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Abstract

This paper shows that in a search model where future employers of trained workers do not benefit from the training in other firms, investment into general training will only be below the competitive level if workers are credit constrained. If workers are credit constrained, then the training firm cannot recover the cost of training since trained workers will search for a better paid job. This does, however, not imply that trainees will benefit from training. Only if the trainee wage is bounded by the workers' credit constraints do trainees gain from training.

JEL Code: J24, J31, J41, J63.
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1 Introduction

Becker (1993) shows that in a competitive labor market workers should pay for general training since they receive the full return to training. In a search model with bargaining, I show that search frictions per se do not necessarily cause underinvestment into general training and thus confirm Becker's result that investment into general training can be efficient if workers are not credit constrained. The underlying reason is that future employers need not profit from the training in other firms, if their profits are driven down to zero as positive profits trigger vacancy creation. Since future employers of trained workers do not benefit from the training in other firms, the training firm and the worker can, if workers are not credit constrained, enter into a long-term contract that guarantees that the training level will be efficient.

This is the difference to models by Acemoglu (1997) or Acemoglu and Pischke (1999), who also explain firm financed general training with search frictions. In their models part of the return goes to future employers as a result of the compressed wage structure and a given separation rate of workers from their training firms. The fact that future employers benefit implies that investment in training will be inefficient, since future employers cannot be part of a training contract.

If workers are credit constrained, then the training firm will still provide some training, because when deciding whether to train an unskilled worker or not, the firm faces the trade off between training an unskilled worker at its own expense or recruiting a skilled worker from the market. The difference in recruitment costs between unskilled workers and skilled workers can be used to pay for the general training, a point already mentioned by Oatey (1970) and Stevens (1994, 2001). While Stevens (1994, 2001) assumed different recruitment cost, the model presented here endogenizes the recruitment cost.[1]

Even if workers are credit constrained, they need not benefit from training, because

[1] Oatey (1970) presents no formal model.

when posting the trainee contract the training firm can lower the trainee wage to take away the trained worker's expected gain from searching for another job provided the implied trainee wage does not become negative. Trainees gain from training only if the trainee wage is bounded by the workers' credit constraints.

The model presented in this chapter follows the line of other research showing that labor market frictions provide an incentive for firms to invest in general training. Acemoglu and Pischke (1999) show that a compressed wage structure is sufficient for firms to pay at least partly for general training and that credit constraints, which are mentioned by Becker (1993) as a reason why firms may pay for general training, are not necessary. This compressed wage structure may be the result of an information asymmetry between training firms and not-training firms about the ability of individual workers as Katz and Ziderman (1990), Chang and Wang (1996) and Acemoglu and Pischke (1998) show. Acemoglu (1997) or Acemoglu and Pischke (1999) model firm financed general training with search frictions. They find that firms can only extract part of the return since future employer benefit from the training in other firms. Hence they are not willing to finance general training up to the efficient level. A third strand of the literature explains general training in combination of firm-specific training.

The plan of the chapter is as follows. Section 2 presents the framework. Section 3 derives the workers' behavior followed by the analysis of labor turnover in the steady state in Section 4. Section 5 derives the firms' vacancy creation decision, the general training condition in a situation where workers are credit constrained and where they are not. Section 6 establishes that multiple labor market equilibria exist if workers are credit constrained. Section 7 presents extensions concerning the wage formation. The chapter concludes by summarizing the main results.

2 The Framework

Firms

The model considers an infinite-horizon, stationary labor market in continuous time. The measure of firms is normalized to unity. Firms are assumed to be risk neutral and to discount future payments by the rate of interest r. All firms live infinitely. Firms search for workers by creating vacancies v_i for the respective labor markets, where $i \in \{s, u\}$. s stands for the labor market of skilled workers and u for the labor market of unskilled workers. The fact that workers of different skill are assumed to search in different markets implies that firms opening a vacancy for one type of worker have no use for another type of worker and can therefore commit not to employ a worker of another type. The advertising cost for a vacancy per time unit is given by adt.

The bargaining wages w_i for skilled and unskilled workers are taken as given by the firm when it chooses the training γ and the promotion rate ρ. The firm offers with probability γdt an employed, unskilled workers a training contract specifying a trainee-wage w_t and the commitment by the firm to pay the education cost c. The general training contract is a take-it-or-leave-it offer by the firm. The large number of unskilled workers per firm implies that the firm has effectively all market power and can therefore offer a contract that makes an unskilled worker exactly indifferent between accepting and rejecting the offer.

Firms produce according to a constant return to scale production function. The output produced over the period dt is given by an strictly increasing, concave and twice continuously differentiable function

$$ydt = F(l_u, l_s + l_t)\, dt.$$

Since training is instantaneous, trainees are able to work as skilled labor. Therefore, the skilled labor force $(l_s + l_t)$ is given by the sum of skilled workers and trainees. The unskilled labor force is given by l_u.

Firms promote trainees to a full skilled job with a respective market wage at rate ρdt. Furthermore, I assume that the firm is able to commit to its promotion promise.

