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Abstract

Since the seminal article of Bates and Granger (1969), a large number of theoretical and empirical studies have shown that pooling different forecasts of the same event tends to outperform individual forecasts in terms of forecast accuracy. However, the results remain heterogenous regarding the size of gains. As there are numerous sources for the large variation of the resulting gains, it is difficult to estimate the improvement in accuracy based on empirical findings. Consequently, we use Monte Carlo techniques which enable us to identify the gains of pooling from VAR forecasts under lab conditions. In particular, the results are allowed to vary with respect to sample size, forecast horizon, number of pooled forecasts, weighting scheme and structure of the model economy. Given strict lab conditions, our setup of the experiment yields a quantification of the virtues that can be obtained in almost any forecast situation. The analysis shows that pooling leads to a substantial reduction of MSE of about 20%, which is comparable to the elimination of estimation uncertainty. Most notably, this reduction is already obtained with an average of about four different forecasts.

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Keywords: Pooling of forecasts, model uncertainty, VAR model, Monte Carlo Study.

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1 Introduction

Since the seminal article of Bates and Granger (1969), a large number of theoretical and empirical studies have shown that pooling different forecasts of the same event tends to outperform individual forecasts in terms of forecast accuracy. Clemen (1989) summarizes the empirical evidence in the literature on forecast combinations as "The results have been virtually unanimous: combining multiple forecasts leads to increased forecast accuracy [...] in many cases one can make dramatic performance improvements by simple averaging the forecasts." In addition to assessing the predominance of pooled predictions, a number of papers quantify their empirical gains compared to single forecasts. Although the vast majority of articles confirms the predominance of pooled forecasts, the results remain heterogenous regarding the size of gains. A broad range of detrimental effects such as structural breaks or outliers and revisions of data potentially impact empirical findings and account for the differences of gains reported. Armstrong (2001) reviews 30 empirical studies on forecast combinations and reports an average reduction in forecast errors of 12.5 percent, ranging between 3 and 24 percent each. As there are numerous sources for the large variation of the resulting gains, it is difficult to estimate the improvement in accuracy in a given forecast situation based on empirical findings.

To draw more common conclusions, we employ a Monte Carlo study and simulate the data from a standard DSGE model. Built on the simulated data sets, we specify parsimonious VAR models and dynamically forecast the target series. Monte Carlo techniques enable us to estimate the gains of pooling under various conditions and, thus, answer a number of interesting questions. First, we analyze the effects of pooling regarding the number of forecasts included. Second, we also track the gains as the forecast horizon grows. Third, we explicitly assess the effects of estimation uncertainty by varying the estimation sample size. Finally, we also contrast a simple average with optimal pooling techniques and answer the question in which situations the former outperforms the latter. To obtain robust conclusions we also modify the underlying structural parameters of the data generating process (DGP). This leads to conclusions about the effect economic structures exert on the virtues from pooling in a given forecast situation. To better understand why pooling yields a higher forecast accuracy, we decompose the mean squared error (MSE) and analyze how different parts of the forecast error are affected by pooling.

Employing a DSGE model, we are able to imitate the main characteristics of a typical economy but keep control of the DGP. In particular, our results rely on well–defined forms of mis–specification on part of the forecasting models. This gains practical relevance as the forms of mis–specification we assume are likely to occur in any real forecasting situation. By contrast, we exclude any accidental effects that may bias the results in favor of combination approaches and possibly lead to misleading conclusions. Thus, given strict lab conditions, our experiment yields a quantification of the gains that can be realized in almost any forecast situation. In that sense the reported results can be interpreted as some sort of minimum gain that is obtained from pooling of forecasts. Our analysis shows that pooling leads to a substantial reduction of MSE of about 20%. Most notably, this reduction is already obtained with an average of about four different forecasts.

The structure of the paper is given as follows. Section 2 gives the framework for the theoretical and empirical gains of pooling of forecasts. Section 3 describes the model economy as our data generating process. Section 4 introduces the VAR forecast framework and presents the combination schemes used. Section 5 describes the settings of our Monte Carlo experiment. Section 6 presents the results and section 7 concludes.

2 The gains from pooling

2.1 Why do we gain from pooling of forecasts?

The use of pooled forecast is mostly motivated by portfolio diversification or hedging arguments, guaranteeing insurance against very large forecast errors. As stated by Bates and Granger (1969) pooled forecasts can even dominate the best individual device.

It is quite clear that, if the true DGP is known to the forecaster, there are no gains from pooling the forecasts from different models. Instead, as argued by Timmermann (2005) pooling the information and thus constructing one *super model* yields the best forecast performance. However, in practice none of the models at hand coincides with the unobservable DGP. Moreover, Diebold and Lopez (1995) point out, that "…it is must be recognized that in many forecasting situations, particularly in real time, pooling of information sets is either impossible or prohibitively costly." Thus, some form of mis–specification, mis–estimation or non–stationarity will be present and contribute to the resulting forecast error.

Hendry and Clements (2004) describe a set of potential explanations for the gains achieved by combining individual forecasts regarding the forecast MSE. The most obvious benefit is based on the assumption of biased single forecasts. If the single predictions are differently biased – i.e. upwards biased and downwards biased – pooling them might improve forecast accuracy. However, reasonably constructed forecast models prevent systematically biased predictions. A source of improvement more relevant in practice results from unexpected breaks in the DGP. As each forecast model is affected differently by breaks, i.e. each model is differentially mis–specified, pooling the resulting predictions might again improve forecast accuracy. Another potential source of gain follows from a reduction of parameter proliferation due to overfitting. Forecasts from the true but estimated DGP do not encompass forecasts from competing mis–specified models in general, especially when the sample size is short compared to the number of parameters to be estimated in the DGP. As a result, pooling the forecasts from parsimonious models that omit a subset of explanatory variables might still outperform the forecasts from the DGP. Timmermann (2005) argues that an additional potential source of gain results from the fact that the single forecasts possibly build on different loss functions, even if they use the same information set. The present paper examines if there are gains from pooling even in the absence of the potential causes described by Hendry and Clements (2004). As we exclude structural breaks and other forms of non–stationarity as well as effects due to sample estimation uncertainty, improvements in forecast accuracy inevitably result from the use of separate sources of information in the different forecast models. By pooling the forecasts from multiple models, the combined prediction is based on a broader information set compared to each single forecast.

2.2 How much do we gain?

Batchelor and Dua (1995) present an expression of the gain from pooling in a more general setting that builds the background for our analysis. In the following, we aim to forecast the h–periods ahead future value of some target variable y whose realization is given as y_{t+h}. The forecast based on some model i is denoted as $\hat{y}_{i,t+h}$, and the resulting forecast error is given as $e_{i,t+h} = \hat{y}_{i,t+h} - y_{t+h}$. The loss function underlying our experiment is MSE loss. Assuming unbiased predictions, i.e. $E(e_{i,t}) = 0$, MSE equals the variance of the forecast errors:[1]

$$MSE_i = \sigma_i^2 \tag{1}$$

Given a total number M of different single forecasts, the expected error variance of a single randomly selected forecast k can thus be expressed as the average of the error variances of all single forecasts, i.e.

$$E(\sigma_k^2) = \overline{\sigma}^2 = \frac{1}{M}\sum_{i=1}^{M}\sigma_i^2, \tag{2}$$

The expected error variance of an equally weighted average of a set of m randomly selected single forecasts can be calculated as

$$E(\sigma_m^2) = \overline{\tau}^2(m) = \frac{1}{m}\overline{\sigma}^2 + \frac{m-1}{m}\overline{s}, \tag{3}$$

where $\overline{s}$ is the average bi–variate covariance between all pairs of single forecast errors.[2] The percentage reduction in expected error variance by pooling m forecasts is thus given as

$$\frac{\overline{\sigma}^2 - \overline{\tau}_m^2}{\overline{\sigma}^2} = \frac{m-1}{m}(1 - \frac{\overline{s}}{\overline{\sigma}^2}). \tag{4}$$

Since $\overline{\sigma}^2 \geq \overline{s}$, this reduction is non–negative and takes a value of zero only if the forecast errors are perfectly correlated. Figure 1 shows the percentage reduction in the expected error variance for $\overline{\sigma}^2 = 1$ as a function of the number m of forecasts combined and of the average bi–variate covariance $\overline{s}$ of the forecast errors.

The improvement in forecast accuracy increases with the number of forecasts combined and with a decreasing average bi–variate covariance of the forecast errors and converges to $1-\frac{\overline{s}}{\overline{\sigma}^2}$ for large values of m. Figure 1 supports the conclusion of Armstrong (2001), that the combination of five forecasting methods is sufficient to achieve most of the possible reduction in forecast error variance and that the inclusion of further forecasts generates only minor additional

[1]As the results are independent of a specific forecast horizon, we skip the corresponding index in the following.

