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Optimal higher education enrollment and productivity externalities in a two-sector model

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Abstract

We investigate externalities in higher education enrollment over the course of development in a two-sector model. Each sector works with only one type of labor, skilled or unskilled, and individuals are differentiated according to their cost of acquiring human capital. Both sectors exhibit productivity externalities in the size of the skill-specific labor and in the average human capital of workers. When skill-biased technological change prevails, it may well be the case that intermediate stages of development witness underenrollment in higher education, while highly developed economies experience overenrollment.

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1 Introduction

Is the number of college graduates in the population too high or too low? Over the last decades, the number of workers who hold an academic degree has increased tremendously. Nowadays, around 40 per cent of a birth cohort graduate from a theory-based program of tertiary education in OECD countries, ranging from graduation rates around 20% in Mexico and Turkey to rates exceeding 50% in Poland, Iceland and the Slovak Republic (OECD, 2011). High and even increasing skill premia in terms of lifetime income for people holding academic degrees in recent decades (Acemoglu, 2002; Mitchell, 2005), even in many developing countries (Ripoll, 2005), underpin this trend. On the other hand, there are worries about future shortages of semi-skilled workers like nurses and technicians, raising doubts about whether or not the current share of students in higher education is already detrimental for growth or welfare. In a theoretical perspective, overeducation arises if the present value of the productivity gain for the marginal, least talented, individual, corrected for possible externalities, falls short of the education cost. As this requires to predict the future evolution of the productivity of the marginal individual, overeducation is hard to identify. Indeed, hinting to the average skill premium is not convincing because it will substantially exceed the productivity increase of the marginal individual. The phenomenon of overeducation has been discussed in the empirical literature, though not in a conclusive way (Sicherman, 1991; Büchel, 2003; Chevalier, 2003). They are looking at individuals being employed in an occupation that does not require the actual formal qualification of the worker at some given point in time. However, this observation does not necessarily indicate overeducation. It may easily go along with a substantial positive return to human capital investment in higher education in a lifetime perspective.

Our paper addresses the question of which pattern of overenrollment and underenrollment can be expected over the course of development. More specifically, the stylized facts suggest a move from underenrollment in some intermediate stage of industrialization to overenrollment in advanced economies. Underenrollment at intermediate stages seems a prevalent phenomenon in view of many contributions stressing borrowing constraints (eg Galor and Zeira, 1993; Gary-Bobo and Trannoy,

2008) and estimates suggesting that social returns to education are considerably higher than private returns in less advanced stages of development (Psacharopulos 1981, 1994). We try to explain this pattern within a simple structure of production externalities that are stationary in terms of parameters describing the externality, where the growth process is driven by some exogenous skilled-biased technological change. When dealing with the issue of whether undereducation or overeducation prevails, our focus lies on externalities of the enrollment decision. We discuss why market forces lead to overinvestment or underinvestment in higher education, justifying government intervention in that sector. Though only a minority of the population takes tertiary education, it is subsidized to a large extent in many countries, where policies toward tuition fees are far from uniform. While tuition fees are either negligible or even absent in many continental European countries, they can take values clearly exceeding average cost at several US universities. In the absence of market failures, standard considerations state that individual decisions to study will not be distorted with a proportional income tax when the subsidy rate of the direct cost coincides with the income tax rate (Trostel, 1993; Nielsen and Sorensen, 1997). This is true as the income tax reduces the returns to education and its opportunity cost by the same factor. Overenrollment can easily turn out in the absence of externalities if an almost proportional income tax is matched with heavy subsidization of tuition. This may reasonably well describe the current policy parameter settings in many European countries, taking account of coexisting progressive income taxes, social insurance contributions and income transfer withdrawal rates.

Our contribution focuses on externalities suggested by endogenous growth theory. Individuals are differentiated according to ability, which translates into differences in the cost of acquiring a university degree. Such a heterogeneity can be attributed to direct costs, like need for additional tuition, opportunity costs, e.g. need to repeat some exams, or even psychic costs, as learning with lower ability will be harder. Although our formulation describes such psychic costs, generalizations would be straightforward. The structure of our model builds on the analyses of educational standards by Costrell (1994) and Betts (1998), using a similar mechanism of sorting by ability. Having asymmetric information on individual productivity as source of market failure, their focus rests on political economy perspectives of the choice of the standard. By contrast, we are concerned with net impacts of technological externalities over the course of development. Moreover, we show that the government generally would like to affect enrollment by additional instruments even if the quality standard of education is optimally set.

We embed the endogenous enrollment decision in a simple model of a production economy with two sectors employing one type of labor - either skilled or unskilled - together with skill-specific technologies. We abstract from neoclassical scarcity effects from diminishing returns as they will typically not be a source of an externality. We also ignore the argument that when it comes to bargaining at the individual level, workers will only get a share of the productivity gain by education or training, thus pointing to underinvestment in human capital (Acemoglu, 1996; Acemoglu and Pischke, 1999).

Two main sources of market failure are considered, (i) an average human capital externality, and (ii) a size externality. Productivity in each sector depends on average human capital of the workers in the spirit of Lucas (1988). When the marginal individual decides to go to college, he disregards that average human capital will go down in each sector. This average human capital externality is clearly a source of overeducation from the point of view of a social planner. A similar overenrollment phenomenon would occur in a matching framework where lower average human capital levels would reduce investment of firms (Charlot and Decreuse, 2005). Productivity of a sector also depends on the size of the sector, which may reflect learning by doing or productivity gains through improved division of labor (Arrow, 1962; Lucas, 1988)[1]. When enrollment in higher education increases, the skilled sector becomes larger and the unskilled sector becomes smaller. Hence, there is a negative externality on the unskilled sector and a positive externality on the skilled sector. Consequently, the net effect of a change in enrollment depends on the interplay between these externalities.

Over the course of development, the size of the skilled sector tends to grow, for example due to skilled-biased technological change. It may well be the case that the net effect of the enrollment on the aggregate welfare is negative in poor economies,

[1] For empirical evidence on agglomeration economies, stressing a positive correlation between productivity and size of an industry, see Ciconne and Hall (1996), and Combes et al. (2012).

positive in some medium range, and again negative in rich economies. This structure of externalities may give rise to a pattern of overinvestment in education in early and late stages of development and underenrollment in between.

The remainder of the paper is organized as follows. Section 2 introduces the model, and Section 3 deals with its equilibria and comparative statics. Optimal enrollment is discussed in Section 4. The following two sections deal with alternative frameworks and extensions, where Section 5 focuses on the political sphere, and Section 6 considers generalizations of the production technology. The final Section 7 concludes and indicates directions for future research.

2 The Model

2.1 Individuals and wages

Each individual lives for one period. Upon learning her ability type, she chooses whether or not to enroll in higher education. All university students graduate and work in the skilled sector, the other individuals work in the unskilled sector. Individuals are heterogeneous in ability a. For simplicity, let ability a be uniformly distributed on $[0, 1]$. The population size is normalized to unity. Wages reflect productivity differences proportionally. In the unskilled sector, the income of an individual of ability level a is given by $y_u(a) = w_u a$, where w_u is a standard wage in the unskilled sector that would be paid to an individual with the highest ability $a = 1$. In the skilled sector, a worker of ability a earns $y_s(a) = w_s a$.

To keep the analysis tractable, utility is assumed to be logarithmic in income, $U(y) = \log(y)$. Acquiring skills is associated with a utility cost $C(a) = \log(1/a)$. Thus, individuals with the highest ability have utility cost of zero, and individuals with the lowest ability level will face an infinite cost. This ensures that the endogenous ability threshold that separates the skilled workers from the unskilled is interior whenever $w_s > w_u$. Individuals possess perfect foresight with respect to their prospective wage. An individual of ability a enrolls in education when net utility from doing so exceeds utility from remaining unskilled, that is, if $\log(w_s a) - \log(1/a) > \log(w_u a)$ holds. This implies that an agent will enroll if ability a exceeds the threshold level

a^*, with

$$a^* = \frac{w_u}{w_s}. \tag{1}$$

2.2 Production

The economy under consideration consists of two sectors. For simplicity, each sector exclusively uses one type of labor, which is either skilled and unskilled. Both sectors are assumed to work under linear production functions: $Y_j = A_j H_j$, with $j \in \{s, u\}$, where H_j is the aggregate sector specific human capital. The coefficient A_j represents the level of technology. Since firms in each sector behave competitively, there is no residual income. One unit of human capital, corresponding to the highest ability level, is paid according to marginal productivity:

$$w_j = A_j. \tag{2}$$

The technology in each sector is determined by

$$A_j = \overline{A}_j \left(\widetilde{h}_j\right)^{\phi_j} (1 + N_j)^{\delta_j}, \tag{3}$$

with $\phi_j, \delta_j \in (0, 1)$. The term $\overline{A}_j$ expresses the exogenous productivity level of sector j in the period under consideration. Although it seems plausible that the current technology depends on the level of the previous period, or historical enrollment levels, we ignore such intertemporal spillovers. The term $\left(\widetilde{h}_j\right)^{\phi_j}$ displays an average human capital externality - the higher the average quality of workers in that sector, the more productive any unit of human capital is. Such an externality may occur if production takes place in teams, where a higher team quality in terms of human capital increases output of each worker in the team. Finally, $(1 + N_j)^{\delta_j}$ describes the size externality, expressing that productivity of each worker increases in the size of the sector N_j. At given enrollment threshold a^*, the average human capital levels $\widetilde{h}_u$ and $\widetilde{h}_s$ in the unskilled and skilled sectors are:

$$\widetilde{h}_u = \frac{a^*}{2} \tag{4}$$

$$\widetilde{h}_s = \frac{a^* + 1}{2} \tag{5}$$

As the population size is normalized to unity, we have sector sizes $N_u = a^*$ and $N_s = 1 - a^*$. We impose $\overline{A}_s > 2^{(\delta_u - \phi_u)}\overline{A}_u$. This condition ensures that $w_s > w_u$ holds at $a^* = 1$. Thus, if everybody plans to work in the unskilled sector, there is an incentive for the most talented type to enroll in university education.