Workers

New market entrants start their working career as unskilled workers, whose measure is defined by m. If they are trained by the firm, they become skilled worker. Workers are assumed to be risk neutral and to discount future payments at rate r. If workers are credit constrained, they cannot make any payment to the firm. A worker's stay in the labor market is exponentially distributed with parameter $\delta > 0$. If a worker exits the labor market, he is replaced by a new individual.

All unskilled workers start searching as unemployed in the labor market for unskilled. During that period they receive unemployment income normalized to zero. Individuals only search if the expected gain is strictly positive. Thus, only trained workers that are not promoted start to search for a skilled job. The labor markets for skilled and unskilled are separated. For simplicity, I assume that employed workers cannot become unemployed. Employment ends with a positive probability per period (here δdt) because of workers exiting the labor market.

Matching

Define s_i as the measure of workers searching in a particular labor market. The labor market tightness is defined as the ratio of vacancies to searching workers, $\theta_i \equiv v_i/s_i$. Define $M(v_i, s_i)$ as a Pissarides-type matching function, where $M(0, s_i) = M(v_i, 0) = 0$. It is assumed to be increasing, twice continuously differentiable, concave and linearly homogeneous. It hence has constant returns to matching and can be written in terms of the labor market tightness $M(v_i, s_i) \equiv s_i q(\theta_i)$. The properties of $M(v_i, s_i)$ imply that $q(\theta_i)$ is an increasing function of θ_i and satisfies the Inada conditions:

$$\text{i) } q(0) = 0, \qquad \text{ii) } \lim_{\theta_i \to +0} q(\theta_i)' = \infty, \qquad \text{iii) } \lim_{\theta_i \to +\infty} q(\theta_i)' = 0.$$

A searching worker meets a vacancy at the Poisson rate $M(v_i, s_i)/s_i = q(\theta_i)$. A vacancy is in turn contacted by a worker at the Poisson rate $M(v_i, s_i)/v_i = q(\theta_i)/\theta_i$. For notational reasons I define:

$$\lambda_i \equiv q(\theta_i), \qquad \text{and} \qquad \eta_i \equiv q(\theta_i)/\theta_i.$$

Bargaining

Wages are negotiated by unions and an employers' association. The unions' bargaining power is given by β. Thus, for each skill level $i \in \{s, u\}$ the agreed wage is given by

$$w_i = \beta F'_{l_i}\left(l_u, l_s + l_t\right), \tag{1}$$

where $F'_{l_i}\left(l_u, l_s + l_t\right)$ denotes the marginal product of a worker with skill level i. Firms and workers take these wages as given when they make their decisions. In Section 7, I allow for individual bargaining.

3 Individuals' Behavior

As new market entrants are unemployed, they start to search for a job. Once employed the individual can be offered training. This enables him to search for a skilled job afterwards if he is not promoted by his current employer. The flow value of being unemployed as unskilled worker is given by $(r+\delta)\,U$. At the rate λ_u he meets an unskilled job vacancy and gets the wage w_u.

$$(r+\delta)\,U = \lambda_u \max\left[V_u(w_u) - U, 0\right]. \tag{2}$$

The value of being employed as an unskilled worker at wage w_u is given by $V_u(w_u)$,

$$(r+\delta)\,V_u(w_u) = w_u + \gamma \max\left[V_t(w_t) - V_u(w_u), 0\right], \tag{3}$$

where the current employer offers the worker a training contract at rate γ. A trainee is promoted with probability ρ by the current employer. At the same time he can search for a skilled job vacancy at another firm (and matches with probability λ_s). The implicit assumption that the firm matches the outside wage when promoting its trainee is without loss of generality. Promoting and paying a wage less than w_s cannot be optimal since the trainees would still search and leave at the same rate λ_s as before. Paying a higher wage would reduce the firms profit. The value of being employed as trainee at wage w_t is thus given by

$$(r+\delta)\,V_t(w_t) = w_t + (\lambda_s+\rho)\max\left[V_{ts}(w_s) - V_t(w_t), 0\right]. \tag{4}$$

The value for a former trainee to be employed as skilled worker at wage w_s is given by:

$$(r+\delta)\,V_{ts}(w_s) = w_s. \tag{5}$$

The four Bellman equations (2), (3), (4), and (5) can be used to derive the conditions under which it is profitable for a worker to change status and hence to start actively searching for a vacancy in the corresponding labor market.

For a trainee to search for a skilled job vacancy, it has to be true that the wage for a skilled worker has to exceed the wage earned as trainee:

$$V_{ts}(w_s) > V_t(w_t) \quad \Leftrightarrow \quad w_s > w_t. \tag{6}$$

For an employed unskilled worker to accept the training contract, the value of being employed as trainee $V_t(w_t)$ must be at least as great as the value of being employed as unskilled worker $V_u(w_u)$[2]

$$V_t(w_t) \geq V_u(w_u) \quad \Leftrightarrow \quad \frac{(r+\delta)w_t + (\lambda_s+\rho)\,w_s}{r+\delta+\lambda_s+\rho} \geq w_u. \tag{7}$$

In other words, the expected wage income from starting as a trainee and later being employed (with probability $\lambda_s+\rho$) as a skilled worker has to exceed or be equal to the current wage earned as an unskilled worker.