[2]For the derivation of the above expression, see Appendix A

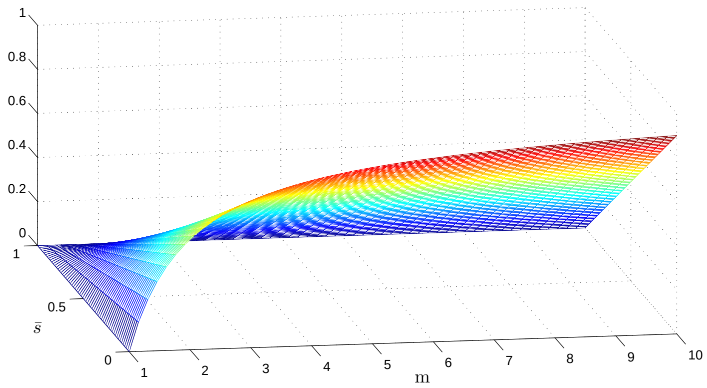


Figure 1: Percentage reduction in expected error variance

gains. Batchelor and Dua (1995) use a panel of 22 US economic forecasters to examine the quantity and probability of gains due to pooling. However, it seems worthwhile to modify their approach and use forecasts solely based on econometric models. The random draw of one or more forecasts from a broader set reflects the situation of a practical forecaster who is faced with choosing between a possibly large number of different forecast models and thus forecasts. As her decision is typically based on some form of ad–hoc in–sample or out–of–sample selection criteria randomly chosen according to certain preferences, the choice of the forecast itself can be regarded as being random.

2.3 Where do the gains come from?

To better understand why pooling of forecasts reduces MSE in a general setting, it seems worthwhile to analyze the structure of forecast errors by breaking down the forecast MSE into its three components, the bias part, the variance part and the covariance part. Following Theil (1966), the h–step forecast MSE can be written as:

$$MSE_{t+h} = (\bar{\hat{y}}_{t+h} - \bar{y}_{t+h})^2 + (\sigma_{\hat{y}_{t+h}} - \sigma_{y_{t+h}})^2 + 2(1-\rho)\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}}, \quad (5)$$

where $\hat{y}_{t+h}$ and y_{t+h} are the forecasted and actual values of some variable y for period $t+h$, $\bar{y}_{t+h}$, $\bar{\hat{y}}_{t+h}$, $\sigma_{y_{t+h}}$ and $\sigma_{\hat{y}_{t+h}}$ are theirs means and standard deviations and ρ is the correlation between them.[3]

The first term on the right hand side shows the deviation of the mean of the forecast from the mean of the actual series and is denoted the bias part. The second term, which is called the variance part, reports the deviation of the variation of the forecast from the variation of the actual series. The last term on the right hand side is named the covariance part and reflects the general co–movement of forecasted and realized values. Having quantified the overall improvement in MSE, the total reduction can be disaggregated into the different shares. To understand how the different parts of MSE are affected by pooling of forecasts is interesting, as one can easily imagine loss functions that depart from an equal weighting of these parts. A loss function that puts a higher weight on the covariance part relative to the variance part would be reasonable when the focus lies on predicting changes in the direction of some target series. An example would be the prediction of turning points. In contrast, if the average amplitude of the target series is of major interest, the variance part plays the crucial role whereas errors due to a lower covariance of the movements of the forecasted and the realized values are of minor interest. For the analysis in the present paper, however, we will stick to conventions and focus on MSE loss.

3 The model economy

For our controlled experiment, we choose a common New–Keynesian type model to simulate economic relationships. Generally speaking, New-Keynesian DSGE models have been developed to replicate distinct features of economic data on a business cycle frequency. Consequently, they are most commonly estimated on quarterly data of aggregate measures such as real GDP, inflation and money–market rates. In recent years, they have become a popular tool for central banks not only for policy analysis, but also for forecasting.[4] In contrast to VAR models, the behavior of all variables is traceable to a set of fundamental assumptions about the underlying structure of the model economy. In other words, the forecasts lend themselves to an economic interpretation as the dynamics of the variables is the result of economic decisions taken at the micro–level. Thus, given that economic theory is in any case meaningful, employing a New-Keynesian DSGE model to simulate the data guarantees that the forecasting experiment is based on data which shares the distinct features of aggregate economic data on a business cycle frequency.

The model we choose is very similar to the one presented in Canova and Paustian (2007). Featuring staggered wages and prices, it is very much in the spirit of Erceg, Henderson, and Levin (2000). We allow for habit formation as in Fuhrer (2000) and for indexation as in Rabanal and Rubio-Ramirez (2005). The model also captures interest rate smoothing of the central bank as in Clarida, Galí, and Gertler (2000). The approach has the advantage that many simpler models are nested in our baseline scenario and we can vary the degree of persistence of the system by changing well specified parameter values that have a structural

[3]For the derivation of the above expression see Appendix B.

[4]See among others Smets and Wouters (2003), Harrison et al. (2005) or Murchison and Rennison (2006).

interpretation. The linearized model equations are (in log deviations from steady state):

$$
\begin{aligned}
\lambda_t &= E_t\lambda_{t+1} + E_t[r_t - \pi_{t+1}] && (6)\\
\lambda_t &= \xi_t^b - \frac{\sigma_c}{(1-h)}(y_t - hy_{t-1}) && (7)\\
y_t &= \xi_t^z + (1-\alpha)n_t && (8)\\
mc_t &= w_t + n_t - y_t + err_2 && (9)\\
mrs_t &= -\lambda_t + \gamma n_t && (10)\\
w_t &= w_{t-1} + \pi_t^w - \pi_t + err_1 && (11)\\
\pi_t^w - \mu_w\pi_{t-1} &= \kappa_w(mrs_t - w_t) + \beta(E_t\pi_{t+1}^w - \mu_w\pi_t) && (12)\\
\pi_t - \mu_p\pi_{t-1} &= \kappa_p(mc_t + \xi_t^\mu) + \beta(E_t\pi_{t+1} - \mu_p\pi_t) && (13)\\
r_t &= \rho_r r_{t-1} + (1-\rho_r)(\gamma_\pi\pi_t + \gamma_y y_t) + \xi_t^r && (14)\\
\xi_t^b &= \rho_b\xi_{t-1}^b + \nu_t^b \quad \nu_t^b \sim N(0,\sigma_b^2) && (15)\\
\xi_t^z &= \rho_z\xi_{t-1}^z + \nu_t^z \quad \nu_t^z \sim N(0,\sigma_z^2) && (16)\\
\xi_t^\mu &\sim N(0,\sigma_\mu^2) && (17)\\
\xi_t^r &\sim N(0,\sigma_r^2) && (18)
\end{aligned}
$$

Equations (6) and (7) describe the demand side of the economy where λ_t is the marginal utility of consumption which depends on the expected real interest rate as the difference of the nominal rate r_t and inflation π_{t+1} and h measures the degree of habit formation in total demand y_t. Moreover, demand is subject to a taste shock ξ_t^b. Equation (8) is the linearized production function with labor share in production $1-\alpha$ and n_t being labor input (hours worked). The process is subject to a productivity shock ξ_t^z. Equations (9) and (10) define marginal cost and the marginal rate of substitution, respectively. Here, w_t is the real wage and γ measures the substitution effect of a change in hourly wages on labor supply. The real wage is defined in equation (11), with real wage inflation being the difference between nominal wage inflation π_t^w and price inflation π_t. The wage Phillips curve is presented in equation (12), where indexation is measured by μ_w and κ_w is the slope. The parameter that determines the dynamics of the wage equation κ_w can also be calculated from deep parameters; i.e. by the probability of keeping wages fixed $1-\zeta_w$, the discount factor β, the elasticity of the labor bundler ψ and the elasticity of labor supply with respect to wages γ:

$$\kappa_w = \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w(1+\psi\gamma)}. \tag{19}$$

Analogously, equation (13) defines the Phillips curve for prices with indexation parameter μ_p and slope κ_p. The slope of the Phillips curve can be shown to depend upon the probability of keeping prices fixed $1-\zeta_p$, the discount factor β, the elasticity of the goods bundler ϵ and the labor share in production $1-\alpha$ in the following way:

$$\kappa_p = \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}\frac{1-\alpha}{(1-\alpha+\alpha\epsilon)}. \tag{20}$$