3 Equilibrium and comparative statics

Definition. *A competitive equilibrium is characterized by an enrollment threshold $a_m = a^*$ such that (i) all workers are paid their respective marginal products and (ii) all individuals maximize utility by either enrolling and working in the skilled sector or by working in the unskilled sector.*

An interior equilibrium is defined by a market enrollment threshold $a_m = a^*$ satisfying equations (1)-(5). Notice that an additional equilibrium exists at $a^* = 0$. This trivial equilibrium is however unstable, as demonstrated below. Proposition 1 shows sufficient conditions under which a unique interior market enrollment rate always exists.

Proposition 1 *A market enrollment threshold $a_m \in (0,1)$ always exists. The market enrollment threshold is unique if $\phi_u + \delta_s + \dfrac{\delta_u - \phi_s}{2} \leq 1$ when $\delta_u - \phi_s > 0$, or $\phi_u + \delta_s < 1$ when $\delta_u - \phi_s \leq 0$.*

Proof. See Appendix A. □

In order to derive a stability condition for comparative static analysis, we consider the related differential equation

$$\dot{a}^* \equiv \frac{da^*}{dt} = f\left(\frac{w_u}{w_s} - a^*\right) = f(Z - a^*) \tag{6}$$

where $f(0) = 0$ and $f' > 0$. Hence, if the marginal individual would lose from enrolling, the enrollment threshold will go up, and vice versa. An equilibrium a_m will be stable if and only if $\frac{d\dot{a}^*}{da^*} \leq 0$. The sufficient stability condition requires that the strict inequality has to hold, which is equivalent here to $\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} < 1$. This condition is fulfilled if the uniqueness condition from Proposition 1 holds. Since the latter is always met if the coefficients describing the externality are sufficiently small, it is not particularly restrictive. In the following we assume that the condition of Proposition 1 is met, ensuring uniqueness and stability of the interior market equilibrium. Notice that the equilibrium at $a^* = 0$ is always unstable due to $\lim_{a^* \to 0} \dfrac{\partial Z}{\partial a^*} = \infty$ and can therefore be neglected. The comparative static properties of the interior market equilibrium a_m with respect to the technology parameters $\overline{A}_u$ and $\overline{A}_s$ can be derived in a straightforward fashion.

Proposition 2 *A higher* $\overline{A}_s/\overline{A}_u$ *decreases the market enrollment threshold* a_m *and increases the average skill premium* $\pi = \dfrac{A_s \widetilde{h}_s}{A_u \widetilde{h}_u}$.

Proof. See Appendix B. □

The comparative static properties are easily understood. A lower $\overline{A}_u/\overline{A}_s$ indicates a stronger relative technological advantage of the skilled sector. As this will be translated into a higher skill premium at any enrollment level, the enrollment incentives are increased. This in turn leads to a lower enrollment threshold a_m and a higher enrollment rate $1 - a_m$. The skill premium π increases mainly because the lower average human capital content in the skilled sector in combination with a smaller quality externality in that sector is more than compensated by a corresponding decline in average human capital in the unskilled sector. At the same time, the size externalities and the quality externality in the unskilled sector reinforce the tendency toward an increasing skill premium.

Over the course of development, $\overline{A}_s/\overline{A}_u$ is assumed to increase, continuously reducing the market enrollment threshold a_m. Figure 1 then shows that as time t elapses and the exogenous productivity trends bring down the relative productivity of the unskilled $\overline{A}_u/\overline{A}_s$, we will observe an increasing skill premium π.

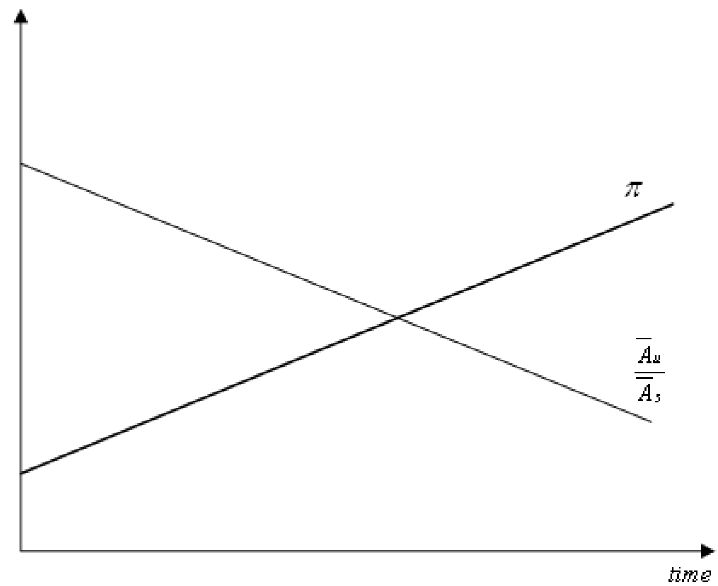

Figure 1: Evolution of relative productivities and skill premium

4 Optimal enrollment

Welfare W is represented by a Benthamite utilitarian welfare function, aggregating utility from wage income minus utility losses due to acquiring human capital:

$$W = \int_0^{a^*} \log(w_u a) f(a) da + \int_{a^*}^1 \log(w_s a) f(a) da - \int_{a^*}^1 \log(1/a) f(a) da \tag{7}$$

Thus, aggregate welfare is derived by adding utility from income of the unskilled, $\int_0^{a^*} \log(w_u a) f(a) da$, to utility from income of the skilled, $\int_{a^*}^1 \log(w_s a) f(a) da$, net of the aggregate utility cost of higher education, $\int_{a^*}^1 \log(1/a) f(a) da$.

We are interested in how the net externality at the market enrollment evolves over the course of development. The development process is described by skilled-biased technological change, where $\overline{A}_u/\overline{A}_s$, the relative standard productivity of the unskilled sector, is falling over time. Accordingly, market enrollment rates are increasing and threshold abilities are decreasing. We consider market enrollment rates shrinking from almost unity to almost zero. Proposition 3 summarizes the results.

Proposition 3 *(i) If $a_m \to 1$, there is overenrollment, that is, $\frac{\partial W}{\partial a^*}(a_m) > 0$. (ii) If $a_m \to 0$, there is overenrollment, that is, $\frac{\partial W}{\partial a^*}(a_m) > 0$ provided that $\phi_s + \phi_u - \frac{1}{2}\delta_s > 0$. (iii) If there is overenrollment for $a_m \to 0$, underenrollment may occur for some $a_m \in \{0, 1\}$.*

Proof. Rewriting the welfare function yields

$$
\begin{aligned}
W &= \int_0^{a^*} \log(w_u a) f(a) da + \int_{a^*}^{1} \log(w_s a) f(a) da - \int_{a^*}^{1} \log(1/a) f(a) da \qquad (8) \\
&= \int_0^{a^*} \log(w_u) f(a) da + \int_0^{a^*} \log(a) f(a) da + \int_{a^*}^{1} \log(w_s) f(a) da + 2\int_{a^*}^{1} \log(a) f(a) da \\
&= a^* \log(w_u) + (1 - a^*) \log(w_s) + \int_{a^*}^{1} \log(a) f(a) da + \int_0^{1} \log(a) f(a) da \\
&= a^* \log\left[\overline{A}_u \left(a^*/2\right)^{\phi_u} \left(1 + a^*\right)^{\delta_u}\right] + (1 - a^*) \log\left[\overline{A}_s \left((a^* + 1)/2\right)^{\phi_s} \left(2 - a^*\right)^{\delta_s}\right] \\
&\quad + \int_{a^*}^{1} \log(a) f(a) da + \int_0^{1} \log(a) f(a) da
\end{aligned}
$$

Increasing the cutoff ability, thus decreasing enrollment, affects welfare as follows:

$$
\begin{aligned}
\frac{\partial W}{\partial a^*} &= \log(w_u) + \frac{a^*}{w_u} \frac{\partial w_u}{\partial a^*} - \log(w_s) + \frac{1 - a^*}{w_s} \frac{\partial w_s}{\partial a^*} - \log(a^*) \qquad (9) \\
&= \log \frac{w_u}{a^* w_s} + \frac{a^*}{w_u} \left[\frac{\phi_u}{a^*} w_u + \frac{\delta_u}{1 + a^*} w_u\right] + \frac{1 - a^*}{w_s} \left[\frac{\phi_s}{1 + a^*} w_s - \frac{\delta_s}{2 - a^*} w_s\right] \\
&= \log \frac{w_u}{a^* w_s} + \phi_u + \delta_u \frac{a^*}{1 + a^*} + \phi_s \frac{1 - a^*}{a^* + 1} - \delta_s \frac{1 - a^*}{2 - a^*}.
\end{aligned}
$$

When evaluating $\frac{\partial W}{\partial a^*}$ at the market solution a_m, the first term becomes zero, according to (1):

$$\frac{\partial W}{\partial a^*}(a_m) = \phi_u + \phi_s \frac{1-a_m}{1+a_m} + \delta_u \frac{a_m}{1+a_m} - \delta_s \frac{1-a_m}{2-a_m}. \tag{10}$$

Claim (i) then is immediate from $\lim_{a^m \to 1} \frac{\partial W}{\partial a^*}(a_m) = \phi_u + \frac{1}{2}\delta_u > 0$. Considering $\lim_{a^m \to 0} \frac{\partial W}{\partial a^*}(a_m) = \phi_u + \phi_s - \frac{1}{2}\delta_s$ yields claim (ii). The final claim (iii) can be proved, for example, by considering the situation in which half of the population opts for working in the skilled sector, $\frac{\partial W}{\partial a^*}(\frac{1}{2}) = \phi_u + \frac{1}{3}\delta_u + \frac{1}{3}\phi_s - \frac{1}{3}\delta_s$. Should $\phi_s = \frac{1}{2}\delta_s$, while at the same time ϕ_u and δ_u are comparatively small, we arrive at a scenario with underenrollment in intermediate stages of development, switching again to overenrollment late. □

Decomposing the total change in welfare when there is a marginal positive change in a_m, that is, lower enrollment, yields the result shown in equation (10). The first term ϕ_u shows the positive effect on the average ability of the unskilled workers, while the second term $\phi_s \frac{1-a_m}{1+a_m}$ captures the positive impact on the average ability of the skilled workers. The size externalities have counteracting signs. While $\delta_u \frac{a_m}{1+a_m}$ expresses the positive impact on the market size of the unskilled technologies, the final term $\delta_s \frac{1-a_m}{2-a_m}$ shows a negative effect on the market size of the skilled technologies.