[2]This condition does not require a strict inequality, since workers are offered training contracts without the necessity to participate in search.

Since it will be optimal for the firm to offer a wage w_t such that the worker is indifferent between accepting and rejecting condition (7) will hold with equality if workers are not credit constrained. If workers are credit constrained, then the training wage is bounded below by zero, i.e. $w_t \geq 0$. Furthermore, the firm can choose its promotion strategy ρ, which allows the firm to determine the expected wage of becoming a trainee. By increasing the promotion rate the firm is thus able to lower the wage w_t acceptable to a trainee. Returning to the individual's behavior, it follows from condition (7) that condition (6) is satisfied as long as $w_s > w_u$.

4 Steady State Turnover

Unemployment Measures

For every individual who leaves the labor market, a new individual enters unemployment as an unskilled worker. Thus, the measure δm of individuals enter the unemployment pool as unskilled workers. The measure $\lambda_u u_u$ of unemployed exit into employment. In addition, there are some individuals, i.e. δu_u, that exit the labor market before finding a job. The steady state unemployment measures of unskilled workers is

$$u_u = \frac{\delta}{\lambda_u + \delta} m. \tag{8}$$

Employment Measures

Since only one wage prevails in each labor market, workers cannot improve their situation by searching for an identical job. Consequently, only unemployed and trainees search.

The inflow into employment out of unemployment is given by $\lambda_u u_u$. Workers of every type exit employment at the rate δl_i. From the unskilled labor force γl_u become trainees, so that the measure of employed unskilled workers is given by

$$l_u = \frac{\lambda_u}{\gamma + \delta} u_u = \frac{\lambda_u}{\lambda_u + \delta} \frac{\delta}{\gamma + \delta} m. \tag{9}$$

The outflow from unskilled labor γl_u equals the inflow into the measure of trainees. The outflow from the trainee status is made up by the sum of individuals who exit the labor market altogether (i.e. δl_t), and by the individuals who find a skilled job vacancy at another firm or are promoted by their current firm (i.e. $(\lambda_s + \rho) l_t$). The measure of trainees is hence given by

$$l_t = \frac{\gamma}{\delta + \lambda_s + \rho} l_u = \frac{\delta}{\delta + \lambda_s + \rho} \frac{\lambda_u}{\lambda_u + \delta} \frac{\gamma}{\gamma + \delta} m. \tag{10}$$

Skilled workers are recruited internally and externally. From the pool of employed trainees $\lambda_s l_t$ are recruited externally and ρl_t internally. Given the outflow of δl_s from skilled labor the total measure of skilled labor is

$$l_s = \frac{\lambda_s + \rho}{\delta} l_t = \frac{\lambda_s + \rho}{\delta + \lambda_s + \rho} \frac{\lambda_u}{\lambda_u + \delta} \frac{\gamma}{\gamma + \delta} m. \tag{11}$$

Note, that the sum of trainees and skilled workers is independent of ρ, since promotion alters the status of the workers but not their role in production

$$l_t + l_s = \frac{\lambda_u}{\lambda_u + \delta} \frac{\gamma}{\gamma + \delta} m. \tag{12}$$

For later analysis, let us briefly focus on the ratio of skilled to unskilled labor, which determines the marginal product of the respective labor forces and hence their wages in equilibrium

$$\frac{l_t + l_s}{l_u} = \frac{\gamma}{\delta}. \tag{13}$$

The ratio increases with γ, the rate at which unskilled workers are recruited as trainees, but is independent of the promotion strategy ρ and the labor market frictions of either market. If a firm does not train while all other firms do but recruits skilled workers from the external market, it is able to achieve a labor ratio of

$$\frac{l_s}{l_u} = \frac{\lambda_s + \rho}{\delta + \lambda_s + \rho} \frac{\gamma}{\delta}, \tag{14}$$

which depends on the other firms training γ and promotion rate ρ.

Measure of Searching Individuals

The measure of individuals searching for unskilled job vacancies are the unskilled unemployed, i.e. $s_u = u_u$. Employed unskilled workers have no incentive to search for an identical job at another firm, since they would just earn the same wage.

The measure of workers searching for skilled job vacancies are the trainees, i.e.

$$s_s = l_t = \frac{\delta}{\delta + \lambda_s + \rho} \frac{\lambda_u}{\lambda_u + \delta} \frac{\gamma}{\gamma + \delta} m. \tag{15}$$

Firms influence s_s through γ and ρ without taking it into account. By granting more unskilled workers general training, firms increase the pool of people searching for skilled job vacancies. This makes it easier for other firms to recruit skilled labor. The resulting externality does not automatically lead to inefficient investment into training, as shown in the next section.