In addition, marginal cost is also driven by an exogenous mark–up shock ξ_t^μ. The nominal interest rate is set by the central bank according to a Taylor–type rule (14) with interest rate smoothing of degree ρ_r. γ_π and γ_y capture the response of the central bank to inflation and output, respectively. The remaining four equations define the emergence of exogenous shocks, where some persistence is allowed for the taste shock and the productivity shock. Moreover, to avoid perfect multicollinearity, real wages and marginal cost are assumed to be measured with error err_1 and err_2, respectively. The parameter values assigned during the experiment are reported in table 1:

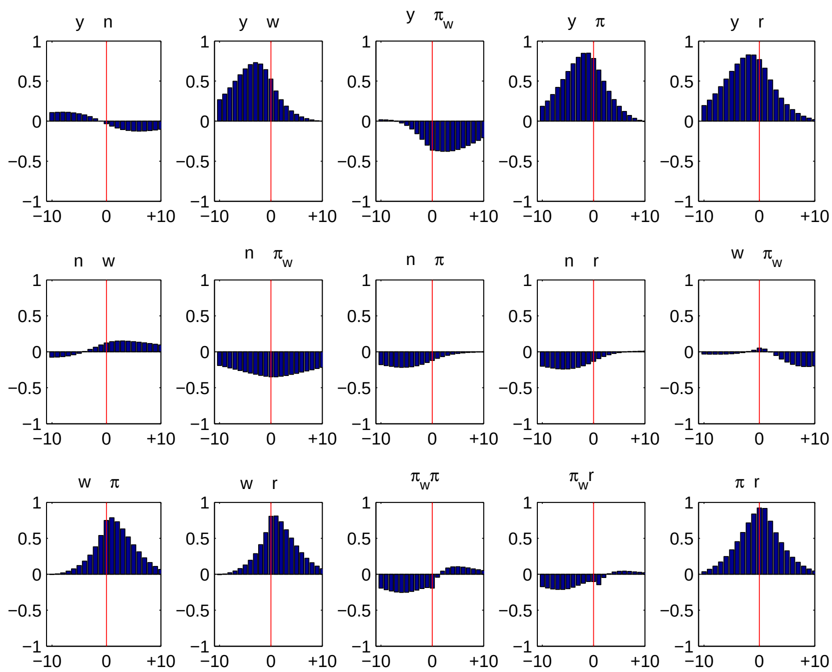
Model parameter		*Model I*	*Model II*	*Model III*
β	discount factor	0.99	0.99	0.99
ϵ	elasticity (goods)	6.00	6.00	6.00
ψ	elasticity (labor)	6.00	6.00	6.00
σ_c	risk aversion	8.33	8.33	8.33
γ	inverse Frish elasticity of labor supply	1.74	1.74	1.74
h	degree of habit formation	0.90	0.00	0.00
ζ_p	1-probability of keeping prices fixed	0.75	0.75	0.75
ζ_w	1-probability of keeping wages fixed	0.62	0.62	0.62
μ_p	rule–of–thumb price setters	0.70	0.70	0.00
μ_p	rule–of–thumb wage setters	0.80	0.80	0.00
α	1-labor share in production function	0.36	0.36	0.36
ρ_r	interest rate smoothing	0.74	0.74	0.74
γ_y	reaction to output in Taylor rule	0.26	0.26	0.26
γ_π	reaction to inflation in Taylor rule	1.08	1.08	1.08
ρ_b	persistence of taste shock	0.82	0.82	0.82
ρ_z	persistence of productivity shock	0.74	0.74	0.74
σ_b	std of taste shock	0.1188	0.1188	0.1188
σ_z	std of productivity shock	0.0388	0.0388	0.0388
σ_μ	std of markup shock	0.3167	0.3167	0.3167
σ_r	std of monetary policy shock	0.0033	0.0033	0.0033
σ_{err1}	std of measurement error 1	0.0001	0.0001	0.0001
σ_{err2}	std of measurement error 2	0.0001	0.0001	0.0001

Table 1: Calibration of the model economy

In principle, we adopt the values from Canova and Paustian (2007) but allow for three different types of economies. In *Model I*, we allow for a considerable degree of backward–lookingness and habit formation. This results in a more sluggish response of GDP to all kinds of shocks when compared to *Model II* and *III.*[5] The setup of *Model II* abstracts from habit formation, i.e. $h = 0$, and, thus, output is less persistent when compared to *Model I.* A productivity shock results in a more pronounced, but somewhat earlier response. *Model III* mimics a more flexible economy where neither prices nor wages are subject to indexation, i.e. $h = 0$, $\mu_p = 0$ and $\mu_w = 0$. This leads to quite similar responses of GDP as in Model II. However, differences occur with respect to price and wage reactions to shocks. As intended, prices tend to adjust quicker – especially in response to markup–shocks.

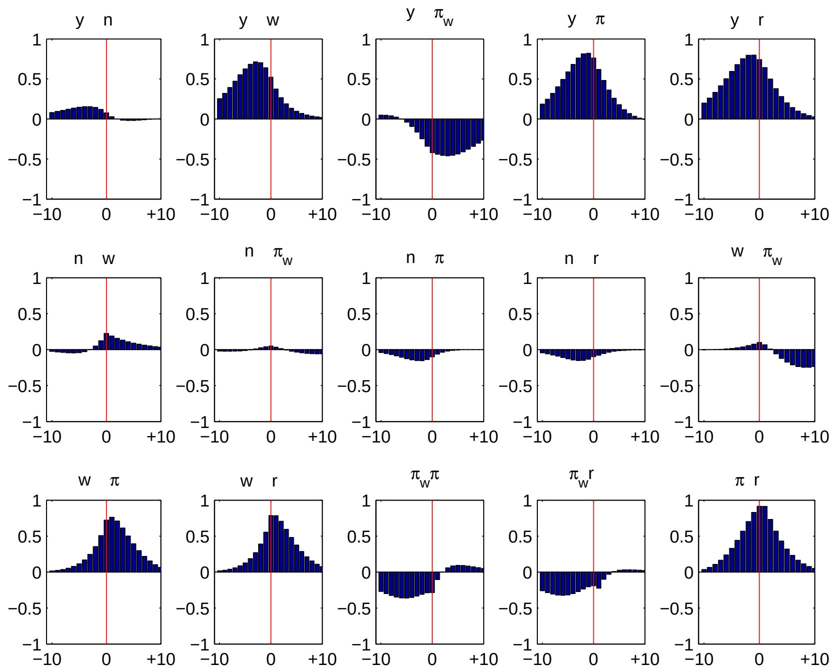
[5]See figures 5 to 8 in appendix C for impulse response functions and figures 9 to 14 in appendix D for a representation of the data and of the autocorrelation and partial autocorrelation patterns.

The differences between *Models I* to *III* also stem from the fact that interdependencies between variables differ. As lags and leads are important when it comes to forecasting the respective processes, we also depict the cross–correlations of the relevant variables in figures 2 to 4 for ± 10 lags. In *Model I*, for instance, employment (n) depends negatively on wage inflation (π_w), whereas in *Model II*, this correlation is almost zero. Note, that in *Model III* employment is almost solely determined by output. In turn, wages do not have a contemporary effect on output. On the whole, the procedure introduces well–defined cross–correlation structure between variables and economically meaningful cross–equation restrictions.



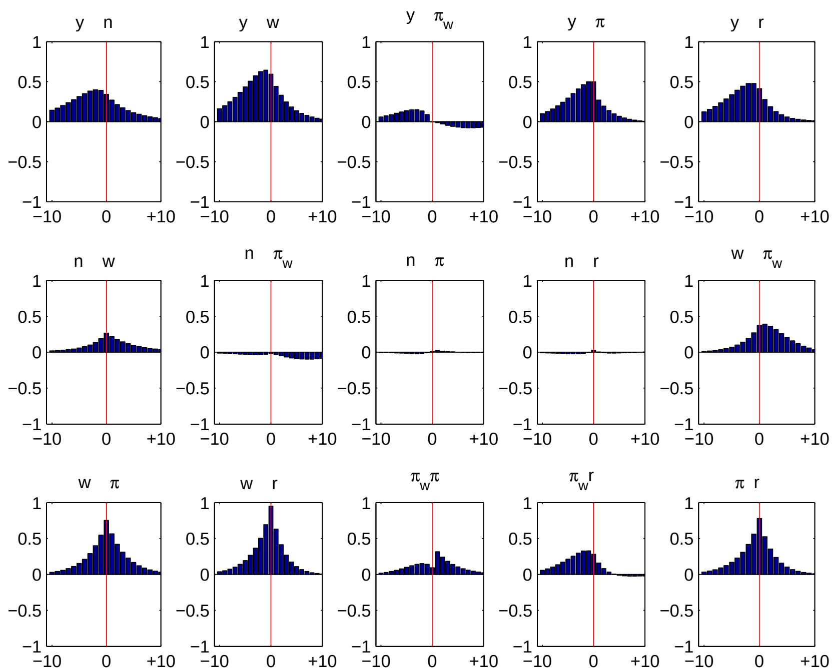
Notes: The plots show the correlation of the first variable with the lags of the second variable, e.g. corr(y_t, n_{t+k}). Lags are given on the abscissa and correlations are depicted on the ordinate, where the numbers represent the mean of a Monte–Carlo simulation based on 10.000 replications of the data set.