Figure 2 illustrates the evolution of the market enrollment threshold a_m and the socially optimal enrollment threshold $\widetilde{a}$. In the early stages of development, a_m is close to unity, where the impacts on the unskilled technology dominate. As enrollment decisions are made ignoring the negative size and quality effects on the unskilled sector, we arrive at overeducation, that is, $\widetilde{a} > a_m$. The market enrollment threshold diminishes over the course of development as time t elapses. In late stages, when enrollment is pretty high, the size effect on the unskilled sector vanishes. In such a situation, the negative quality effect of enrollment on the skilled technology is already strong. This may be different in some intermediate range, as the quality externality in the skilled sector is more sensitive to the enrollment level than the size externality. In the example, the externalities in the skilled sector tend to offset each other in late stages of development, whereas the size externality dominates in the intermediate range. At the same time, the externalities in the unskilled sector

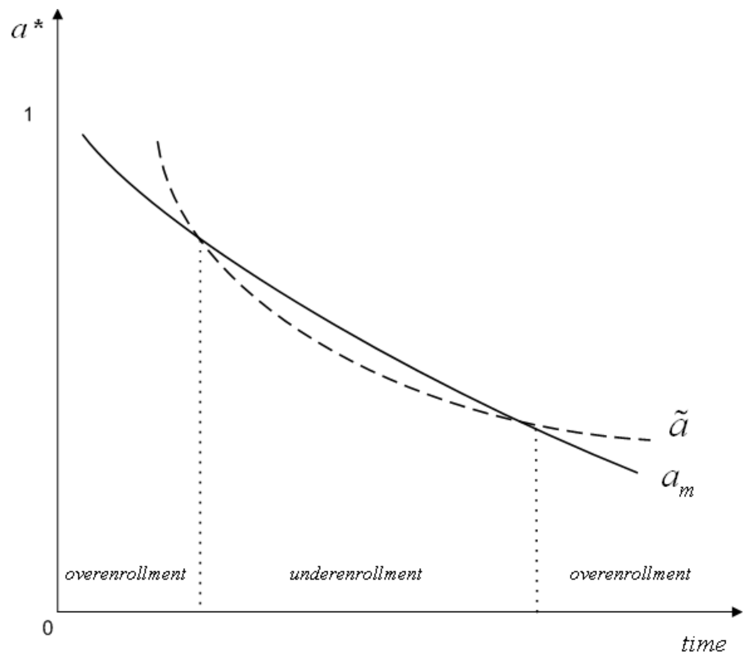

Figure 2: Market enrollment and socially optimal enrollment thresholds

remain comparatively small. In that event, as depicted in Figure 2, we arrive at a scenario with underenrollment in intermediate stages of development, switching again to overenrollment late.

A possible objection is that the underenrollment outcome in intermediate stages requires an unrealistic high coefficient δ_s relative to ϕ_s. But recalling that we are still ignoring the underinvestment argument from the matching literature, stressing that productivity gains will not be fully reflected in the wage growth at the individual level, a scenario becomes likely with moving from an overenrollment situation in a developing country stage to underenrollment under intermediate industrialization, and back to overenrollment again in advanced countries.

An important special case is found when the size externality does not exist. Unsurprisingly, this yields an unambiguous outcome:

Corollary. *In the absence of the size externality, overenrollment arises at any stage of development.*

Proof. With $\delta_u = \delta_s = 0$, it follows from (10) that $\dfrac{\partial W}{\partial a^*}(a_m) = \phi_u + \phi_s \dfrac{1 - a_m}{1 + a_m} >$

0. $\square$

Next, we study conditions under which the enrollment threshold that maximizes the aggregate welfare is interior and unique. Proposition 4 displays a sufficient existence condition on the minimal superiority of the skilled technology over the unskilled technology. If the coefficients describing the externalities are small, it is only slightly more demanding than set of the conditions ensuring the existence and uniqueness of market equilibria.

Proposition 4 *If $\overline{A}_s/\overline{A}_u > 2^{\delta_u-\phi_u}\exp\left(\phi_u+\frac{\delta_u}{2}\right)$, then there exists an interior enrollment threshold that maximizes welfare. This threshold is unique if $\delta_u-\phi_s\leq 0$.*

Proof. See Appendix C. $\square$

While the sufficient uniqueness condition $\delta_u-\phi_s\leq 0$ seems to be somewhat restrictive at first sight, it is not necessary. When inspecting the proof of Proposition 4, it transpires that $\delta_u-\phi_s>0$ will often go along with uniqueness of the welfare optimum. In fact, it is rather difficult to construct examples in which there are multiple solutions to the first-order condition characterizing an interior welfare optimum, though not impossible.

It is obvious that the government can improve welfare by introducing a subsidy to higher education in the case of underenrollment or a tax in the case of overenrollment, where tax revenue may be returned to all citizens in a lump-sum fashion. By appropriate choice of the level of the subsidy or tax, it can also induce any enrollment threshold $\widetilde{a}$ that maximizes welfare W.

Proposition 5 *If a unique welfare-maximizing enrollment threshold $\widetilde{a}\in(0,1)$ exists, it can be implemented by setting a tuition fee $\theta w_s(a^*)$ with*

$$\theta=1-\exp\left\{-\left[\phi_u+\delta_u\frac{\widetilde{a}}{1+\widetilde{a}}+\phi_s\frac{1-\widetilde{a}}{1+\widetilde{a}}-\delta_s\frac{1-\widetilde{a}}{2-\widetilde{a}}\right]\right\}.$$

Proof. The welfare-maximizing enrollment threshold $\widetilde{a}$ satisfies the first-order condition

$$\frac{\partial W}{\partial a^*}(\widetilde{a})=\log\frac{w_u}{a^*w_s}+\phi_u+\delta_u\frac{a^*}{1+a^*}+\phi_s\frac{1-a^*}{a^*+1}-\delta_s\frac{1-a^*}{2-a^*}=0. \tag{11}$$

With a tuition fee of $\theta w_s(a^*)$, the marginal individual with ability a^* will be characterized by

$$\log\left(w_s(a^*) - \theta w_s(a^*)\right) - \log(1/a^*) = \log\left(w_u(a^*)\right) \tag{12}$$

It remains to be shown that this tuition fee indeed induces $a^* = \widetilde{a}$. Rearranging (12) yields

$$\log(1-\theta) = \log\frac{w_u}{w_s a^*} \tag{13}$$

Using the definition of θ and equation (13) shows that the induced enrollment threshold indeed satisfies (11), that is, $a^* = \widetilde{a}$. □

The optimal tuition fee is related to the resulting market equilibrium standard skilled wage $w_s(a^*)$, where the "tax rate" θ is designed so as to internalize the externalities at the welfare-maximizing enrollment level. Taking into account the externalities, the decentralized market solution then coincides with the social optimum. When the net externalities of the enrollment decision at the social optimum $\widetilde{a}$ are negative, $\phi_u + \delta_u\frac{\widetilde{a}}{1+\widetilde{a}} + \phi_s\frac{1-\widetilde{a}}{1+\widetilde{a}} - \delta_s\frac{1-\widetilde{a}}{2-\widetilde{a}} > 0$, the Pigouvian tuition fee is positive, $\theta > 0$, while a positive net externality at the welfare optimum requires a subsidy, $\theta < 0$.

5 Alternative political environment

5.1 Income maximizing social planner

As an alternative to our specification of a Benthamite government, the social planner may pursue the goal of maximizing aggregate income. As possible justification, the government may aim at maximizing the cake available for redistributive or tax-collecting purposes. Following the approaches of Costrell (1994) and Betts (1998), the government then simply neglects any opportunity cost of acquiring skills like the value of foregone leisure. This leads to different conclusions of evaluating the market outcome. The government now maximizes aggregate income I, defined as

$$I = \int_0^{a^*} (w_u a)\, f(a)da + \int_{a^*}^{1} (w_s a) f(a)da. \tag{14}$$

Increasing the enrollment threshold a^* changes aggregate income as follows:

$$\frac{\partial I}{\partial a^*} = [w_u(a^*) - w_s(a^*)]\, a^* + \left[\frac{\phi_u}{a^*} + \frac{\delta_u}{1+a^*}\right] w_u \frac{1}{2}(a^*)^2 \qquad (15)$$
$$+ \left[\frac{\phi_s}{a^*} - \frac{\delta_s}{2-a^*}\right] w_s \left[\frac{1}{2} - \frac{1}{2}(a^*)^2\right].$$

If the externality coefficients are zero, the income-maximizing social planner will increase enrollment until wages of skilled and unskilled workers are equalized. In that situation, the market equilibrium is always perceived as exhibiting underenrollment because $w_u(a_m) - w_s(a_m) < 0$. It may easily happen that an income maximizing social planner chooses the boundary solution to enroll everybody - which turns out for sufficiently small externality coefficients and $w_u(a^* = 0) < w_s(a^* = 0)$.

Comparing (15) to the corresponding expression (9) of the welfarist planner in the previous section, it transpires that additional weights are attached to the externality coefficients, reflecting both wage rates and aggregate human capital levels in the respective sectors. Summing up, promoting rather than deterring enrollment is much more likely for an income maximizing planner than for a welfarist planner.

5.2 Egalitarian social planner

Another suggestion to predict the political outcome when the government tries to influence enrollment is to use a welfare function that captures preferences for equity. Following Betts (1998), an egalitarian social planner uses a welfare function

$$\Omega = \int_0^{a^*} \psi\left(\log(w_u a)\right) f(a)da + \int_{a^*}^{1} \psi\left(\log(w_s a) - \log(1/a)\right) f(a)da \qquad (16)$$

with $\psi' > 0 > \psi''$, ensuring that the government gives relatively more weight to the unskilled poor than in our Benthamite utilitarian benchmark.