5 Firms' Behavior

Firms maximize their present value. The instruments at hand are to create vacancies v_i for unskilled and skilled workers, to offer unskilled workers general training contracts at rate γ, to determine the trainee-wage w_t and to decide how many trainees ρ are promoted and given a full skilled worker's contract. The firm takes the wages for skilled and unskilled workers as given. Formally

$$\max_{v_i,\gamma,\rho} \pi = \int_0^\infty \left(F\left(l_u, l_s + l_t\right) - \sum_{i \in \{s,u\}} \left[w_i l_i + a v_i\right] - w_t l_t - \gamma l_u c \right) e^{-rt} dt \tag{16}$$

$$
\begin{aligned}
\text{s.t.}\quad \dot{l}_u &= \eta_u v_u - (\gamma+\delta)\, l_u \\
\dot{l}_t &= \gamma l_u - (\delta+\lambda_s+\rho)\, l_t \\
\dot{l}_s &= \eta_s v_s + \rho l_t - \delta l_s \\
w_t &= \begin{cases} \max\left[w_u - (\lambda_s+\rho)\dfrac{w_s-w_u}{r+\delta}, 0\right], \\ \text{if workers are credit constrained and} \\ w_u - (\lambda_s+\rho)\dfrac{w_s-w_u}{r+\delta}, \ \text{if not.} \end{cases} \cdot
\end{aligned}
$$

The total training costs for a firm is $\gamma l_u c$, which equals the inflow of new trainees multiplied by the cost of education. The firm contacts a worker with probability η_i per vacancy, so that the inflow out of unemployment into the skilled and unskilled labor force is given by $\eta_i v_i$.

Note that the marginal product of a trainee is the same as the marginal product of a skilled worker, since I assume that training is instantaneous. Denote x_i as the co-state variable associated with (16). Then the resulting Euler-conditions are:

$$
\begin{aligned}
\frac{\partial H}{\partial v_u} &: \quad a = \eta_u x_u \qquad\qquad \frac{\partial H}{\partial \rho} : \ x_t = x_s + \frac{w_s-w_u}{r+\delta} \\
\frac{\partial H}{\partial v_s} &: \quad a = \eta_s x_s \qquad\qquad \frac{\partial H}{\partial \gamma} : \ c = x_t - x_u \\
\frac{dx_u}{dt} &= x_u r - F'_{l_u}(l_u, l_s+l_t) + w_u + c\gamma + x_u(\delta+\gamma) - x_t\gamma \\
\frac{dx_t}{dt} &= x_t r - F'_{l_s}(l_u, l_s+l_t) + w_t + x_t(\delta+\lambda_s+\rho) - x_s\rho \\
\frac{dx_s}{dt} &= x_s r - F'_{l_s}(l_u, l_s+l_t) + w_s + x_s\delta.
\end{aligned}
$$

Recruitment Cost

The steady state solution to this problem gives the vacancy creation condition for each labor market, i.e.

$$
a = \eta_i^* \frac{F'_{l_i}(l_u, l_s+l_t) - w_i}{r+\delta} \quad \text{for } i \in \{s,u\}. \tag{17}
$$

The vacancy creation condition requires that the cost of creating a vacancy a equals the expected return of a match. In the simple Pissarides (2000) model the vacancy creation condition determines together with the zero profit condition the number of firms (vacancies) in equilibrium. Here, the measure of firms is fixed to unity, so that the vacancy creation condition determines the size of a firm. This also guarantees that the value of creating a vacancy is equal to zero.

Proposition 1: *Given all other firms train, the recruitment cost for skilled labor is higher than for unskilled labor.*

Proof: Define $\Phi(\theta_i) \equiv a\theta_i/q(\theta_i)$. Given the properties of the matching function, it follows that

$$\Phi'(\theta_i) > 0, \;\; \Phi''(\theta_i) < 0, \;\; \lim_{\theta_i \to +0} \Phi(\theta_i) = 0 \;\text{ and }\; \lim_{\theta_i \to +\infty} \Phi(\theta_i) = \infty.$$

Hence, $\Phi(\theta_i)$ is a strictly increasing and concave function of θ_i, with domain $[0, \infty)$ and range $[0, \infty)$.

Denote $\widetilde{F}'_{l_i}(\theta_s, \gamma) \equiv F'_{l_i}(l_u, l_t + l_s)$, where γ is the training rate of all other firms. From (14) and the properties of the production function, it follows that the marginal product of an unskilled worker is increasing in θ_s and the marginal product of a skilled worker is decreasing in θ_s if the firm does not train. If it trains the marginal product for each skill level is independent of search frictions, see equation (13). Hence, a strictly positive and unique θ_i^* exists, where $\Phi(\theta_i^*) = \widetilde{F}'_{l_i}(\theta_s^*, \gamma)$.