Figure 2: Mean of Cross-Correlation Functions (*Model I*)



Notes: See figure 2.

Figure 3: Mean of Cross-Correlation Functions (*Model II*)



Notes: See figure 2.

Figure 4: Mean of Cross-Correlation Functions (*Model III*)

4 The forecasting approach

4.1 VAR forecast framework

To analyze the gains from pooling of forecasts, we predict the target series y employing VAR models. The use of VAR models for macroeconomic forecasting was first introduced by Sims (1980) to address the common structural identification problem inherent in simultaneous equation models. It follows the idea of exploiting the dynamic correlation patterns among observed time series without imposing strong restrictions relating to the structure of the economy. As all variables are determined endogenously, no a priori knowledge is used except to decide which variables should enter the system. Thus, the VAR approach is often referred to as being atheoretical. Due to their comparable good forecast record, the relative low computational effort involved and their ability to generate iterative multi–step predictions VAR–models are frequently used in macroeconomic forecasting.[6] An important feature of VAR forecasts is their unbiasedness. Dufour (1985) has shown that, as long as the data generating process (DGP) process considered has an autoregressive representation and satisfies the symmetry condition, "any vector autoregression estimated by least squares will yield unbiased forecasts even if the orders of the autoregressive models fitted are lower than the actual ones and even if explanatory variables are missing in some or all the equations".

We aim at isolating the gains in forecast accuracy due to the reduction of the mis–specification problem by pooling as described in section 2. We use the simplest form of forecasting model at hand, as, in our experimental setup, the forecasting performance of small–scale models should not be much worse than that of larger VARs. We also fix estimation uncertainty at a constant level by restricting the analysis to small scale VAR–models with $K = 2$ endogenous variables and a lag length of $p = 1$. Most important, in doing so, we ensure that virtues from pooling are directly comparable across different setups. In particular, in opposition to empirical studies, a change in MSE is directly attributable to pooling of forecasts and results are not distorted by detrimental effects like different degrees of underlying parameter uncertainty. Consequently, each VAR–model only incorporates the target series plus one of the remaining 5 variables. We assume that y, r, n, w, π and π^w are directly observable by the forecaster. Note, that, as we forecast the economy with a VAR type model, any cross equation restrictions from the DSGE model are omitted. Thus, information contained in cross–correlations of the additional observable variables with output is necessarily inefficiently used by the forecaster. Moreover, a bivariate VAR will omit information present in cross–correlations among observable variables that are not included and the variable to be forecast. We thus depart from the optimal situation where the DGP is known and end up with a total of $M = 5$ necessarily mis–specified forecast models. Note again, that this approach is very much in line with a typical forecasting situation. Hence, given an estimation sample of size T, VAR models with $T - K^2p + K$ degrees of freedom are estimated and 1- to h-step predictions of variable y are derived.

[6]Examples of small scale VAR–systems used to forecast output, prices and interest rates are numerous, including Litterman (1986), Del-Negro and Schorfheide (2004), Favero and Marcellino (2005) and Clark and McCracken (2006). See Lütkepohl (2006) for a detailed discussion of VARs in macroeconomic forecasting.

To quantify the gains due to pooling of forecasts, we modify the idea of Batchelor and Dua (1995) and compare the forecast performance of one randomly chosen single VAR model to the performance of a pooled prediction from m randomly chosen VAR models with $1 \leq m \leq M$.

4.2 Pooling techniques

One crucial issue when pooling different forecasts is the weighting scheme that defines how single predictions are combined into one pooled forecast of the target variable. Various methods for the estimation of appropriate weighting schemes have been proposed and their relative performance depends on the underlying assumptions. As Winkler (1989) points out: "The better we understand which sets of underlying assumptions area associated with which combining rules, the more effective we will be at matching combining rules to forecasting situations." We focus on two schemes somehow defining the upper and the lower bound. On the one hand we introduce optimal weighting of forecasts, which relies on the covariance structure of forecast errors stemming from different models. On the other hand, we employ a simple average which is particularly easy to implement and no additional information is needed for the computation of weights. All remaining weighting schemes discussed in literature rely on some form of trade–off between the advantages of these two schemes.

Optimal weighting scheme Assuming a MSE based loss function that exclusively depends on the forecast error of the pooled forecast, $e^c = y_{t+h} - g(y_{i,t+h}; \omega)$, optimal weights are chosen to solve the problem:

$$\boldsymbol{\omega}^* = \arg \min_{\omega} [\boldsymbol{\omega}' \boldsymbol{\Sigma}_e \boldsymbol{\omega}], \tag{21}$$

which gives

$$\boldsymbol{\omega}^* = \frac{\boldsymbol{\Sigma}_e^{-1} \mathbf{1}_N}{\mathbf{1}_N' \boldsymbol{\Sigma}_e^{-1} \mathbf{1}_N}. \tag{22}$$

where Σ_e denotes the covariance matrix of the forecast errors e_i of the single models. Thus, the optimal weights are a function of the covariance matrix of the forecast errors. In practice, the elements of Σ_e are unknown and have to be estimated. Assuming linear relationships Granger and Ramanathan (1984) propose to directly estimate the optimal weights by OLS, regressing realizations of the target variable on the vector of forecasts. This can result in unstable combination weights when the forecast errors are highly correlated and when the number of forecasts to pool and thus the covariance matrix of the forecast errors is too large.[7]

Equal weighting scheme A simplification of the optimal pooling approach is the use of equal weights, which particularly solves the computation problem. In empirical applications, simple averages of forecasts tend to outperform more elaborated weighting schemes, a phenomenon often referred to as *forecast combination puzzle* (see e.g. Stock and Watson (2004)).

[7]See e.g. Winkler and Clemen (1992)

As Timmermann (2005) shows, equal weights are theoretically optimal if the individual forecast errors have the same variance and identical pair–wise correlations, i.e. $\omega^* = \frac{1}{N}\iota$, where ι is an $N \times 1$ column vector of ones. This case gains relevance by the fact that forecasts of a certain variable based on differently specified VAR models often show similar characteristics. As forecasts converge to the unconditional mean of the process when the forecast horizon grows, the respective moments, variances and pair–wise correlations of the forecast errors also converge.

5 Monte Carlo simulations

We employ each of the DSGE Models described in section 3 to simulate the path of the six observable variables (y, r, n, w, π, π^w) for 1100 periods within a Monte–Carlo experiment of 10.000 draws. For each draw, we derive $h = 1, \ldots, 100$ steps ahead forecasts of the target variable y built on an estimation sample of size T that form the basis for the calculation of the pooled predictions. As the decomposition of the resulting MSE builds on the assumption that the VAR operators are stable and the processes and thus the forecasts are stationary, we exclude iterations that yield VAR models with unstable roots.[8]

The single VAR forecasts are pooled employing equal weights as well as optimized weights building on the covariance matrix of the forecast errors as described in section 4.2. In the latter case, for each forecast horizon h, the covariance matrix is estimated employing a subset of N forecast errors of the total of 10.000 draws. However, in order to rule out any coincidence in the estimation of the covariance matrix of the forecast errors, we report the mean over 1.000 randomly drawn subsets for each of the descriptives. As for the single VAR forecasts, the MSE of the pooled forecasts is decomposed as described in section 2.3.

6 Discussion of the results

Tables 2 to 5 show the results for *Model I* to *Model III* in turn. The respective absolute and relative figures are presented for sample size T, number of pooled VARs m and forecast horizon h. As laid out in section 2.3, we split MSE into its bias, its variance and its covariance parts. We first report the results for the single randomly chosen VAR followed by the simple average and the optimally pooled forecasts. Note, that the bias part can be neglected because it is approximately zero for the present setup.

[8]Note: If an unstable root is detected in any of the single VAR models, the entire draw is excluded from further analyses.