With such a political objective function, increasing the enrollment threshold -

thus restricting enrollment - affects welfare as follows.

$$\begin{aligned}\frac{\partial\Omega}{\partial a^*} &= \psi\left[\log\left(w_u(a^*)a^*\right)\right] - \psi\left[\log w_s(a^*)a^* - \log(1/a^*)\right] \\ &+ \int_0^{a^*} \frac{\psi'\left[\log(w_u(a^*)a)\right]}{w_u(a^*)a} a \frac{\partial w_u}{\partial a^*} f(a)da \\ &+ \int_{a^*}^{1} \frac{\psi'\left[\log(w_s(a^*)a) - \log(1/a)\right]}{w_s(a^*)a} a \frac{\partial w_s}{\partial a^*} f(a)da.\end{aligned} \tag{17}$$

Note that the first line on the RHS of (17) is equal to zero at the market equilibrium. The second and the third line show that, compared to the Benthamite welfare function, externalities related to the unskilled (second line) get a higher weight relative to the externalities falling on the skilled (third line). With evaluating derivatives,

$$\begin{aligned}\frac{\partial\Omega}{\partial a^*} &= \psi\left[\log\left(w_u(a^*)a^*\right)\right] - \psi\left[\log w_s(a^*)a^* - \log(1/a^*)\right] \\ &+ \left[\frac{\phi_u}{a^*} + \frac{\delta_u}{1+a^*}\right] \int_0^{a^*} \psi'\left[\log(w_u(a^*)a)\right] f(a)da \\ &+ \left[\frac{\phi_s}{1+a^*} - \frac{\delta_s}{2-a^*}\right] \int_{a^*}^{1} \psi'\left[\log(w_s(a^*)a) - \log(1/a)\right] f(a)da\end{aligned} \tag{18}$$

Compared to the Benthamite planner, the egalitarian planner tends to choose a more restrictive enrollment policy. This is due to the fact that he balances negative and positive impacts on wages differently, attaching higher weights to wage losses of the unskilled. In the extreme case of a Rawlsian social planner caring exclusively for individuals at the bottom of the utility distribution, the planner would choose to close the skilled sector down. These results are very much in line with Betts (1998) arguing that an egalitarian social planner tends to choose high standards in order to narrow the income distribution.

5.3 Political economy

Let us now consider a median voter approach with the enrollment threshold as political choice variable. First of all, we can determine most preferred enrollment rates. Due to the lack of redistributive policies towards the poor, everybody who expects to remain unskilled chooses the highest possibly threshold $a^* = 1$. For those who expect to enroll, matters are more complicated as quality externality and size externality work in opposite directions. Maximizing utility as skilled worker is tantamount to maximizing

$$\left(\frac{1+a^*}{2}\right)^{\phi_s} (2-a^*)^{\delta_s}$$

with respect to a^* subject to $a \geq a^*$. Taking logs yields $\phi_s \log\left[(1+a^*)/2\right] + \delta_s \log(2-a^*)$. Differentiating the last expression with respect to a^* gives as first-order condition for an interior solution $\phi_s / (1+a^{**}) - \delta_s / (2-a^{**}) = 0$. Solving this yields $\phi_s(2-a^{**}) = \delta_s(1+a^{**})$, being equivalent to $a^{**} = (2\phi_s - \delta_s)/(\phi_s + \delta_s)$.

Let the preferred enrollment threshold as skilled worker be denoted by a_s. Recalling the restrictions $0 \leq a_s \leq a$, we obtain

$$a_s = \begin{cases} 0 & \text{if} \quad a^{**} \leq 0, \\ a^{**} & \text{if} \quad 0 < a^{**} < a, \\ a & \quad \text{else.} \end{cases}$$

Three possible scenarios can arise. If $\delta_s \geq 2\phi_s$, the size externality is dominant, resulting in $a_s = 0$, that is, universal enrollment is the preferred outcome. If $\delta_s < \phi_s/2$, the quality externality is dominant, resulting in $a^{**} > 1$ and $a_s = a$ for any ability level a. Every skilled worked is then eager to be the least able among the skilled to maximize the skilled wage. Finally, if $2\phi_s > \delta_s > \phi_s/2$, we get $0 < a^{**} < 1$ and $a_s = \min\{a_s, a\}$. In that event, the preferred threshold lies in the interior, independent of own ability for the very top ability individuals, while medium ability types again have to ensure that they themselves are the least able workers in the skilled sector. An extension of the skilled sector beyond that level is pointless as it will reduce the wage of the skilled due to the quality externality.

Figure 3 depicts the preferences of individuals with different abilities for a scenario in which a^{**} lies in the interior. It can be seen that preferences over enrollment

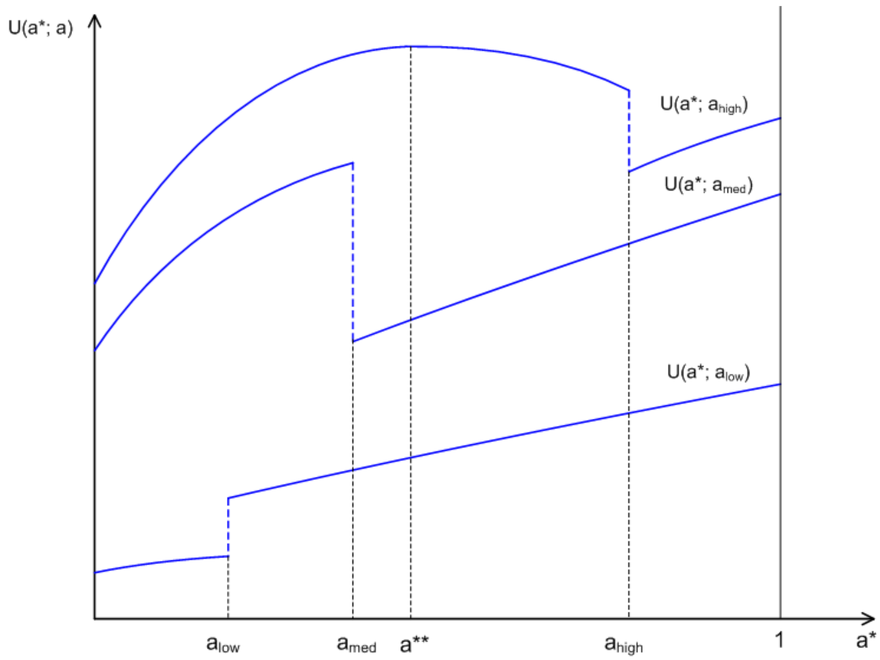

Figure 3: Preferences on enrollment thresholds

thresholds of more able individuals are typically double peaked, with one peak at a^{**} (or at 0 or at a, see above) and the other at 1. The preference curves of low ability agents exhibit an upward jump at their own ability level, see the lower curve. This happens because utility of a low ability type is higher when remaining unskilled. Conversely, the preference curves of the high ability types display a downward jump where the enrollment level coincides with their ability type, as in the two upper curves in Figure 3. There is no discontinuity of the preference curve when the ability type equals the market enrollment threshold ($a = a_m$). At this enrollment threshold, the individual with ability a_m is indifferent between becoming skilled or remaining unskilled.

The most preferred enrollment threshold is given by

$$\widetilde{a}^* = \begin{cases} a_s & \text{if} \quad \log(w_s\,(a^* = a_s)\,a) - \log(1/a) > \log(w_u\,(a^* = a_s)\,a), \\ \\ 1 & \qquad\qquad\qquad\qquad\qquad \text{else.} \end{cases}$$

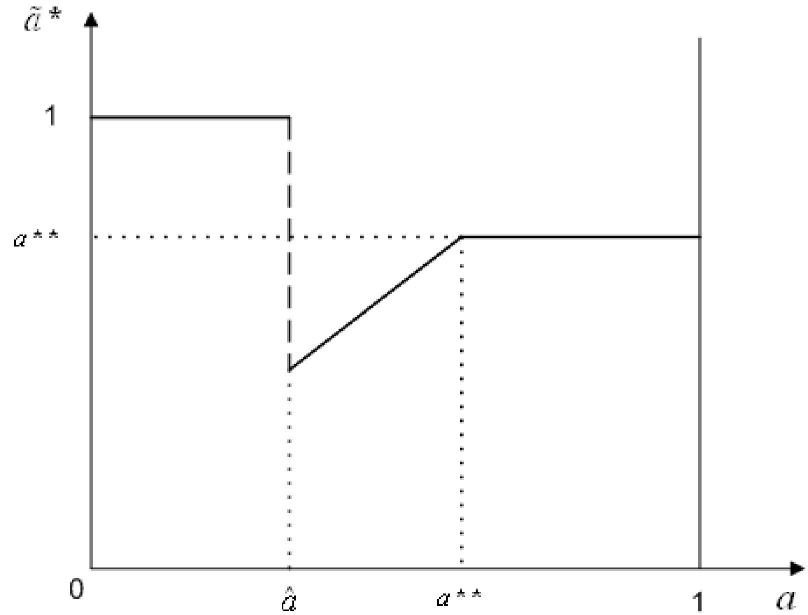

Figure 4: Most preferred enrollment thresholds

Figure 4 illustrates most preferred enrollment thresholds for some medium stage of development scenario. Low ability types ($a \in [0, \widehat{a})$) are always in favor of the highest feasible enrollment threshold $a^* = 1$ as they are harmed otherwise. The highest ability types ($a \in (a^{**}, 1]$) pick a^{**}, where productivity of the skilled sector is at its maximum. Some medium ability types ($a \in (\widehat{a}, a^{**})$) prefer they own type as enrollment threshold to ensure that they are included in the skilled sector, with displaying lowest ability. Notice that the level of a^{**} only depends on the externality coefficients and therefore by assumption stays constant over the course of development. However, when the relative exogenous productivity of the skilled sector $\overline{A}_s/\overline{A}_u$ rises, some marginal type individuals choose $a_s = \min\{a^{**}, a\}$ rather than 1 as most preferred enrollment threshold. In Figure 4, the ability of the marginal medium type $\widehat{a}$ moves to the left.