Since all other firms train, the training rate γ is such that $\widetilde{F}'_{l_s}(\theta_s, \gamma) > \widetilde{F}'_{l_u}(\theta_s, \gamma)$. Given equation (1) it follows that $F'_{l_s}(l_u, l_s + l_t) - w_s > F'_{l_u}(l_u, l_s + l_t) - w_u$ whether the firm trains or not. Thus, according to equation (17) the recruitment cost of a skilled worker is higher than for an unskilled worker, i.e.

$$\frac{av_s}{M(v_s, s_s)} > \frac{av_u}{M(v_u, s_u)}.$$

□

Rearranging equation (17) shows that the recruitment cost per match equals the discounted marginal revenue of a matched worker.

$$\frac{av_i}{M(v_i, s_i)} = \frac{F'_{l_i}(l_u, l_s + l_t) - w_i}{r + \delta}$$

In equilibrium the cash flow (i.e. $F'_{l_i}(l_u, l_s + l_t) - w_i$) of a skilled worker is greater than the cash flow of an unskilled worker. The firm will therefore pay more for the recruitment of a skilled worker than for an unskilled worker. While for a firm it is harder to find skilled workers than unskilled workers (i.e. $\eta_s^* < \eta_u^*$), the matching technology implies that it is easier for searching skilled individuals to find a vacancy than for unskilled individuals (i.e. $\lambda_s^* > \lambda_u^*$).

Firms make zero profit, since they pay one part of the marginal product for recruitment and the other part in wages to workers themselves. Thus, firms that recruit trained workers pay them their effective marginal product and hence do not profit from recruiting trained workers. The fact that the future employer of a trained worker does not benefit from the training in other firms implies that search frictions per se need not cause underinvestment in training.

Promotion Decision

The firm can use promotion to prevent trainees from searching for a skilled job vacancy at another employer. The promotion condition requires that the shadow value of a trainee equals the shadow value of a skilled worker plus the discounted value of the promotion, which equals the discounted wage difference between a skilled and an unskilled worker

$$x_t = x_s + \frac{w_s - w_u}{r + \delta}. \tag{18}$$

The value of a trainee x_t after substituting the trainee wage w_t out is given by

$$x_t = \frac{F'_{l_s}(l_u, l_s + l_t) - w_u + (\lambda_s + \rho)\dfrac{w_s - w_u}{r + \delta} + \rho x_s}{r + \delta + \lambda_s + \rho}, \tag{19}$$

for workers that are not credit constrained and for workers that are credit constrained by

$$x_t = \frac{F'_{l_s}\left(l_u, l_s + l_t\right) - \max\left[w_u - \left(\lambda_s + \rho\right)\dfrac{w_s - w_u}{r + \delta}, 0\right] + \rho x_s}{r + \delta + \lambda_s + \rho}. \tag{20}$$

Thus, the promotion condition (18) can only be satisfied if unskilled workers are not credit constrained and if the promotion rate is infinity (see equation 19). This implies that in turn for the promotion the firm demands a lump-sum transfer of the worker that is equivalent to the value of the promotion (i.e. $(w_s - w_u) / (r + \delta)$), because the firm can extract all rent from an unskilled worker when posting the training contract.

Training Decision

Firms promote all trainees if workers are not credit constrained, and thereby keep them off the skilled labor market. This implies that workers do not benefit from training, since they pay for their promotion up front. At the same time future employers do not benefit from the training of other firms either, since all skilled workers stay with their training firm. Thus, if workers are not credit constrained the training level will be equal to the level of training in a competitive market, where workers pay for training.

Proposition 2: *If workers are not credit constrained, firms will train up to the competitive level, i.e.* $(r + \delta) c = \widehat{F}'_{l_s}(\gamma^*) - \widehat{F}'_{l_u}(\gamma^*)$, *where* $\widehat{F}'_{l_i}(\gamma) \equiv F'_{l_i}(l_u, l_t + l_s)$.

Proof: The rent extracted from the difference in recruitment cost is

$$\frac{\widehat{F}'_{l_s}(\gamma^*) - w_s}{r + \delta} - \frac{\widehat{F}'_{l_u}(\gamma^*) - w_u}{r + \delta}.$$

The lump-sum payment equivalent to the value of the promotion is given by

$$\frac{w_s - w_u}{r + \delta}.$$

Adding up gives the same training condition as in a competitive market, where worker pay for training.

$$(r + \delta) c = \widehat{F}'_{l_s}(\gamma^*) - \widehat{F}'_{l_u}(\gamma^*). \tag{21}$$

Note that the difference in the marginal products between skilled and unskilled workers is higher for firms that do not train than for training firms, since $(l_t + l_s)/l_u > l_s/l_u$ according to equation (13) and (14). The return to training will therefore exceed the cost of training so that training is optimal. $\square$

If workers are credit constrained, then firms cannot extract all the rent from workers by promoting them immediately. Firms will therefore not promote at all, i.e. $\rho = 0$ and pay only a positive trainee wage if the probability λ_s that a trained worker leaves to another firm is small enough to ensure that

$$w_t = w_u - \lambda_s \frac{w_s - w_u}{r + \delta} > 0.$$

Otherwise, the firm will pay nothing to trainees.

The difference in recruitment costs can still be used to pay for the general training of some unskilled workers. This can be seen by looking at the Euler equation, which implies that the difference in the shadow value of a trainee and the shadow value of employing an unskilled worker has to equal the cost of training (i.e. $c = x_t - x_u$). In other words, the cost of general training has to equal the discounted cash flows between trainees and unskilled workers. Using condition (7) to substitute the trainee-wage and rearranging gives

$$(r + \delta) c = \frac{(r + \delta) F'_{l_s}(l_u, l_s + l_t) + \min[\lambda_s w_s, (r + \delta + \lambda_s) w_u]}{r + \delta + \lambda_s} - F'_{l_u}(l_u, l_s + l_t), \quad (22)$$

where $\lambda_s w_s$ applies if the training wage is positive and $(r + \delta + \lambda_s) w_u$ if not.