		MODEL I														
		MSE (10e-3)					VARIANCE (10e-3)					COVARIANCE (10e-3)				
	T	25	50	100	200	1000	25	50	100	200	1000	25	50	100	200	1000
h	1	0.022	0.020	0.019	0.019	0.019	0.000	0.000	0.000	0.000	0.001	0.022	0.020	0.018	0.019	0.018
	2	0.054	0.050	0.044	0.044	0.042	0.000	0.001	0.001	0.001	0.002	0.053	0.048	0.043	0.042	0.040
	3	0.094	0.086	0.074	0.073	0.071	0.001	0.004	0.003	0.003	0.005	0.093	0.082	0.071	0.070	0.066
	4	0.140	0.123	0.110	0.104	0.103	0.002	0.006	0.006	0.006	0.009	0.139	0.117	0.104	0.098	0.093
	5	0.181	0.154	0.138	0.134	0.135	0.002	0.008	0.009	0.010	0.016	0.179	0.145	0.129	0.124	0.119
	6	0.220	0.187	0.164	0.163	0.160	0.003	0.012	0.014	0.015	0.024	0.216	0.175	0.150	0.149	0.136
	7	0.251	0.210	0.188	0.186	0.178	0.003	0.014	0.019	0.021	0.032	0.248	0.196	0.169	0.165	0.146
	8	0.279	0.228	0.205	0.206	0.190	0.004	0.016	0.024	0.029	0.040	0.275	0.212	0.181	0.178	0.150
	9	0.299	0.242	0.215	0.219	0.200	0.005	0.018	0.029	0.035	0.049	0.294	0.224	0.187	0.183	0.151
	10	0.317	0.248	0.222	0.225	0.204	0.006	0.019	0.032	0.041	0.056	0.310	0.230	0.190	0.184	0.148
	25	0.317	0.269	0.233	0.216	0.199	0.007	0.033	0.060	0.080	0.116	0.309	0.236	0.173	0.136	0.084
	50	0.333	0.251	0.221	0.212	0.198	0.007	0.037	0.073	0.103	0.139	0.326	0.214	0.148	0.109	0.059
	100	0.328	0.244	0.232	0.205	0.195	0.005	0.037	0.084	0.109	0.152	0.323	0.206	0.147	0.096	0.043
		MODEL II														
		MSE (10e-3)					VARIANCE (10e-3)					COVARIANCE (10e-3)				
	T	25	50	100	200	1000	25	50	100	200	1000	25	50	100	200	1000
h	1	0.022	0.020	0.019	0.018	0.018	0.000	0.000	0.000	0.000	0.001	0.022	0.020	0.019	0.018	0.018
	2	0.054	0.049	0.045	0.043	0.042	0.001	0.001	0.001	0.002	0.002	0.053	0.048	0.044	0.041	0.040
	3	0.091	0.084	0.075	0.071	0.068	0.001	0.003	0.003	0.004	0.004	0.090	0.081	0.072	0.067	0.063
	4	0.130	0.118	0.105	0.101	0.096	0.002	0.005	0.006	0.007	0.009	0.128	0.112	0.100	0.094	0.088
	5	0.165	0.147	0.132	0.129	0.125	0.003	0.008	0.008	0.011	0.015	0.162	0.139	0.123	0.117	0.111
	6	0.196	0.174	0.157	0.153	0.149	0.004	0.011	0.013	0.017	0.022	0.192	0.163	0.144	0.136	0.127
	7	0.221	0.195	0.179	0.172	0.167	0.004	0.013	0.018	0.024	0.030	0.217	0.182	0.161	0.148	0.137
	8	0.247	0.214	0.197	0.190	0.180	0.005	0.017	0.024	0.031	0.038	0.241	0.197	0.173	0.159	0.142
	9	0.263	0.224	0.206	0.198	0.188	0.006	0.019	0.029	0.037	0.046	0.257	0.205	0.178	0.161	0.143
	10	0.273	0.232	0.211	0.205	0.193	0.006	0.021	0.033	0.044	0.054	0.267	0.211	0.178	0.161	0.140
	25	0.295	0.244	0.220	0.198	0.189	0.006	0.035	0.068	0.091	0.131	0.289	0.208	0.152	0.106	0.058
	50	0.305	0.238	0.211	0.197	0.185	0.007	0.038	0.075	0.104	0.145	0.298	0.200	0.136	0.093	0.040
	100	0.315	0.240	0.217	0.194	0.187	0.007	0.040	0.079	0.104	0.147	0.307	0.201	0.138	0.090	0.040
		MODEL III														
		MSE (10e-4)					VARIANCE (10e-4)					COVARIANCE (10e-4)				
	T	25	50	100	200	1000	25	50	100	200	1000	25	50	100	200	1000
h	1	0.197	0.178	0.168	0.167	0.158	0.006	0.009	0.011	0.011	0.011	0.191	0.169	0.157	0.156	0.147
	2	0.365	0.343	0.315	0.309	0.298	0.014	0.028	0.035	0.036	0.039	0.351	0.316	0.280	0.273	0.260
	3	0.528	0.486	0.439	0.436	0.407	0.023	0.049	0.059	0.073	0.073	0.506	0.436	0.380	0.362	0.335
	4	0.646	0.601	0.537	0.540	0.510	0.029	0.067	0.086	0.120	0.122	0.617	0.534	0.451	0.420	0.388
	5	0.740	0.679	0.601	0.613	0.595	0.036	0.089	0.112	0.164	0.177	0.704	0.590	0.489	0.449	0.418
	6	0.819	0.743	0.664	0.650	0.635	0.041	0.105	0.147	0.197	0.224	0.779	0.638	0.517	0.453	0.411
	7	0.865	0.780	0.732	0.671	0.668	0.040	0.115	0.184	0.226	0.268	0.825	0.664	0.548	0.446	0.401
	8	0.920	0.810	0.762	0.710	0.692	0.046	0.124	0.205	0.260	0.309	0.874	0.686	0.556	0.450	0.383
	9	0.937	0.821	0.758	0.715	0.687	0.048	0.130	0.214	0.276	0.334	0.889	0.690	0.544	0.438	0.353
	10	0.975	0.837	0.753	0.727	0.697	0.052	0.139	0.224	0.301	0.370	0.923	0.698	0.529	0.427	0.327
	25	1.076	0.858	0.786	0.743	0.717	0.052	0.170	0.309	0.412	0.563	1.024	0.688	0.477	0.331	0.154
	50	1.029	0.874	0.769	0.759	0.724	0.047	0.183	0.303	0.426	0.574	0.981	0.690	0.466	0.333	0.150
	100	1.094	0.854	0.795	0.726	0.715	0.051	0.177	0.320	0.406	0.571	1.043	0.676	0.475	0.320	0.144

Table 2: Decomposition of MSE of a single VAR

6.1 Forecast Errors from single randomly chosen VAR models

Table 2 reports the results which can be inferred from the single randomly chosen VARs. We generally observe that MSE rises with the forecast horizon for all DGPs and sample sizes T. Of course, this is in line with theoretical considerations, as MSE should approach the variance of the process if forecasts are unbiased.

For all simulated economies, we also observe that for shorter horizons up to about $h = 10$ (Model I and II) and $h = 3$ (Model III) the rise of MSE is largely attributed to a rise of the covariance part. Interestingly, the covariance part does not increase monotonically with the forecast horizon. For larger estimation samples it shows a peak during the first 10 forecasting periods and declines to some lower values afterwards. The larger the estimation sample, the steeper is the decline of the covariance part for longer horizons. In contrast, the variance part increases monotonically until a certain level is reached. In the absence of estimation uncertainty, its contribution to MSE even dominates that of the covariance part for horizons greater than $h = 19$ (Model I), $h = 16$ (Model II) and $h = 10$ (Model III). Thus, if we wish to forecast on a business cycle frequency, we should focus on the reduction of the covariance part of the forecast error. In contrast, for longer horizons the variance part plays the crucial role as far as forecast accuracy is concerned.

We also observe that MSE decreases with an increasing estimation sample size T. This is, of course, due to a decline of estimation uncertainty driving the forecast. The reduction in MSE is quite considerable when increasing the estimation sample from $T = 25$ to $T = 50$ and to $T = 100$ and converges to 15% (Model I) and 20% (Model II and III) for $T = 1.000$ and $h = 1$. As the forecast horizon grows, the benefits from precisely estimated model coefficients accumulate to 41% (Model I and II) and 35% (Model III) for $h = 100$.