The prediction of the political outcome is difficult because the median voter theorem cannot be applied since, similar to Epple and Romano (1996a), neither single-peakedness of preferences nor the single-crossing property hold in general. Nevertheless, some observations can be made. In early stages of development, when the majority remains unskilled anyway, the enrollment threshold preferred by a majority of voters is unity, that is, there is no skilled sector. Thus, compared to choice of the Benthamite social planner, the policy under majority voting will be more restrictive.

As before, the course of development is described by rising relative productivity of the skilled $\overline{A}_s/\overline{A}_u$, where externality coefficient do not change. Therefore, a^{**} stays constant over time, and more types prefer to become skilled. In particular, the ability of the marginal type $\widehat{a}$ is decreasing in $\overline{A}_s/\overline{A}_u$. Thus, in late stages of development a majority of individuals have preferred enrollment threshold levels below unity.

Considering an advanced country with $\widehat{a} < 1/2$, i.e. voters who prefer to be enrolled are a majority, we can distinguish some cases. First, if $\delta_s \geq 2\phi_s$, we obtain $a^{**} \leq 0$, and the preferred threshold of the majority is $a^* = 0$. Second, if $2\phi_s > \delta_s > \phi_s/2$, it turns out that case $a^{**} \in (0, 1)$. If moreover $\delta_s > \phi_s$, then $a^{**} < 1/2$, where the most preferred threshold of a majority is $a^{**} = (2\phi_s - \delta_s) / (\phi_s + \delta_s)$. Should, however $\phi_s > \delta_s$ hold, then $a^{**} \in (1/2, 1)$, and the enrollment threshold of a^{**} does not have immediate support by a majority. In that event, an ends-against-the middle coalition in the style of Epple and Romano (1996a,b) can be formed, where the unskilled and the most talented individuals opt for a relatively high enrollment threshold, which presumably lies considerably above the market outcome. This is indicated by the property that some medium type individuals would prefer lower enrollment thresholds to enter the skilled sector themselves - which makes sense if they would also become skilled in the market equilibrium. Finally, when $\delta_s < \phi_s/2$, we have $a^{**} > 1$ and thus the preferred threshold of skilled agents is their own ability. Again, like in the previous case, the most able can form coalitions with the unskilled, resulting in comparatively high enrollment thresholds.

Summing up, the political economy outcome is always more restrictive than the choice of the welfarist social planner in early stages of development, while the opposite may hold later on.

6 Generalized structures of production

6.1 CES production function

As an alternative to our specification of perfect substitutability between goods produced by the skilled and the unskilled sector, consider a constant elasticity of substitution (CES) environment with

$$Y = \left[(1-\alpha)(Y_u)^\gamma + \alpha (Y_s)^\gamma\right]^{1/\gamma} \tag{19}$$

with $0 < \alpha < 1$ and $\gamma \leq 1$, where Y_u and Y_s are sector-specific intermediate good outputs and Y stands for the final good. The production functions in the intermediate good sectors are $Y_j = A_j H_j$, where $j = u, s$. Intermediate good prices are then given by

$$\begin{aligned} P_u &= \frac{\partial Y}{\partial Y_u} = \left[(1-\alpha)(Y_u)^\gamma + \alpha (Y_s)^\gamma\right]^{1/\gamma - 1}(1-\alpha)(Y_u)^{\gamma-1} && (20) \\ &= \frac{Y}{Y^\gamma}(1-\alpha)(Y_u)^{\gamma-1} = (1-\alpha)\left(\frac{Y}{Y_u}\right)^{1-\gamma}, \\ P_s &= \frac{\partial Y}{\partial Y_s} = \left[(1-\alpha)(Y_u)^\gamma + \alpha (Y_s)^\gamma\right]^{1/\gamma - 1}\alpha (Y_s)^{\gamma-1} && (21) \\ &= \frac{Y}{Y^\gamma}\alpha (Y_s)^{\gamma-1} = \alpha \left(\frac{Y}{Y_s}\right)^{1-\gamma}, \end{aligned}$$

The relative price generally depends on intermediate good output levels:

$$P_u/P_s = \frac{(1-\alpha)}{\alpha}\left(\frac{Y_s}{Y_u}\right)^{1-\gamma}. \tag{22}$$

In this setting, $\sigma = 1/(1-\gamma)$ is the elasticity of substitution. While $\gamma = 1$ represents the case of perfect substitutes, $\gamma \to 0$ would correspond to a Cobb-Douglas scenario. When $\gamma < 1$, the relative price of the intermediate good in the unskilled sector increases when enrollment rises, everything else equal. The intermediate goods are gross substitutes if $\sigma > 1$, corresponding to $\gamma \in (0, 1)$, while being gross complements if $\sigma < 1$, being equivalent to $\gamma < 0$. The limiting case $\gamma \to -\infty$ describes a Leontief technology.

Wages are given by $w_i = P_i A_i$ and the market equilibrium enrollment threshold satisfies

$$a_m = \frac{w_u}{w_s} = \frac{P_u A_u}{P_s A_s}. \tag{23}$$

or

$$a_m = \frac{(1-\alpha)}{\alpha} \left(\frac{A_s}{A_u}\right)^{-\gamma} \left(\frac{H_s}{H_u}\right)^{1-\gamma} \tag{24}$$

Unlike the basic model, the last term in equation (24) shows a neoclassical scarcity effect. Ignoring the impacts via A_s/A_u, a higher relative input of skilled labor H_s/H_u increases the relative wage of the unskilled at any $\gamma < 1$.

A welfarist social planner again maximizes (7). Differentiating this welfare function leads to some modifications:

$$\begin{aligned}\frac{\partial W}{\partial a^*} &= \log(w_u) + \frac{a^*}{w_u}\frac{\partial w_u}{\partial a^*} - \log(w_s) + \frac{1-a^*}{w_s}\frac{\partial w_s}{\partial a^*} - \log(a^*) \\ &= \log \frac{w_u}{a^* w_s} + \frac{a^*}{w_u}\frac{\partial (P_u A_u)}{\partial a^*} + \frac{1-a^*}{w_s}\frac{\partial (P_s A_s)}{\partial a^*} \\ &= \log \frac{w_u}{a^* w_s} + \frac{a^*}{w_u}\left(P_u \frac{\partial A_u}{\partial a^*} + A_u \frac{\partial P_u}{\partial a^*}\right) + \frac{1-a^*}{w_s}\left(P_s \frac{\partial A_s}{\partial a^*} + A_s \frac{\partial P_s}{\partial a^*}\right)\end{aligned} \tag{25}$$

Notice that only the terms related to the impacts of the market equilibrium on the intermediate goods prices are new. At the market equilibrium we obtain

$$\begin{aligned}\frac{\partial W}{\partial a^*}(a_m) = {} & \phi_u + \phi_s \frac{1-a_m}{1+a_m} + \delta_u \frac{a_m}{1+a_m} - \delta_s \frac{1-a_m}{2-a_m} \\ & + \frac{a_m}{P_u}\frac{\partial P_u}{\partial a^*}(a_m) + \frac{(1-a^*)}{P_s}\frac{\partial P_s}{\partial a^*}(a_m).\end{aligned} \tag{26}$$

Thus, two additional externality terms enter in the second line of (26) that were absent in the baseline perfect substitutability scenario. These will typically be of opposite sign. Recalling aggregate human capital levels $H_u = (a^*)^2/2$ and $H_s =$

$(1-(a^*)^2)/2$, calculating these terms yields

$$\begin{aligned}\frac{a^*}{P_u}\frac{\partial P_u}{\partial a^*} &= \frac{a^*}{P_u}\left[\frac{\partial P_u}{\partial Y_u}\left(\frac{\partial Y_u}{\partial H_u}\frac{\partial H_u}{\partial a^*}+H_u\frac{\partial A_u}{\partial a^*}\right)\right. \\ &\quad \left.+\frac{\partial P_u}{\partial Y_s}\left(\frac{\partial Y_s}{\partial H_s}\frac{\partial H_s}{\partial a^*}+H_s\frac{\partial A_s}{\partial a^*}\right)\right] \\ &= \frac{a^*}{P_u}\left[\frac{\partial P_u}{\partial Y_u}A_u\frac{\partial H_u}{\partial a^*}+\frac{\partial P_u}{\partial Y_s}A_s\frac{\partial H_s}{\partial a^*}\right] \\ &\quad +\frac{a^*}{P_u}\left[H_u\frac{\partial P_u}{\partial Y_u}\frac{\partial A_u}{\partial a^*}+H_s\frac{\partial P_u}{\partial Y_s}\frac{\partial A_s}{\partial a^*}\right]\end{aligned} \tag{27}$$

and

$$\begin{aligned}\frac{(1-a^*)}{P_s}\frac{\partial P_s}{\partial a^*} &= \frac{1-a^*}{P_s}\left[\frac{\partial P_s}{\partial Y_u}\left(\frac{\partial Y_u}{\partial H_u}\frac{\partial H_u}{\partial a^*}+H_u\frac{\partial A_u}{\partial a^*}\right)\right. \\ &\quad \left.+\frac{\partial P_s}{\partial Y_s}\left(\frac{\partial Y_s}{\partial H_s}\frac{\partial H_s}{\partial a^*}+H_s\frac{\partial A_s}{\partial a^*}\right)\right] \\ &= \frac{1-a^*}{P_s}\left[\frac{\partial P_s}{\partial Y_u}A_u\frac{\partial H_u}{\partial a^*}+\frac{\partial P_s}{\partial Y_s}A_s\frac{\partial H_s}{\partial a^*}\right] \\ &\quad +\frac{1-a^*}{P_s}\left[H_u\frac{\partial P_s}{\partial Y_u}\frac{\partial A_u}{\partial a^*}+H_s\frac{\partial P_s}{\partial Y_s}\frac{\partial A_s}{\partial a^*}\right]\end{aligned} \tag{28}$$