Proposition 3: *For* $0 < \lambda_s \leq \overline{\lambda_s}$*, where* $\overline{\lambda_s} = (r + \delta) w_u / (w_s - w_u)$*, the training level* γ^1 *is below the competitive level* γ^* *(since trainees leave their training firm). For* $\lambda_s > \overline{\lambda_s}$ *the training level* $\gamma^2 < \gamma^1 < \gamma^*$ *is even lower, since unskilled workers receive part of the return to training.*

Proof: For $0 < \lambda_s \leq \overline{\lambda_s}$ and after substituting the trainee wage and the wage for skilled workers out, the training condition is according to equation (22) given by

$$(r+\delta)\,c = \frac{r+\delta+\lambda_s\beta}{r+\delta+\lambda_s}\widehat{F}'_{l_s}(\gamma^1) - \widehat{F}'_{l_u}(\gamma^1). \tag{23}$$

Comparing this condition to the competitive level

$$(r+\delta)\,c = \widehat{F}'_{l_s}(\gamma^*) - \widehat{F}'_{l_u}(\gamma^*),$$

and noting that the marginal product of a skilled worker is decreasing in γ and the marginal product of a unskilled worker is increasing in γ as well as noting that $\beta \in (0,1)$, it follows that $\gamma^1 < \gamma^*$.

For $\lambda_s > \overline{\lambda_s}$ the training condition is according to equation (22) given by

$$(r+\delta)\,c = \frac{r+\delta}{r+\delta+\lambda_s}\widehat{F}'_{l_s}(\gamma^2) - (1-\beta)\,\widehat{F}'_{l_u}(\gamma^2).$$

Substituting $\overline{\lambda_s}$ for λ_s implies

$$(r+\delta)\,c < \widehat{F}'_{l_s}(\gamma^2) - (2-\beta)\,\widehat{F}'_{l_u}(\gamma^2). \tag{24}$$

Substituting in equation (23) gives

$$(r+\delta)\,c \geq \widehat{F}'_{l_s}(\gamma^1) - (2-\beta)\,\widehat{F}'_{l_u}(\gamma^1). \tag{25}$$

Comparing equation (24) and (25) gives $\gamma^2 < \gamma^1$.

The fact that workers get part of the return to training can be seen by looking at equations (2) to (5) and comparing (i) $w_t = w_u - \lambda_s \frac{w_s - w_u}{r+\delta}$ with (ii) $w_t = 0$. In the case of (i) it follows according to condition (7) that $V_t(w_t) = V_u(w_u)$ which implies $U|_{(i)} = \frac{\lambda_u}{r+\delta+\lambda_u} w_u$. In case of (ii) it follows that

$$U|_{(ii)} = \frac{\lambda_u}{r+\delta+\lambda_u}\frac{w_u}{r+\delta} + \frac{\lambda_u}{r+\delta+\lambda_u}\frac{\gamma^2}{r+\delta+\gamma^2}\frac{\lambda_s}{r+\delta+\lambda_s}\frac{w_s}{r+\delta}.$$

□

Although future employers do not benefit from employing trained workers, training will be inefficient as Proposition 3 shows, because workers are credit constrained. The reason is that for the training firm to recover its training expenses fully, all trained workers would have to stay with their training firm for their entire working life and receive the wage of an unskilled worker. Outside firms are, however, willing to pay them the wage of a skilled worker. This induces trained workers to search for another employer.

The training firm can prevent workers from starting to search by promoting them immediately and paying them the market wage of a high skilled worker. If workers are not credit constrained, then the firm can make the worker indifferent between being unskilled or becoming a trainee. The reason is that the firm temporarily possesses all the bargaining power when offering the trainee contract. It can therefore demand the value of the promotion as a lump-sum payment up-front. This guarantees that a training firm gets all the return from training and will therefore invest efficiently.

If workers are credit constrained, then the training firm cannot recover the cost of the promotion via a lump-sum transfer. The training firm will therefore not promote and will accept that trained workers search for another job. Thus, training will be inefficient.

Trained workers need not benefit from training, because when posting the trainee contract the training firm can lower the trainee wage to take away the trained worker's expected gain from searching for another job provided the implied trainee wage does not become negative. If the trainee wage had to be negative in order to extract the whole rent from the trainees, credit constraints on the worker's side imply that they cannot pay a negative wage. Thus, trained workers will be better off compared to unskilled workers since they get part of the return to training. This, however, implies that firms will train even less.

6 Labor Market Equilibrium

The aim of this section is to show that in an economy with credit constrained workers there may be multiple training equilibria. If workers are not credit constrained, promotion in turn for an equivalent lump-sum payment from the trainee to the training firm prevents trainees from quitting and leads to a unique labor market equilibrium.