Comparing the predictability of the three models, the levels of MSE decrease from Model I to Model III independent of the estimation sample size and the forecast horizon. This hints to the fact that a forecasting exercise is more difficult in less flexible economies.[9]

6.2 Forecast Errors from pooled randomly chosen VAR models - equal weights

Now, we turn to the notion of pooling and first focus on equally weighted forecasts. We look at the performance of the different pooling schemes relative to the performance of the single VAR. In order to better understand how the virtues from pooling emerge, we also give the relative change in MSE with respect to the variance and the covariance part. The numbers in tables 3 to 5 represent changes as percentage of MSE associated with the single VAR. Thus, the variance and covariance parts are given as contributions to total MSE change. To be more precise, we look at the percentage gains regarding four dimensions: sample size T, forecast horizon h, number of pooled forecasts m and the type of the simulated economy (*Model I* to *Model III*).

Even the small set of up to five pooled VAR forecasts yields a significant reduction of MSE. The results are very much in line with the theoretical gains described in section 2.2, as a combination of only $m = 4$ single predictions already guarantees the major proportion of improvement. Most notably, for shorter horizons, the decline of MSE by pooling $m = 5$ forecasts is comparable to the decline achieved by an extension of the sample size from $T = 25$ to $T = 1000$ as described in section 6.1. Thus, the gain from pooling is approximately

[9]Note: All DGPs considered fluctuate around mean zero.

comparable to the extinction of estimation uncertainty. This gains practical relevance when forecasting quarterly macroeconomic aggregates, as one usually has to deal with a maximum number of observations no more than 200 to 250. This makes pooling clearly advantageous.

In terms of the variance part, pooling is not favorable as it reduces the variance of the forecast compared to the single VAR prediction. Although this is intended in most applications – because pooling is often seen as an insurance against extreme forecast errors – it increases the variance part of MSE. This means, that the second moment of the target variable is forecasted with a larger error. By contrast, pooling considerably reduces the covariance part. This over–compensates for the increase of the variance part and leads to the decline of total MSE.

With respect to the different DGPs, the reduction in MSE in general is highest for *Model I*, followed by *Model II* and *Model III*. This indicates that pooling of forecasts is particularly beneficial in less flexible economies that are harder to forecast.

For all models, we find that the relative gain increases for smaller samples. First, we consider $T = 25$. Here we find that MSE reduction due to pooling of five VARs ranges between 16% and 9% for the first horizon. It becomes clear that the gain is quite substantial and it increases with the number of pooled forecasts. With growing estimation samples, the gains decrease and reach 11% to 3.5% for $T = 1.000$.

Interestingly, irrespective of the DGP and the estimation sample size, the gain reaches a maximum for $h = 2$ and gradually decreases with the forecast horizon. Whereas the gain remain significant for small samples even for $h = 100$, it disappears for larger estimation samples and long forecast horizons. Generally speaking, the benefit of pooling increases up to the second horizon and then slowly declines until all individual forecasts converge to their unconditional mean. Interestingly, as there remains a small sample bias in the estimation of the unconditional mean for each model, the gain does not disappear for $T < 1.000$. Thus, averaging long–term forecasts can be regarded as an insurance against a bias in the estimation of the unconditional means of the DGPs. On the contrary, when the estimation sample is large, all forecasts of bi–variate VAR(1) processes asymptotically converge to an unconditional mean of zero and thus, pooling these predictions does not reduce MSE compared to single forecasts.

The variance part of MSE increases and the covariance part decreases monotonically with the number m of pooled predictions. The reduction in the covariance part again has a peak for $h = 2$ and then declines slowly – a pattern that can hence be observed for the reduction of MSE as stated above. The reduction of total MSE due to pooling can thus be attributed to a better forecasting performance with respect to the covariance of the forecast and the target variable.

6.3 Forecast Errors from pooled randomly chosen VAR models - optimal weights

Tables 3 to 5 additionally report the relative gains of pooling 5 forecasts employing optimal weights in the sense of Bates and Granger (1969). The single columns represent the gains for different numbers N of forecast errors used to estimate the covariance matrix Σ_e. N is referred to as the size of the *optimization window* and grows from $N = 10$ to $N = 1.000$ observations. In a practical forecast exercise, one has to use ex–post forecast errors to estimate Σ_e. Thus, the total length of the data set available has to be split into an estimation sample used to estimate the models' coefficients and an optimization sample used to estimate Σ_e This can be done in a recursive manner or based on a rolling window approach. As the total number of observations is typically rather short when forecasting macroeconomic aggregates, sizes of the optimization window of $N = 10$ to $N = 50$ gain practical relevance.[10] In contrast, a size of $N = 1.000$ imitates a situation without estimation uncertainty regarding Σ_e and thus the estimated optimal weights approximate their true values.

As expected, the gain of optimal pooling increases with N, because the estimated weights converge to their true values. This holds for all sizes T and for all forecast horizons h. For $N = 10$, optimal pooling yields a MSE that lies far beyond its single VAR counterpart for all models and all estimation samples. The relative inferiority increases with the forecast horizon from 80% for $h = 1$ to 220% for $h = 100$ for all sample sizes T and all DGPs. This probably results in empirical studies finding that pooling forecasts based on optimized weights in the sense of Bates and Granger (1969) frequently yields a poor forecast performance.[11]

The efficiency of optimal pooling improves considerably with larger optimization windows. At shorter horizons, the optimal pooling approach dominates the single VARs for $N = 25$ ($N = 50$) and beats the equally weighted average for $N = 50$ ($N = 100$) for Model I and Model II (Model III). However, these gains fade away for longer horizons. In the absence of estimation uncertainty, i.e. $N = 1.000$, optimally pooling 5 single VAR forecasts improves forecast accuracy for $h = 1$ compared to a single forecast by about 20% on average for Models I and II and by about 10% on average for Model III. These gains slightly increase for $h = 2$ and then decline with the forecast horizon. However, even for $h = 10$, an improvement of about 10% on average for Models I and II and of about 3% on average for Model III remains. Thus, the use of optimized weights based on the covariance matrix of the forecast errors is advisable only for an adequate length of the optimization window of $N \geq 50$ and only for predictions up to 10 periods ahead. Interestingly, the optimized framework works best for less flexible economies that are harder to forecast in general.

For short horizons, the variance part of MSE is slightly smaller for optimized weights when compared to a simple average, but it increases faster as h grows. Interestingly, we observe, that a larger optimization window N comes along with a bigger variance part. However, it converges to the value of the benchmark approach only in the absence of uncertainty about

[10] See Clark and McCracken (2006) for an empirical application.

[11] Starting their recursive *optimization window* with a size of $N = 17$, Clark and McCracken (2006) find that pooling based on optimized weights yields a root mean squared forecast error (RMSE) twice as high as an equally weighted average.

Σ_e. For values of $N \leq 200$ the variance part stays below these levels even for $h = 100$. Thus, employing optimized weights in practice, i.e. for $N \leq 50$, likely reduces the variance part of the forecast errors provided that the underlying estimation samples are sufficiently long.

The benefits of using optimized weights mainly results from an reduction of the covariance part of MSE. Given that the estimated parameters of Σ_e coincide with their true values, the covariance part is considerably reduced for short– and mid–term forecasts, i.e. for $h = 1$ to $h = 10$ by up to 30% and converges to the values for the single VARs and the equally weighted average for large horizons. However, this reduction decreases with smaller values of N and is negative for $N < 25$ (Model I and II) and $N < 50$ (Model III). Thus, the gain from estimating optimal weights based on the covariance matrix of the forecast errors in practice most likely originates from a reduction of the variance part. In contrast, the covariance part plays the crucial role only in the absence of estimation uncertainty with respect to optimized weights.

MODEL I

MSE (%)

