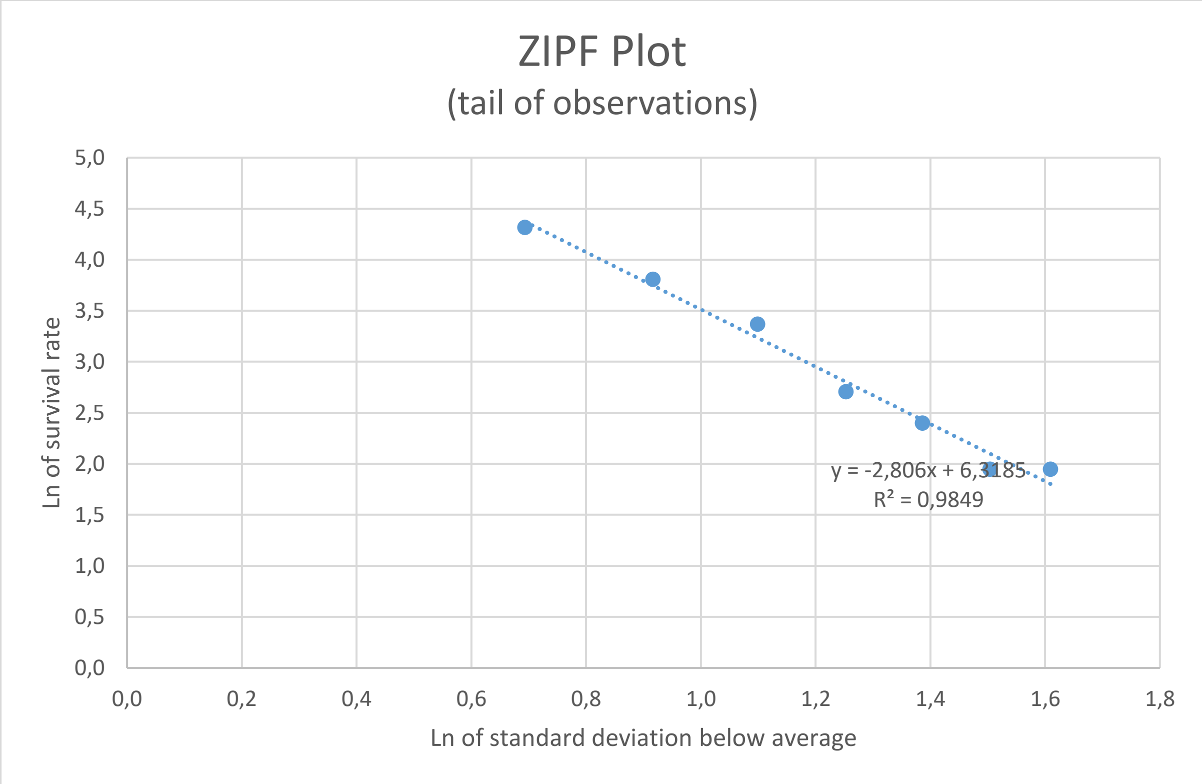


Source: Own calculations based on Mauro et al., 2013

There is thus strong evidence for fat tails. Cirillo and Taleb (2020) and Taleb (2020) argue that for a fat-tailed distribution, the most important information is in the tails and that one should not try to estimate what other distribution could fit the entire dataset. Instead, one should look only at the tail of the distribution (for an application to the economics of climate change, see Nordhaus 2011).

Figure 2 shows the ZIPF plot, which depicts the natural logarithm of the survival rate (the number of observations below a threshold) and the natural logarithm of the growth rate in terms of standard deviations below the average. Only the left-hand tail, i.e., the points with a growth shortfall of at least 2.5 times the standard deviation, are shown.

<u>Figure 2</u>



Source: Own calculations based on Mauro et al., 2013

The data shows a clear linear relationship between (the logarithm of) the survival function and (the logarithm of) the position in the tail, which is typical of the Pareto distribution. The estimated coefficient of 7.8 is (just) below 3, which means that the alpha parameter of the Pareto distribution would be equal to 1.8. This is below 2, implying that the variance of the distribution of growth rates does not exist.

4. Concluding remarks

Much of the literature on the desirability or acceptability of higher debt ratios starts from the observation that a low, perhaps negative value for the interest rate−growth differential (usually denoted by r−g) can render even very high debt ratios sustainable. The historical record does not support this argument, however, since defaults seem to have been as frequent during periods of a favorable interest rate−growth differential as during periods when interest rates were above growth rates (Mauro 2019).

But the argument that low interest rates should allow for high debt overlooks two other fundamental aspects: uncertainty and the convex nature of the cost of public debt. The Covid-19 crisis has called our attention to the importance of uncertainty. Rare but high-impact events do in fact occur from time to time.