Definition: Labor Market Equilibrium

In a labor market equilibrium, firms create vacancies according to (17), offer general training at rate γ satisfying (22) if workers are credit constrained and (21) if workers are not credit constrained and are promoted immediately. Workers follow an optimal search strategy according to (2) - (5) and bargaining wages are formed according to (1).

Proposition 4: *If workers are not credit constrained, then a unique labor market equilibrium exists.*

If workers are credit constrained, multiple equilibria with inefficient training can exist, where a high training equilibrium is sustained by a low matching rate for trainees and vice versa, i.e. for any two equilibria a and b, we have $\lambda_s^a < \lambda_s^b$ and $\gamma^* > \gamma^a > \gamma^b$.

Proof: Part 1: Existence and uniqueness if workers are not credit constrained.

Since $\widehat{F}'_{l_s}(\gamma) - \widehat{F}'_{l_u}(\gamma)$ goes to infinity for $\gamma \to 0$ and to zero for $\gamma \to \infty$, a unique $\gamma^* > 0$ for the training rate in equation (21) exists. The wages are w_u^*, w_s^* and the market tightness θ_u^*, θ_s^* are functions of γ^* via the marginal product of a worker but not vice versa. Thus, the vacancy creation condition (17) implies a unique market tightness θ_i^* for each market. Wages are uniquely determined by equation (1) via the marginal product.

Part 2: Existence and multiplicity, if workers are credit constrained.

Again the property of the production function implies that $\gamma^j > 0$ for $j = 1, 2$ for the training rate in equation (22) exists. To establish the possibility of multiplicity it is sufficient to show that there are multiple (θ_s^*, γ^j) that satisfy the vacancy creation condition (17) for skilled workers and the training equation (22). The training condition can be written as

$$(r+\delta)\,c = f_j\left(\theta_s^*\right)\overline{F}'_{l_s}(\gamma^j) - h_j\overline{F}'_{l_u}(\gamma^j), \tag{26}$$

where

$$\begin{aligned} f_1\left(\theta_s^*\right) &= \frac{r+\delta+\lambda_s\beta}{r+\delta+\lambda_s} \text{ and } h_1 = 1 \text{ for } j=1 \text{ and} \\ f_2\left(\theta_s^*\right) &= \frac{r+\delta}{r+\delta+\lambda_s} \text{ and } h_2 = 1-\beta \text{ for } j=2. \end{aligned}$$

Note that $f_j\left(\theta_s^*\right)$ is decreasing in θ_s^* and the rhs of equation (26) is decreasing in γ^j. In the vacancy creation condition (17) the rhs is decreasing in θ_s^* and in γ^j. Thus, multiple equilibria a and b can exist for $\gamma^a > \gamma^b$ and $\theta_s^{*a} < \theta_s^{*b}$. □

If workers are credit constrained, firms are deprived of the promotion instrument, and general training generates a search externality since firms do not take into account that by training they increase the pool s_s of people searching for skilled job vacancies – compare equation (15) – and that by doing so it becomes harder for other trainees to find a job. This lower separation rate increases the firm's return to general training, which sustains a high training level and a low market tightness for skilled labor. On the other side, a low training equilibrium can exist where the probability for trainees to find a job at another firm is high. This decreases the return to general training such that firms train less, which sustains a high matching rate for trainees.

Only if unskilled workers are not credit constrained can the current firm extract the whole rent from general training and prevent its trainees from searching. This eliminates this externality and leads to an efficient investment in general training.

7 Extensions

Individual Bargaining

Assume that wages are negotiated after a worker contacted a firm. Firms take these wages as given then; they choose the number of vacancies, the training rate and the promotion rate. Nature chooses with probability β the worker to make an offer and with probability $1-\beta$ the firm. Workers and firms are assumed to have some bargaining power (i.e. $0 > \beta > 1$). If the other party accepts the offer, a wage contract is written and production starts immediately thereafter. If the offer is rejected, the respondent can leave the negotiation table and continue searching (both parties), or he can wait for the bargaining game to start again next period.

During this period the worker receives the flow-utility of leisure normalized to zero, since an employed worker has to take a day leave while bargaining with a different firm. The firm makes no loss or gain, since it does not advertise the job vacancy during negotiations.

At the same time there is a positive probability δdt that the worker exits the labor market. This could result in a breakdown of the negotiations, where the worker receives a flow utility of zero and the firm continues searching with the unfilled vacancy, which has a value of zero due to free entry. The firm's payoff while negotiations are postponed is also zero, as mentioned above.

The outside options of the workers are to take another day leave which gives him zero utility. The outside option for a firm is to walk away and to search for another worker. Since the value of a vacancy (i.e. searching) is zero in equilibrium, the outside option of the firm has a value of zero.

In case of a breakdown, payoffs are zero. The outside and the inside options are not binding so that the bargaining model simplifies to a random proposer Rubinstein model. Furthermore, the fact that the discount rates for firms and workers are identical implies that the bargaining power is equivalent to the probability of being chosen by nature to make an offer. Muthoo (1999, ch. 3.2 and 7.2.4) shows that the solution to the bargaining scenario - as $dt \rightarrow 0$ - is given by

$$w_i^* = \beta F'_{l_i}\left(l_u, l_s + l_t\right).$$

The assumption that an employed worker receives only the value of leisure and not his wage while negotiations are postponed ensures a single wage for each type of labor. This implies that employed workers do not gain by searching for an identical job at another firm. Therefore, only the unemployed and trainees will search. This assumption is relaxed below.