		T = 25										T = 50									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-9.72	-12.49	-14.25	-15.83	77.66	-3.53	-14.62	-18.44	-20.10	-21.38	-9.02	-11.96	-13.26	-13.86	58.25	-4.68	-14.38	-18.40	-20.02	-21.27
	2	-10.55	-14.70	-16.49	-18.16	80.46	-6.14	-16.70	-20.21	-21.89	-23.00	-11.34	-14.57	-15.92	-16.94	58.50	-10.47	-19.77	-23.42	-25.04	-26.19
	3	-9.66	-14.32	-15.58	-17.26	92.00	-0.28	-12.84	-16.92	-18.55	-19.79	-10.16	-13.46	-14.68	-15.62	69.74	-7.45	-17.39	-21.00	-22.62	-23.79
	4	-8.82	-12.88	-14.24	-15.82	101.86	2.37	-9.39	-13.74	-15.52	-16.75	-9.35	-12.16	-13.22	-14.19	70.21	-2.43	-13.40	-16.92	-18.68	-20.00
	5	-8.59	-12.24	-13.74	-15.23	125.96	4.16	-7.13	-11.88	-13.73	-15.08	-8.98	-11.50	-12.73	-13.62	85.96	-0.79	-10.59	-15.02	-16.81	-18.03
	6	-8.09	-11.31	-12.79	-14.18	144.18	8.46	-4.46	-9.53	-11.51	-12.96	-8.19	-10.88	-11.98	-12.86	104.25	3.75	-8.05	-12.63	-14.32	-15.62
	7	-7.81	-10.55	-12.15	-13.34	160.31	11.83	-2.62	-7.49	-9.95	-11.32	-8.13	-10.80	-11.77	-12.54	104.01	5.01	-7.17	-11.21	-13.20	-14.48
	8	-7.39	-9.61	-11.24	-12.21	180.30	13.92	-0.21	-5.39	-7.91	-9.51	-8.33	-10.90	-11.66	-12.44	110.43	5.78	-6.71	-11.02	-12.70	-14.02
	9	-6.51	-8.73	-10.12	-10.99	191.05	18.41	1.14	-4.05	-6.08	-7.92	-8.17	-10.88	-11.58	-12.34	108.14	6.31	-6.02	-10.32	-12.34	-13.65
	10	-6.06	-8.14	-9.49	-10.16	207.48	17.86	2.09	-2.79	-4.92	-6.83	-7.88	-10.68	-11.30	-12.04	121.45	7.23	-5.75	-10.08	-11.92	-13.23
	25	-4.85	-5.89	-6.50	-7.17	236.74	14.55	0.02	-3.56	-4.32	-5.00	-4.04	-5.21	-6.18	-6.49	197.33	14.97	-2.14	-6.48	-8.02	-8.91
	50	-6.09	-8.33	-8.40	-9.09	256.68	11.26	-1.11	-4.62	-5.21	-2.68	-2.17	-2.91	-3.47	-3.77	190.78	17.18	0.50	-3.67	-4.96	-5.43
	100	-5.29	-7.59	-8.29	-8.91	301.10	26.45	8.19	12.45	11.49	27.00	-2.18	-2.88	-3.48	-3.86	201.03	16.99	1.09	-3.48	-4.74	-5.29

VARIANCE (%)

		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	0.25	0.43	0.40	0.44	5.67	0.13	0.27	0.36	0.40	0.44	0.79	0.90	0.97	1.00	1.14	-0.11	0.24	0.45	0.53	0.59
	2	0.75	1.31	1.29	1.41	14.60	0.36	0.84	1.06	1.22	1.32	1.55	1.88	2.06	2.15	4.31	0.11	0.84	1.20	1.40	1.54
	3	1.08	1.91	1.91	2.10	17.79	0.53	1.24	1.70	1.93	2.12	2.41	2.97	3.30	3.47	6.52	0.35	1.78	2.51	2.81	3.09
	4	1.38	2.42	2.46	2.71	24.78	0.60	1.55	2.23	2.59	2.87	2.90	3.63	4.07	4.29	8.74	0.49	2.53	3.61	4.25	4.61
	5	1.51	2.63	2.72	3.01	42.10	0.82	1.64	2.45	2.90	3.27	3.24	4.11	4.65	4.91	13.74	0.96	3.19	4.67	5.31	5.92
	6	1.64	2.80	2.93	3.24	52.37	0.91	1.47	2.50	3.02	3.48	3.58	4.59	5.22	5.53	21.37	0.44	3.56	5.56	6.27	7.02
	7	1.65	2.76	2.94	3.24	62.50	1.32	1.42	2.30	2.93	3.39	3.75	4.86	5.57	5.91	24.52	0.40	4.06	5.89	6.97	7.82
	8	1.71	2.79	3.01	3.30	76.37	1.33	1.20	2.09	2.72	3.26	3.90	5.10	5.87	6.25	22.88	0.61	4.10	6.35	7.48	8.49
	9	1.77	2.81	3.06	3.35	84.86	1.77	1.15	1.99	2.45	3.03	3.99	5.24	6.06	6.46	23.43	0.27	3.99	6.36	7.81	8.86
	10	1.79	2.80	3.07	3.34	92.32	1.61	0.95	1.70	2.18	2.76	4.07	5.37	6.23	6.66	28.89	0.14	4.17	6.73	8.07	9.18
	25	1.20	1.79	1.98	2.20	125.24	2.08	1.16	1.45	1.62	1.64	3.19	4.15	4.93	5.32	67.98	-3.64	1.83	4.81	6.15	7.03
	50	1.30	2.01	2.20	2.41	138.23	3.84	3.17	2.60	2.36	4.04	2.20	2.96	3.51	3.78	55.77	-4.08	1.08	4.10	5.31	5.87
	100	1.23	1.95	2.15	2.40	170.50	13.64	11.13	14.02	15.52	27.49	1.99	2.70	3.26	3.53	64.99	-4.46	0.74	3.82	4.92	5.37

COVARIANCE (%)

		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-9.98	-12.94	-14.66	-16.28	71.98	-3.67	-14.89	-18.81	-20.51	-21.83	-9.82	-12.86	-14.23	-14.85	57.11	-4.57	-14.61	-18.84	-20.54	-21.86
	2	-11.33	-16.04	-17.79	-19.58	65.84	-6.52	-17.55	-21.29	-23.12	-24.34	-12.90	-16.45	-17.98	-19.09	54.18	-10.59	-20.61	-24.62	-26.44	-27.73
	3	-10.75	-16.23	-17.49	-19.36	74.21	-0.82	-14.09	-18.62	-20.48	-21.92	-12.57	-16.43	-17.98	-19.09	63.22	-7.80	-19.18	-23.51	-25.43	-26.87
	4	-10.21	-15.30	-16.70	-18.53	77.08	1.77	-10.94	-15.97	-18.11	-19.62	-12.25	-15.80	-17.29	-18.48	61.46	-2.93	-15.94	-20.54	-22.94	-24.62
	5	-10.11	-14.87	-16.45	-18.23	83.86	3.35	-8.77	-14.33	-16.63	-18.34	-12.23	-15.62	-17.39	-18.54	72.19	-1.78	-13.80	-19.72	-22.15	-23.97
	6	-9.72	-14.11	-15.72	-17.42	91.80	7.54	-5.94	-12.04	-14.53	-16.45	-11.78	-15.48	-17.22	-18.40	82.86	3.29	-11.62	-18.21	-20.60	-22.66
	7	-9.46	-13.31	-15.10	-16.58	97.77	10.50	-4.05	-9.80	-12.90	-14.73	-11.90	-15.67	-17.35	-18.46	79.47	4.60	-11.25	-17.11	-20.18	-22.31
	8	-9.11	-12.41	-14.26	-15.53	103.84	12.56	-1.44	-7.50	-10.65	-12.79	-12.25	-16.00	-17.53	-18.69	87.54	5.17	-10.82	-17.38	-20.19	-22.51
	9	-8.29	-11.56	-13.20	-14.36	106.05	16.59	-0.05	-6.08	-8.58	-10.99	-12.17	-16.13	-17.64	-18.80	84.70	6.04	-10.02	-16.68	-20.15	-22.51
	10	-7.87	-10.97	-12.58	-13.52	114.96	16.20	1.09	-4.53	-7.14	-9.63	-11.96	-16.05	-17.54	-18.70	92.54	7.08	-9.92	-16.82	-19.99	-22.41
	25	-6.05	-7.68	-8.48	-9.37	111.48	12.47	-1.13	-5.01	-5.95	-6.64	-7.24	-9.36	-11.11	-11.81	129.24	18.60	-3.97	-11.29	-14.16	-15.94
	50	-7.39	-10.34	-10.60	-11.50	118.38	7.41	-4.28	-7.21	-7.57	-6.72	-4.38	-5.88	-6.99	-7.55	134.94	21.26	-0.58	-7.77	-10.26	-11.30
	100	-6.52	-9.54	-10.44	-11.32	130.42	12.76	-2.98	-1.61	-4.06	-0.53	-4.17	-5.58	-6.74	-7.39	135.93	21.43	0.34	-7.30	-9.66	-10.66

Table 3: Gains from pooling of forecasts (*Model I*)