The change in enrollment affects the prices of intermediate goods both through its impacts on the sectoral technologies and general equilibrium effects via changes in sectoral labor supply at given technological coefficients. The additional technological externality terms T are defined by

$$\begin{aligned}T \equiv &\frac{a^*}{P_u}\left[H_u\frac{\partial P_u}{\partial Y_u}\frac{\partial A_u}{\partial a^*}+H_s\frac{\partial P_u}{\partial Y_s}\frac{\partial A_s}{\partial a^*}\right] \\ &+\frac{1-a^*}{P_s}\left[H_u\frac{\partial P_s}{\partial Y_u}\frac{\partial A_u}{\partial a^*}+H_s\frac{\partial P_s}{\partial Y_s}\frac{\partial A_s}{\partial a^*}\right],\end{aligned} \tag{29}$$

and general equilibrium effects can be collected in G,

$$\begin{aligned}G \equiv &\frac{a^*}{P_u}\left[\frac{\partial P_u}{\partial Y_u}A_u\frac{\partial H_u}{\partial a^*}+\frac{\partial P_u}{\partial Y_s}A_s\frac{\partial H_s}{\partial a^*}\right] \\ &+\frac{1-a^*}{P_s}\left[\frac{\partial P_s}{\partial Y_u}A_u\frac{\partial H_u}{\partial a^*}+\frac{\partial P_s}{\partial Y_s}A_s\frac{\partial H_s}{\partial a^*}\right].\end{aligned} \tag{30}$$

After some derivations (see Appendix D), the general equilibrium effects evaluated at the market equilibrium a_m can be rewritten as:

$$G(a_m) = \frac{a_m (H_s + H_u) A_s A_u}{w_s \widetilde{h}_s} \frac{\partial P_u}{\partial Y_s} [1 - \pi] < 0,$$

where $\widetilde{h}_s$ is the average human capital in the skilled sector and π the relative skill premium. With $\gamma < 1$, we obtain $G(a_{m)} < 0$, as $\partial P_u / \partial Y_s > 0$ and π always exceeds unity. Thus, reducing enrollment by incresing the enrollment threshold decreases welfare through the general equilibrium terms. Taking the general equilibrium effects in isolation, the social planner generally prefers higher enrollment levels. This obtains because the welfare function is strictly concave, implying a preference for redistribution. A higher enrollment increases the output of the skilled sector and has a positive effect on the price of the intermediate good produced in the unskilled sector. Consequently, the social planner perceives enrollment as too low in any market equilibrium when technological externalities are absent. It should be stressed that general equilibrium effects are irrelevant when using linear utility in the welfare function, like the income maximizing planner. In that event, general equilibrium effects would be ignored as they always cancel out against each other in our framework.

The new technological externality terms T can be rewritten as follows:

$$T = T_u \frac{\partial A_u}{\partial a^*} + T_s \frac{\partial A_s}{\partial a^*} \tag{31}$$

with

$$\begin{aligned} T_u &= \frac{a^*}{P_u} H_u \frac{\partial P_u}{\partial Y_u} + \frac{1 - a^*}{P_s} H_u \frac{\partial P_s}{\partial Y_u} \\ &= \frac{H_u}{Y_u} \left[a^* \epsilon_{u,u} + (1 - a^*) \, \epsilon_{s,u} \right] \end{aligned} \tag{32}$$

where

$$\begin{aligned} \epsilon_{u,u} &\equiv \frac{Y_u}{P_u} \frac{\partial P_u}{\partial Y_u} = \frac{1}{(1-\alpha) \, Y^{(1-\gamma)} Y_u^{\gamma}} \frac{\partial P_u}{\partial Y_u} \\ &= -\alpha \, (1-\gamma) \, \frac{Y_u^{(-2)} Y_s^{\gamma}}{(1-\alpha)\,(Y_u)^{\gamma} + \alpha \, (Y_s)^{\gamma}} = -\frac{\alpha \, (1-\gamma)}{Y_u^2 \left[(1-\alpha) \, (Y_u / Y_s)^{\gamma} + \alpha \right]} \leq 0, \\ \varepsilon_{s,u} &\equiv \frac{Y_u}{P_s} \frac{\partial P_s}{\partial Y_u} = \frac{1}{\alpha} \left(\frac{Y_s}{Y} \right)^{1-\gamma} Y_u \frac{\partial P_s}{\partial Y_u} = \frac{(1-\alpha)\,(1-\gamma)}{(1-\alpha) + \alpha \, (Y_s / Y_u)^{\gamma}} \geq 0, \end{aligned} \tag{33}$$

and

$$T_s = \frac{a^*}{P_u} H_s \frac{\partial P_u}{\partial Y_s} + \frac{1-a^*}{P_s} H_s \frac{\partial P_s}{\partial Y_s} = \frac{H_s}{Y_s} \left[a^* \epsilon_{u,s} + (1-a^*) \epsilon_{s,s} \right] \tag{34}$$

where

$$\epsilon_{u,s} \equiv \frac{Y_s}{P_u} \frac{\partial P_u}{\partial Y_s} = \frac{1}{1-\alpha} \left(\frac{Y_u}{Y} \right)^{1-\gamma} Y_s \frac{\partial P_u}{\partial Y_s} = \frac{\alpha (1-\gamma)}{(1-\alpha)(Y_s/Y_u)^{\gamma} + \alpha} \geq 0, \tag{35}$$

$$\epsilon_{s,s} \equiv \frac{Y_s}{P_s} \frac{\partial P_s}{\partial Y_s} = \frac{1}{\alpha Y^{(1-\gamma)} Y_s^{\gamma}} \frac{\partial P_s}{\partial Y_s} = -\frac{(1-\alpha)(1-\gamma)}{Y_s^2 \left[(1-\alpha) + \alpha (Y_s/Y_u)^{\gamma} \right]} \leq 0. \tag{36}$$

The new technological externality terms work through changes in intermediate good prices, where elasticities of these prices with respect to intermediate good outputs matter. With $\gamma < 1$, thus imperfect substitutability, increasing enrollment raises output in the skilled sector and reduces output in the unskilled sector. This increases the intermediate output price in the unskilled sector both through a direct effect ($\partial P_u / \partial Y_u < 0$), as diminishing marginal returns prevail, and though cross effects as cross derivatives of the production function are positive ($\partial P_u / \partial Y_s > 0$). Accordingly, the intermediate output price in the skilled sector declines both through a direct and a cross price effect. The negative and positive externalities of changing enrollment on technological levels now have to be corrected according to these changes in intermediate good marginal productivities.

Summarizing our analysis, incorporating imperfect substitutability yields both additional technological externalities and general equilibrium effects. While the net impact of the former on optimal enrollment is ambiguous in general, the latter does increase enrollment if the social planner cares about the distribution of income.

6.2 Quality choice

Up to now, the quality of education has been treated as fixed. In this section, the social planner also chooses some sort of educational standard. Increasing this quality variable raises both the productivity in the skilled sector and the cost of education. More specifically, let the technology in the skilled sector be

$$A_s = \overline{A}_s \left(\widetilde{h}_s \right)^{\phi_s} (1 + N_s)^{\delta_s} g(q) \tag{37}$$

with $g' > 0$, $g'' \leq 0$ and $g(1) = 1$, where $q \geq 0$ stands for quality. The cost of acquiring education of quality q is $C(a, q) = \log (h(q)/a)$ with $h' > 0$, $h'' \geq 0$ and

$h(1) = 1$. To ensure existence and uniqueness of optimal quality, we assume that $g''(q) - h''(q) < 0$ at any $q > 0$, $g(0) > 0$, $h(0) > 0$, and the Inada conditions $\lim_{q \to 0} g'(q) = \lim_{q \to \infty} h'(q) = \infty$ and $\lim_{q \to \infty} g'(q) = \lim_{q \to 0} h'(q) = 0$ are satisfied. The baseline model analyzes the special case $q = 1$. In the generalized model, the market enrollment rate at given quality satisfies $\log(w_s) - \log(h(q)/a) = \log(w_u)$, which can be solved to arrive at

$$\frac{w_u}{w_s} = \frac{a_m}{h(q)}. \tag{38}$$

Imposing appropriate regularity conditions, a higher quality yields a lower market enrollment rate. Just as in models of Costrell (1994) and Betts (1998), quality can then be perceived as additional policy variable. At given quality, the analysis of externalities of the enrollment decision does not change substantially. However, when choosing quality, the policy-maker determines both the market enrollment rate and the net externality. Maximizing welfare with respect to quality amounts to maximizing

$$W = \int_0^{a^*(q)} \log(w_u a) f(a) da + \int_{a^*(q)}^{1} \log(w_s g(q) a) f(a) da - \int_{a^*(q)}^{1} \log(h(q)/a) f(a) da. \tag{39}$$

The first-order condition is

$$\begin{aligned} \frac{\partial W}{\partial q} =& (1 - a^*(q)) \left[\frac{g'(q)}{g(q)} - \frac{h'(q)}{h(q)} \right] \\ & - \frac{\partial a^*}{\partial q} \left[\log(w_s g(q) a^*) - \log(h(q)/a^*) - \log(w_s g(q) a^*) \right] \\ & + \frac{\partial a^*}{\partial q} \left[a^* \frac{\partial w_u / \partial a^*}{w_u(a^*)} + (1 - a^*) \frac{\partial w_s / \partial a^*}{w_s(a^*)} \right] = 0. \end{aligned} \tag{40}$$

Notice that the second line on the RHS of (40) is zero at the market equilibrium. Without externalities, the optimal quality is found at some $\widetilde{q}$ where the absolutes of the elasticities of $g(q)$ and $h(q)$ with respect to q are the same. However, the third line reveals that the social planner also cares about the impacts on production externalities via ϕ_u, ϕ_s, δ_u and δ_s. In fact, recalling (9), we have

$$\frac{\partial W}{\partial q} = (1 - a^*(q)) \left[\frac{g'(q)}{g(q)} - \frac{h'(q)}{h(q)} \right] + \frac{\partial a^*}{\partial q} \frac{\partial W}{\partial a^*} = 0. \tag{41}$$

Let $\widetilde{q}$ satisfy $g'(q)/g(q) = h'(q)/h(q)$. Hence, if the government does not affect enrollment by other means, a welfare-maximizing social planner will increase quality beyond $\widetilde{q}$ if net externalities of restricting enrollment are positive, and vice versa. In that environment, optimal quality choice depends on the stage of development. Similar to the scenario in the basic model, the quality level $\widetilde{q}$ would be generally perceived as too low in early and late stages of development, while the opposite may hold in between.