On-the-job Search and Search Intensity

In the preceding analysis the bargaining game was chosen such that only unemployed and trainees searched but not the skilled and unskilled workers. If one assumes that the inside option of a worker is his current wage and not the value of leisure, then on-the-job search will arise since workers can increase their wage every time they meet a new employer, i.e.

$$w_{i,e} = (1-\beta)w_{i,e-1} + \beta F'_{l_i}\left(l_u, l_s + l_t\right), \tag{27}$$

where e is an index for the number of employers the worker was/is employed with and $w_{i,e-1}$ indicates the wage at the last employer or in the case of the first employer the value of leisure normalized to zero. Thus, employed workers will continue searching as long as they earn less than their marginal product.

Promotion would keep trainees away from the skilled labor market and lead to efficient investment in general training if workers are not credit constrained, since the training firm can recover the promotion cost up-front via a lump-sum payment for training equivalent to the cost of promotion. If workers are not credit constrained, then the training firm will not promote the trained workers. It can, however, reduce the trainee wage in order to capture the future wage increases the worker expects to get from searching on-the-job.

The result that training firms do not promote, or demand the lowest possible trainee wage, only changes if the search intensity is no longer fixed and costless for workers. To introduce search intensity I follow Pissarides (2000). The matching rate depends not only on the market tightness θ_i, but also on a worker's search intensity $\sigma_{i,e}$, which will vary with his wage and thus with the number of jobs he already occupied, and it will depend on the average search intensity σ_i of all workers from his skill group. The transition rate for a worker is therefore given by

$$\sigma_{i,e}\phi_i \equiv \sigma_{i,e}\frac{q(\sigma_i,\theta_i)}{\sigma_i} = \sigma_{i,e}\frac{M(v_i,\sigma_i s_i)}{\sigma_i s_i}.$$

Assume that the search cost function $k(\sigma_{i,e})$ is convex and $k(0)=0$, then the Bellman equation for a trainee is given by:

$$(r+\delta)V(w_{t,e}) = \max_{\sigma_{t,e}}\left[w_{t,e} - k(\sigma_{t,e}) + \sigma_{t,e}\phi_t\left(V(w_{t,e+1}) - V(w_{t,e})\right)\right].$$

It follows that the optimal search intensity equates the marginal cost of searching with the marginal expected gain from being employed at the new employer at wage $w_{t,e+1}$, i.e.

$$\frac{\partial k(\sigma_{t,e})}{\partial \sigma_{t,e}} = \phi_t\left(V(w_{t,e+1}) - V(w_{t,e})\right).$$

The convex search cost function and the fact that the expected utility gain of changing employer, i.e. $V(w_{t,e+1}) - V(w_{t,e})$, decreases[3] with a higher current wage guarantees that each trainee will search less if his current wage is higher. However, trainees will continue to search as long as they earn less than their marginal product. Nevertheless, firms might be able to extract some rent from their trainees by promoting them immediately after training since the promotion saves the trainees search costs and reduces their incentive to search more intensively. A firm will promote a trainee, i.e. pay him a wage $w_{s,e} > w_{t,e}$, if and only if the lower matching probability compensates the firm

[3]This can easily be seen from equation (27) and the fact that $V(w_{t,e})$ is bounded above by the discounted sum of the workers marginal product.

for the cost of promotion, i.e.

$$\begin{aligned}
&\max_{w_{s,e}\in\left(w_{t,e},F'_{l_s}(l_u,l_s+l_t)\right)}\left[F'_{l_s}\left(l_u,l_s+l_t\right)-w_{s,e}+\sigma_{s,e}\phi_s\left[0-J\left(w_{s,e}\right)\right]\right]\\
> \quad & F'_{l_i}\left(l_u,l_s+l_t\right)-w_{t,e}+\sigma_{t,e}\phi_t\left[0-J\left(w_{t,e}\right)\right],
\end{aligned}$$

where $J\left(w_{i,e}\right)$ is the value of employing a worker at wage $w_{i,e}$. If the worker leaves, then the value to the firm is zero. Provided the convexity of the search cost function is severe enough, then the training firm will promote its trained workers.

8 Conclusion

The model presented in this chapter shows that in a search model where vacancy creation drives profits down to zero such that future employers of trained workers do not benefit from the training in other firms, then firm's investment into general training will only be below the competitive level if workers are credit constrained. The reason is that unskilled workers have to pay their expected gain from training to the training firm in exchange for being trained.

If workers are credit constrained, then the training firm cannot recover the cost of training, since trained workers will search for a better paid job. This, however, does not imply that trainees will benefit from training, since the firm can extract the worker's expected gain from searching for another employer by paying him a low trainee wage as long as the worker stays with the training firm. Only if the trainee wage is bounded by the workers' credit constraints do trainees gain from training.

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