The latter is a key fact of life: the cost of doubling the debt/GDP ratio is much higher than doubling the interest service, because the higher taxes needed to service this debt create more and more distortions. This paper has emphasized the increasingly distortionary nature of taxes, but there are other reasons why the cost of debt increases at a disproportionate rate. One additional reason is that a higher debt ratio leads to a higher risk premium and, objectively, to a higher risk of a debt crisis. This implies that if debt is already high, it becomes very difficult to react to a negative shock and maybe even to service the existing debt (as in the case of Greece).

This should caution countries with high debt ratios not to rely on low interest rates to make their debt sustainable. The cost of encountering the next “once in a lifetime” shock with public debt already at high levels might be extremely high.

References

Adams, P. A., Adrian, T., Boyarchenko, N., & Giannone, D. (2021). Forecasting Macroeconomic Risks. International Journal of Forecasting, 37(3), 1173–1191.

Adrian, T., Boyarchenko, N., & Giannone, D. (2019). Vulnerable Growth. American Economic Review, 109(4), 1263–89.

Alcidi, Cinzia and Daniel Gros. “Public Debt and the Risk Premium: A Dangerous Doom Loop.” VoxEU.org, May 22 (2019) https://voxeu.org/article/public-debt-and-risk-premium.

Cirillo, P. and Taleb, N.N., 2020. Tail Risk of Contagious Diseases. Nature Physics, 16(6), pp.606–613.

Cottarelli, C., Forni, L., Gottschalk, J., & Mauro, P. (2010). “Default in Today’s Advanced Economies: Unnecessary, Undesirable, and Unlikely.” IMF Staff Position Notes, 2010(012).

Dupraz, S., Nakamura, E., and Steinsson, J., 2019. A Plucking Model of Business Cycles (No. w26351). National Bureau of Economic Research.

Giorgio Fagiolo, Mauro Napoletano, and Andrea Roventini. “Are Output Growth-Rate Distributions Fat-Tailed? Some Evidence from OECD Countries.” Journal of Applied Econometrics 23.5 (2008), pp. 639–669.

Kozlowski, Julian with Laura Veldkamp, Venky Venkateswaran (2020). “Scarring Body and Mind: The Long-Term Belief-Scarring Effects of COVID-19,” NBER Working Paper No. 27439, https://www.nber.org/papers/w27439

Paolo Mauro, Rafael Romeu, Ariel Binder, and Asad Zaman, 2013, “A Modern History of Fiscal Prudence and Profligacy,” IMF Working Paper No. 13/5, International Monetary Fund, Washington, DC.

Mauro, Paolo and Jing Zhou (2021). “rg< 0: Can We Sleep More Soundly?” IMF Economic Review 69.1 (2021).

Nordhaus, W. D. (2011). The Economics of Tail Events with an Application to Climate Change. *Review of Environmental Economics and Policy*, 5(2), 240–257.

Schwaab, B., Lucas, A., and Zhang, X., 2021. “Modeling Extreme Events: Time-Varying Extreme Tail Shape.” European Central Bank, Working Paper Series, No 2524 / February 2021.

Taleb, N.N., 2020. Statistical Consequences of Fat Tails: Real World Preasymptotics, Epistemology, and Applications. arXiv preprint arXiv:2001.10488.

https://www.sr-sv.com/the-dangerous-disregard-of-fat-tails-in-quantitative-finance/

Appendix

Details of calculations

The starting point is the expression for the expected value of the social cost of debt:

$$E(L) = \beta d^2 \left\{\frac{(1+\theta^2)}{(1-\theta^2)^2}\right\}$$

To save on notation it will be convenient to work in the squares of the GDP a risk, with σ defined as σ = θ^2 .

$$E(L) = \beta d^2 \left\{\frac{(1+\theta^2)}{(1-\theta^2)^2}\right\} = \beta d^2 \left\{\frac{(1+\sigma)}{(1-\sigma)^2}\right\}$$

One directly finds that:

$$(1)\ \frac{\partial(E(L))}{\partial d} = 2d\beta \left\{\frac{(1+\theta^2)}{(1-\theta^2)^2}\right\} > 0$$

Differentiating with respect to θ yields:

$$(2)\ \frac{\partial(E(L))}{\partial \sigma}\frac{\partial \sigma}{\partial \theta} = \beta d^2 \left\{\frac{(1-\sigma)+2*(1+\sigma)}{(1-\sigma)^3}\right\} 2\theta > 0$$

$$\frac{\partial(E(L))}{\partial \theta} = 2\theta\beta d^2 \left\{\frac{(3+\sigma)}{(1-\sigma)^3}\right\} > 0$$