If both quality and enrollment are policy variables, the social planner solves

$$\max_{\{q,a^*\}} W = \max_{\{q,a^*\}} \left\{ \int_0^{a^*} \log(w_u a) f(a) da + \int_{a^*}^1 \log(w_s g(q) a) f(a) da \right. \tag{42}$$
$$\left. - \int_{a^*}^1 \log(h(q)/a) f(a) da \right\}$$

where $\partial W / \partial a^*$ is zero at any optimal enrollment threshold. In that event, the optimal level of quality is constant over the course of development and equals $\widetilde{q}$. This outcome occurs because the functions g and h stay constant over time and optimal quality does not vary across ability types.

Proposition 6 *If both quality q and the ability threshold a^* are policy parameters, optimal quality is unique at $\widetilde{q}$ satisfying $g'(\widetilde{q})/g(\widetilde{q}) = h'(\widetilde{q})/h(\widetilde{q})$.*

Proof. See Appendix E. □

Taxes and subsidies will then be used to achieve optimal enrollment at quality $\widetilde{q}$. This achieves a higher welfare level than relying on the quality standard alone, as losses would be incurred through not equalizing $g^{'}(q)/g(q)$ and $h^{'}(q)/h(q)$. Hence, while it may make sense to change the educational standard q in the spirit of Costrell (1994) and Betts (1998), a welfarist social planner will prefer to use additional instruments to affect enrollment.

6.3 Endogenous growth

Endogenous growth models explicitly formulate an intertemporal spillover of productivity, like Galor and Moav (2000) and Acemoglu (2002). For simplicity, only the skilled sector is affected. While there may still be an exogenous trend on $\overline{A}_s$ and $\overline{A}_u$, the productivity in the skilled sector may be modeled by $h_s = Ba$ where $B_t = f(1 - a^*_{t-1})$, with $f(0) = 1$ and $f' \geq 0$, where our basic model considers the case $f' = 0$ throughout. Thus, if $f' > 0$, the productivity of a worker in the skilled sector increases with a greater size of the skilled sector in the previous period, which may be a consequence of innovative activities benefiting the skilled sector. A generalization in which also the productivity of the unskilled sector would be affected is straightforward.

In period t, a welfarist social planner would then maximize $\widetilde{W}_t = W_t + \Gamma(1 - a^*_t, \overline{A}_{st}, \overline{A}_{ut})$, where the term $\Gamma(1-a^*_t, \overline{A}_{st}, \overline{A}_{ut})$ represents a reduced form of all future benefits associated with a variation of the enrollment threshold in period t. Changing the enrollment threshold affects the intertemporal welfare function by

$$\frac{\partial \widetilde{W}_t}{\partial a^*_t} = \frac{\partial W_t}{\partial a^*_t} - \frac{\partial \Gamma}{\partial a^*_t} \tag{43}$$

where $-\partial\Gamma/\partial a^*_t < 0$ indicates a negative intertemporal externality of decreasing enrollment in the current period. Taken in isolation, this impact contributes to underinvestment in education. Compared to the outcome of the basic model, optimal taxes on enrollment tend to be lower, and optimal subsidies will be higher.

7 Conclusions

We have analyzed a simple two-sector model with two types of technological externalities where the decision to acquire skill is endogenous. We show that an economy described in this fashion may well switch from overenrollment to underenrollment and back to overenrollment over the course of development without any government intervention. In such a situation, the optimal policy to discourage enrollment or encourage people to take higher education depends on the state of technological development.

We are still ignoring some underinvestment arguments from the literature, in particular that the wage premium of the worker will fall short of the individual productivity gain due to bargaining in an imperfect labor market. Moreover, underinvestment may occur in an intertemporal perspective if technological progress depends on the number of skilled workers in earlier periods.

On the other hand, we also neglect the overinvestment tendencies due to imperfect labor markets when payment by degree prevails, as in Costrell (1994) and Betts (1998). Depending on the extent of collectivity in wage setting institutions, firms tend to pay their workers by their formal qualification level - and not according to the personal human capital endowment of the worker. If this happens, the skill premium reflects the difference in marginal productivities for average workers in the respective sectors. As this skill premium is far larger than the productivity differential for the marginal individual, the incentive to enroll in university is too strong. Hence, payment by degree is a source of overinvestment in human capital. It clearly works in the same direction as the average human capital externality, but may induce a far stronger distortion.

Summing up, it is quite plausible that in an extended model being enriched by these arguments for overinvestment and underinvestment in university education a similar outcome of moving from undereducation to overeducation occurs when reaching late stages of economic development.

Appendix

A: Proof of Proposition 1

As firms behave competitively, the sector-specific wage levels are determined by the zero profit condition:

$$w_u = \overline{A}_u \left(\frac{a^*}{2}\right)^{\phi_u} (1+a^*)^{\delta_u}\,, \tag{44}$$

$$w_s = \overline{A}_s \left(\frac{a^*+1}{2}\right)^{\phi_s} (2-a^*)^{\delta_s}\,. \tag{45}$$

Using (1) we obtain the following expression in the market enrollment threshold:

$$a^* = \frac{\overline{A}_u \left(\frac{a^*}{2}\right)^{\phi_u} (1+a^*)^{\delta_u}}{\overline{A}_s \left(\frac{a^*+1}{2}\right)^{\phi_s} (2-a^*)^{\delta_s}} \tag{46}$$

Denote the RHS of equation (46) as Z. We can see that $\lim_{a^*\to 0} Z = 0$ and

$$\lim_{a^*\to 1} Z = \frac{\overline{A}_u (2)^{\delta_u - \phi_u}}{\overline{A}_s}, \tag{47}$$

being smaller than unity by our assumption $\overline{A}_s > 2^{(\delta_u - \phi_u)}\overline{A}_u$. Further,

$$\frac{\partial Z}{\partial a^*} = Z \left[\frac{\phi_u}{a^*} + \frac{\delta_u - \phi_s}{1+a^*} + \frac{\delta_s}{2-a^*} \right]. \tag{48}$$

Since $0 < \phi_u < 1$, it follows that $\lim_{a^*\to 0} \frac{\partial Z}{\partial a^*} = \infty$, ensuring that equation (46) has a solution $a^* \in (0,1)$. Denote the solution of this equation a_m. Uniqueness of this solution turns out if $\frac{\partial Z}{\partial a^*} < 1$ at any candidate solution a^*. Note that at any candidate solution $Z = a^*$, implying

$$\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} = \phi_u + \frac{(\delta_u - \phi_s)\, a^*}{1+a^*} + \frac{\delta_s a^*}{2-a^*}. \tag{49}$$

Since both $a^*/(1+a^*)$ and $a^*/(2-a^*)$ are strictly increasing in the interval $[0,1]$, the RHS of (49) assumes its maximum at $a^* = 1$ if $\delta_u - \phi_s > 0$. Therefore, $\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} < \phi_u + \delta_s + \frac{\delta_u - \phi_s}{2}$ when $\delta_u - \phi_s > 0$, and $\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} < \phi_u + \delta_s$ when $\delta_u - \phi_s \leq 0$. This suffices to establish uniqueness of a_m.

B: Proof of Proposition 2

The market enrollment threshold is determined by $Z - a^* = 0$ with Z being defined as above. According to the implicit function theorem, $\frac{\partial a_m}{\partial \left(\overline{A}_u/\overline{A}_s\right)} = -\frac{\partial Z/\partial \left(\overline{A}_u/\overline{A}_s\right)}{\frac{\partial Z}{\partial a^*} - 1}$. Since the stability condition $\left.\frac{\partial Z}{\partial a^*}\right|_{a^*=a_m} < 1$ has to be met for meaningful comparative statics, it follows that

$$sgn\left[\frac{\partial a_m}{\partial \left(\overline{A}_u/\overline{A}_s\right)}\right] = sgn\left[\frac{\partial Z}{\partial \left(\overline{A}_u/\overline{A}_s\right)}\right] = sgn\left[Z/\left(\overline{A}_u/\overline{A}_s\right)\right] > 0. \tag{50}$$

The relative skill premium π in the market equilibrium is equal to

$$\pi = \frac{A_s \widetilde{h}_s}{A_u \widetilde{h}_u} = \frac{\overline{A}_s}{\overline{A}_u} \frac{\frac{1+a_m}{2}}{\frac{a_m}{2}} \frac{\left(\frac{1+a_m}{2}\right)^{\phi_s}}{\left(\frac{a_m}{2}\right)^{\phi_u}} \frac{(2-a_m)^{\delta_s}}{(1+a_m)^{\delta_u}}$$

$$= \frac{\overline{A}_s}{\overline{A}_u} \left(1 + \frac{1}{a_m}\right) \frac{\left(\frac{1+a_m}{2}\right)^{\phi_s}}{\left(\frac{a_m}{2}\right)^{\phi_u}} \frac{(2-a_m)^{\delta_s}}{(1+a_m)^{\delta_u}}.$$

Since a higher $\overline{A}_s/\overline{A}_u$ raises $1/a_m$, the expressions $(2-a_m)^{\delta_s} / (1+a_m)^{\delta_u}$ and $(a_m/2)^{-\phi_u}$ will also increase. While $(1+a_m)^{\phi_s}$ falls with declining a^*, this is more than offset by the increase in $(1+1/a_m)$. This can be seen by considering the limiting case with $\delta_s = \delta_u = \phi_u = 0$ and $\phi_s = 1$, where $(1+1/a_m)(1+a_m) = 2 + a_m + 1/a_m$ is clearly decreasing in a_m.

C: Proof of Proposition 4

Given the threshold ability level a^*, welfare can be written as

$$\begin{aligned} W(a^*) = {} & a^* \log\left[\overline{A}_u (a^*/2)^{\phi_u} (1+a^*)^{\delta_u}\right] \\ & + (1-a^*) \log\left[\overline{A}_s ((a^*+1)/2)^{\phi_s} (2-a^*)^{\delta_s}\right] - 2 - a^*(\log a^* - 1). \end{aligned} \tag{51}$$

The first derivative of (51) with respect to a^* is

$$\begin{aligned} \frac{\partial W}{\partial a^*} &= \log \frac{w_u}{a^* w_s} + \phi_u + \delta_u \frac{a^*}{1+a^*} + \phi_s \frac{1-a^*}{a^*+1} - \delta_s \frac{1-a^*}{2-a^*} \\ &= \log \frac{2^{\phi_s - \phi_u} \overline{A}_u}{\overline{A}_s} + (\phi_u - 1) \log a^* + (\delta_u - \phi_s) \log(1+a^*) \\ &\quad - \delta_s \log(2-a^*) + \phi_u + \delta_u \frac{a^*}{1+a^*} + \phi_s \frac{1-a^*}{a^*+1} - \delta_s \frac{1-a^*}{2-a^*}. \end{aligned} \tag{52}$$

Note that $lim_{a^* \longrightarrow 0} \frac{\partial W}{\partial a^*}(a^*) = \infty$. Then we compute

$$\lim_{a^* \to 1} \frac{\partial W}{\partial a^*}(a^*) = \log \frac{2^{\phi_s - \phi_u} \overline{A}_u}{\overline{A}_s} + \phi_u + \delta_u \frac{1}{2} + \delta_u \log 2 - \phi_s \log 2 \tag{53}$$
$$= \log \frac{\overline{A}_u 2^{\delta_u - \phi_u}}{\overline{A}_s} + \phi_u + \frac{\delta_u}{2}$$
$$= -\log \frac{\overline{A}_s}{\overline{A}_u 2^{\delta_u - \phi_u}} + \phi_u + \frac{\delta_u}{2}.$$

Thus, $\lim_{a^* \to 1} \frac{\partial W}{\partial a^*}(a^*) < 0$ iff

$$\log \frac{\overline{A}_s}{\overline{A}_u 2^{\delta_u - \phi_u}} > \phi_u + \frac{\delta_u}{2}. \tag{54}$$

Since $W(a^*)$ is differentiable on the interval $[0,1]$, the properties $\lim_{a^* \to 0} \frac{\partial W}{\partial a^*}(a^*) > \infty$ and $\lim_{a^* \to 1} \frac{\partial W}{\partial a^*}(a^*) < 0$ ensure the existence of an interior welfare maximum. The maximum is unique if $\frac{\partial^2 W}{(\partial a^*)^2} < 0$ holds at any a^* for which $\frac{\partial W}{\partial a^*}(a^*) = 0$.

Straightforward derivation shows

$$\frac{\partial^2 W}{(\partial a^*)^2} = \frac{1}{a^*} \left[\phi_u + \frac{(\delta_u - \phi_s)\, a^*}{1 + a^*} + \frac{\delta_s a^*}{2 - a^*} - 1 \right] \tag{55}$$
$$+. \frac{\delta_u - 2\phi_s}{(1 + a^*)^2} + \frac{\delta_s}{(2 - a^*)^2},$$

which is negative provided that the externality coefficients $\delta_u, \phi_u, \delta_s, \phi_s$ are sufficiently small.

D: Derivations CES

The impacts of output changes on the prices are given by

$$
\begin{aligned}
\frac{\partial P_u}{\partial Y_u} = & \left(\frac{1}{\gamma} - 1\right) \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 2} \gamma (1-\alpha)^2 (Y_u)^{2\gamma - 2} \\
& + \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 1} (1-\alpha)(Y_u)^{\gamma - 2} (\gamma - 1) \\
= & \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 2} (1-\alpha)(Y_u)^{\gamma - 2} (1-\gamma) \\
& * \left[(1-\alpha)(Y_u)^{\gamma} - \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]\right] \\
= & - \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 2} (1-\alpha)(Y_u)^{\gamma - 2} (1-\gamma) \alpha (Y_s)^{\gamma} \\
= & - P_u \frac{(Y_u)^{-1} (1-\gamma) \alpha (Y_s)^{\gamma}}{\left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]} \leq 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial P_u}{\partial Y_s} = & \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 2} (1-\gamma)(1-\alpha)(Y_u)^{\gamma - 1} \alpha (Y_s)^{\gamma - 1} \\
= & P_u \frac{(1-\gamma) \alpha (Y_s)^{\gamma - 1}}{\left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]} = P_s \frac{(1-\gamma)(1-\alpha)(Y_u)^{\gamma - 1}}{\left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]} = \frac{\partial P_s}{\partial Y_u} \geq 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial P_s}{\partial Y_s} = & \left(\frac{1}{\gamma} - 1\right) \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 2} \gamma \alpha^2 (Y_s)^{2\gamma - 2} \\
& + \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 1} \alpha (Y_s)^{\gamma - 2} (\gamma - 1) \\
= & \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 2} \alpha (Y_s)^{\gamma - 2} (1-\gamma) \\
& * \left[\alpha (Y_s)^{\gamma} - \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]\right] \\
= & - \left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]^{1/\gamma - 2} \alpha (Y_s)^{\gamma - 2} (1-\gamma)(1-\alpha)(Y_u)^{\gamma} \\
= & - P_s \frac{(1-\gamma)(1-\alpha)(Y_u)^{\gamma} (Y_s)^{-1}}{\left[(1-\alpha)(Y_u)^{\gamma} + \alpha (Y_s)^{\gamma}\right]} \leq 0.
\end{aligned}
$$

Recalling that the production function is linear homogenous in Y_u and Y_s (also in H_u and H_s at fixed A_u and A_s, that is, if technological externality terms are not taken into account), first derivatives are homogenous of degree zero implying

$$
\begin{aligned}
\frac{\partial P_i}{\partial Y_u} Y_u + \frac{\partial P_i}{\partial Y_s} Y_s = 0, \\
A_i \left[\frac{\partial P_i}{\partial Y_u} A_u H_u + \frac{\partial P_i}{\partial Y_s} A_s H_s\right] = 0,
\end{aligned}
$$

for $i \in \{u, s\}$.

The general equilibrium externality terms can be written as:

$$\begin{aligned} G = & \frac{a^*}{w_u} A_u \left[\frac{\partial P_u}{\partial Y_u} A_u \frac{\partial H_u}{\partial a^*} + \frac{\partial P_u}{\partial Y_s} A_s \frac{\partial H_s}{\partial a^*} \right] \\ & + \frac{(1-a^*)}{w_s} A_s \left[\frac{\partial P_s}{\partial Y_u} A_u \frac{\partial H_u}{\partial a^*} + \frac{\partial P_s}{\partial Y_s} A_s \frac{\partial H_s}{\partial a^*} \right] \\ = & a^* \left\{ \frac{a^*}{w_u} \frac{A_u}{H_u} \left[\frac{\partial P_u}{\partial Y_u} A_u H_u - \frac{\partial P_u}{\partial Y_s} A_s H_u \right. \right. \\ & \left. - \frac{\partial P_u}{\partial Y_s} A_s H_s + \frac{\partial P_u}{\partial Y_s} A_s H_s \right] + \\ & \frac{(1-a^*)}{w_s} \frac{A_s}{H_s} \left[\frac{\partial P_s}{\partial Y_u} A_u H_s - \frac{\partial P_s}{\partial Y_s} A_s H_s \right. \\ & \left. \left. - \frac{\partial P_s}{\partial Y_u} A_u H_u + \frac{\partial P_s}{\partial Y_u} A_u H_u \right] \right\}. \end{aligned} \tag{56}$$

Using the fact that $\frac{\partial P_i}{\partial Y_u} Y_u + \frac{\partial P_i}{\partial Y_s} Y_s = 0$ and $\frac{\partial P_s}{\partial Y_u} = \frac{\partial P_u}{\partial Y_s}$, we arrive at the following expression for G :

$$\begin{aligned} G = & a^* \left\{ \frac{(1-a^*)}{w_s} \frac{A_s}{H_s} \frac{\partial P_s}{\partial Y_u} A_u (H_s + H_u) \right. \\ & \left. - \frac{a^*}{w_u} \frac{A_u}{H_u} \frac{\partial P_u}{\partial Y_s} A_s (H_s + H_u) \right\} \\ = & a^* (H_s + H_u) \frac{\partial P_u}{\partial Y_s} \left[\frac{(1-a^*)}{w_s} \frac{A_s}{H_s} A_u - \frac{a^*}{w_u} \frac{A_u}{H_u} A_s \right] \\ = & a^* (H_s + H_u) \frac{\partial P_u}{\partial Y_s} A_s A_u \left[\frac{(1-a^*)}{w_s H_s} - \frac{a^*}{w_u H_u} \right] \\ = & \frac{a^* (H_s + H_u) A_s A_u}{w_s \widetilde{h}_s w_u \widetilde{h}_u} \frac{\partial P_u}{\partial Y_s} \left[w_u \widetilde{h}_u - w_s \widetilde{h}_s \right] \\ = & \frac{a^* (H_s + H_u) A_s A_u}{w_s \widetilde{h}_s} \frac{\partial P_u}{\partial Y_s} [1 - \pi] < 0. \end{aligned}$$

E: Proof of Proposition 6

For any welfare-maximizing combination of q and a^* in the interior, both $\partial W / \partial q = 0$ and $\partial W / \partial a^* = 0$ have to hold simultaneously. Thus, the optimal quality level $\widetilde{q}$ has to satisfy $g'(\widetilde{q})/g(\widetilde{q}) = h'(\widetilde{q})/h(\widetilde{q})$. Existence of the optimal quality $\widetilde{q}$ is ensured by the Inada conditions, implying $\lim_{q\to 0}[\partial W/\partial q] > 0$ and $\lim_{q\to\infty}[\partial W/\partial q] < 0$ whenever $\partial W/\partial a^* = 0$ holds at the same time. Uniqueness of $\widetilde{q}$ follows directly from strict concavity of W with respect to q, noting that

$$\frac{\partial^2 W}{\partial q^2} = (1 - a^*(q))\,\frac{g''(q)\,h(q) - h''(q)g(q)}{g(q)h(q)} < 0. \tag{57}$$

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