The total differential of the expected loss is then equal to:

$$(10)\Delta[E(L)] = 0 = 2\beta \left\{d\left[\frac{(1+\sigma)}{(1-\sigma)^2}\right]\partial d + 2\theta\beta d^2 \left[\frac{(3+\sigma)}{(1-\sigma)^3}\right]\partial\theta\right\}$$

$$(11)\Delta[E(L)] = 0 = 2\beta d(1-\sigma)^{-3}\{[(1+\sigma)(1-\sigma)]\partial d + \theta d[(3+\sigma)]\partial\theta\}$$

Keeping the expected loss constant and simplifying yields:

$$(12)\frac{\partial d}{\partial \theta}_{\Delta[E(L)=0]} = -\frac{\theta d[(3+\theta^2)]}{(1-\theta^4)}$$

To consider large changes one can use an approximation. Consider a discrete change in the GDP at risk from σ to σ_{pc}. If the expected cost of debt service is to be held constant this requires a lower debt level, indicated by d_{pc}.

$$E(L) = constant\ = \beta d_{pc}^2 \frac{(1+\sigma_{pc})}{\left(1-\sigma_{pc}\right)^2} = \beta d^2 \frac{(1+\sigma)}{(1-\sigma)^2}$$

(13)

$$\frac{d_{pc}^2}{d^2} = \frac{(1+\sigma)\left(1-\sigma_{pc}\right)^2}{\left(1+\sigma_{pc}\right)(1-\sigma)^2}$$

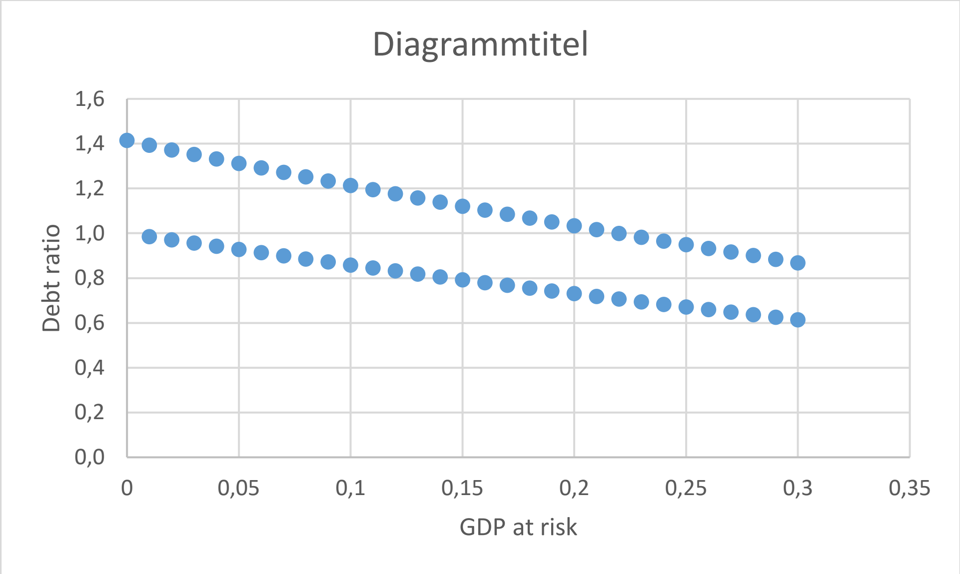
(14)

Taking logs and using the usual approximations for small values of σ that ln(1+σ) is approximately equal to σ.

$$\ln\frac{d_{pc}}{d} \cong \frac{1}{2}\left(\sigma-\sigma_{pc}\right) - \left(-\sigma_{pc}+\sigma\right) = \frac{3}{2}\left(\sigma-\sigma_{pc}\right)$$

(15)

This relationship shows that a more than marginal increase in uncertainty should be reflected in a proportional fall in the sustainable debt level which is 1.5 times as large.



Source: Own calculations based on equation (5) in the text

EconPol Europe

EconPol Europe - The European Network for Economic and Fiscal Policy Research is a unique collaboration of policy-oriented university and non-university research institutes that will contribute their scientific expertise to the discussion of the future design of the European Union. In spring 2017, the network was founded by the ifo Institute together with eight other renowned European research institutes as a new voice for research in Europe.

The mission of EconPol Europe is to contribute its research findings to help solve the pressing economic and fiscal policy issues facing the European Union, and thus to anchor more deeply the European idea in the member states. Its tasks consist of joint interdisciplinary research in the following areas

1) sustainable growth and 'best practice',

2) reform of EU policies and the EU budget,

3) capital markets and the regulation of the financial sector and

4) governance and macroeconomic policy in the European Monetary Union.

Its task is also to transfer its research results to the relevant target groups in government, business and research as well as to the general public.