MODEL I																					
		T = 100										T = 200									
		MSE (%)										MSE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-7.11	-10.00	-11.39	-12.15	72.20	-3.61	-14.15	-18.44	-20.22	-21.58	-6.84	-9.23	-10.03	-10.74	70.50	-3.14	-13.22	-17.18	-19.03	-20.39
	2	-9.14	-12.44	-13.82	-14.74	50.56	-8.83	-19.29	-22.92	-24.64	-25.98	-8.27	-10.83	-11.87	-12.83	54.97	-9.16	-19.06	-22.86	-24.54	-25.89
	3	-8.41	-11.40	-12.86	-13.61	67.74	-7.97	-17.99	-21.74	-23.58	-24.93	-7.45	-10.03	-10.93	-11.79	60.23	-7.33	-17.07	-21.24	-22.99	-24.26
	4	-8.25	-10.30	-11.95	-12.47	70.06	-3.07	-14.47	-18.42	-20.26	-21.73	-6.48	-9.03	-10.09	-10.80	70.81	-3.22	-13.76	-18.16	-19.88	-21.27
	5	-7.45	-9.28	-10.85	-11.37	78.74	0.30	-10.98	-15.49	-17.42	-18.93	-5.42	-7.81	-8.93	-9.48	76.57	0.47	-10.50	-14.73	-16.71	-18.15
	6	-6.29	-7.84	-9.48	-9.96	92.58	5.39	-6.59	-11.55	-13.66	-15.24	-5.45	-7.65	-8.75	-9.08	83.49	2.58	-8.43	-12.63	-14.89	-16.28
	7	-5.93	-7.52	-8.93	-9.45	105.87	7.68	-4.59	-9.77	-11.81	-13.36	-5.24	-7.17	-8.20	-8.48	85.89	3.44	-7.15	-11.56	-13.59	-15.04
	8	-5.47	-7.14	-8.45	-8.96	99.88	8.75	-3.52	-8.56	-10.52	-12.14	-4.97	-6.93	-7.93	-8.19	81.51	5.98	-5.49	-10.01	-12.06	-13.65
	9	-5.24	-6.93	-8.17	-8.61	112.53	11.46	-2.88	-7.52	-9.69	-11.29	-4.55	-6.57	-7.58	-7.82	89.92	7.23	-4.05	-9.18	-11.14	-12.73
	10	-5.14	-6.61	-7.87	-8.29	108.28	11.22	-1.50	-6.57	-8.79	-10.38	-4.41	-6.46	-7.32	-7.58	95.72	9.55	-3.29	-8.35	-10.57	-12.10
	25	-2.73	-3.71	-4.74	-4.92	180.23	18.62	2.85	-2.99	-5.18	-6.57	-3.11	-3.68	-4.17	-4.45	190.35	20.26	4.10	-1.68	-4.09	-5.75
	50	-1.97	-2.49	-2.87	-2.88	174.52	23.09	5.72	-0.13	-2.18	-3.61	-1.39	-1.15	-1.62	-1.67	182.51	26.06	9.05	2.53	0.03	-1.72
	100	-0.82	-0.78	-0.88	-1.04	153.62	20.45	6.44	1.58	-0.51	-1.74	-0.62	-0.66	-0.61	-0.63	130.62	21.04	7.88	2.64	0.59	-0.90
		VARIANCE (%)										VARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	0.13	0.19	0.25	0.27	3.97	-0.81	-0.62	-0.52	-0.47	-0.43	0.38	0.58	0.56	0.60	1.18	-0.54	-0.36	-0.24	-0.19	-0.15
	2	0.60	0.81	0.97	1.05	1.17	-1.19	-0.68	-0.42	-0.30	-0.19	0.91	1.34	1.37	1.47	2.83	-0.67	-0.17	0.09	0.21	0.30
	3	1.04	1.40	1.64	1.78	5.91	-1.10	-0.30	0.22	0.43	0.62	1.41	2.05	2.18	2.32	3.13	-0.50	0.31	0.82	1.19	1.31
	4	1.50	2.00	2.31	2.52	6.83	-1.15	0.48	1.30	1.71	2.07	1.81	2.62	2.83	3.01	6.87	-0.48	1.20	2.20	2.57	2.92
	5	1.95	2.60	2.99	3.25	7.39	-0.98	1.21	2.42	3.02	3.59	2.16	3.11	3.41	3.62	8.38	-0.16	2.30	3.53	4.25	4.82
	6	2.40	3.21	3.67	4.00	10.80	-1.20	1.87	3.82	4.81	5.62	2.48	3.56	3.96	4.20	8.61	0.17	3.25	5.06	6.21	6.93
	7	2.79	3.76	4.27	4.65	16.00	-1.49	2.29	4.90	6.10	7.04	2.89	4.14	4.65	4.92	5.70	-0.05	4.14	6.51	7.92	8.89
	8	3.19	4.33	4.91	5.34	10.92	-1.54	2.88	5.92	7.35	8.62	3.28	4.69	5.32	5.62	3.48	-0.79	4.57	7.78	9.45	10.92
	9	3.58	4.89	5.53	6.01	19.77	-2.28	3.38	6.65	8.58	9.99	3.65	5.23	5.98	6.31	5.62	-1.17	4.59	8.96	10.78	12.55
	10	3.89	5.36	6.05	6.56	12.44	-2.47	3.67	7.47	9.44	11.03	4.05	5.79	6.67	7.05	6.86	-2.63	4.83	9.73	12.22	14.42
	25	4.70	6.31	7.04	7.60	34.43	-9.62	-1.16	4.98	8.22	10.65	5.59	7.67	9.07	9.70	24.22	-16.15	-4.54	3.75	8.65	12.98
	50	2.77	3.61	3.82	4.14	18.25	-16.25	-6.25	0.26	3.34	5.86	2.74	3.70	4.35	4.58	9.47	-27.21	-14.60	-5.02	0.04	4.73
	100	1.70	2.11	2.22	2.43	7.80	-17.01	-7.79	-1.99	1.29	3.68	0.82	1.22	1.36	1.40	-20.19	-28.21	-15.83	-7.42	-2.93	1.02
		COVARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-7.25	-10.19	-11.66	-12.42	68.22	-2.79	-13.53	-17.92	-19.74	-21.14	-7.21	-9.80	-10.58	-11.33	69.30	-2.60	-12.85	-16.93	-18.83	-20.23
	2	-9.76	-13.27	-14.82	-15.81	49.38	-7.64	-18.61	-22.49	-24.33	-25.78	-9.16	-12.15	-13.24	-14.29	52.13	-8.48	-18.88	-22.94	-24.74	-26.18
	3	-9.47	-12.82	-14.52	-15.41	61.82	-6.88	-17.69	-21.96	-24.01	-25.54	-8.86	-12.08	-13.10	-14.11	57.08	-6.83	-17.36	-22.05	-24.17	-25.57
	4	-9.76	-12.31	-14.27	-15.00	63.22	-1.93	-14.95	-19.73	-21.98	-23.80	-8.29	-11.65	-12.92	-13.81	63.91	-2.75	-14.97	-20.36	-22.46	-24.19
	5	-9.41	-11.90	-13.85	-14.64	71.33	1.26	-12.20	-17.93	-20.45	-22.54	-7.58	-10.92	-12.34	-13.11	68.15	0.63	-12.81	-18.27	-20.96	-22.96
	6	-8.70	-11.08	-13.16	-13.97	81.75	6.56	-8.48	-15.39	-18.48	-20.88	-7.94	-11.22	-12.71	-13.28	74.86	2.42	-11.68	-17.69	-21.08	-23.19
	7	-8.73	-11.29	-13.21	-14.11	89.83	9.14	-6.90	-14.69	-17.93	-20.42	-8.13	-11.32	-12.85	-13.41	80.16	3.49	-11.29	-18.07	-21.51	-23.92
	8	-8.68	-11.49	-13.37	-14.31	88.91	10.27	-6.42	-14.50	-17.88	-20.78	-8.26	-11.63	-13.25	-13.82	78.00	6.77	-10.06	-17.78	-21.50	-24.56
	9	-8.83	-11.83	-13.71	-14.63	92.71	13.72	-6.28	-14.20	-18.29	-21.30	-8.22	-11.82	-13.58	-14.15	84.29	8.41	-8.62	-18.13	-21.90	-25.26
	10	-9.03	-11.96	-13.92	-14.86	95.80	13.67	-5.18	-14.05	-18.23	-21.42	-8.48	-12.29	-14.02	-14.65	88.83	12.19	-8.11	-18.07	-22.78	-26.51
	25	-7.41	-10.01	-11.76	-12.51	145.80	28.25	4.03	-7.95	-13.38	-17.21	-8.70	-11.36	-13.24	-14.15	166.08	36.41	8.64	-5.43	-12.74	-18.73
	50	-4.73	-6.10	-6.68	-7.01	156.26	39.35	11.98	-0.39	-5.52	-9.46	-4.14	-4.85	-5.97	-6.25	173.02	53.27	23.65	7.56	0.00	-6.45
	100	-2.51	-2.88	-3.09	-3.46	145.82	37.47	14.24	3.58	-1.79	-5.41	-1.44	-1.88	-1.97	-2.02	150.78	49.25	23.71	10.06	3.52	-1.91

Table 3 continued: Gains from pooling of forecasts (*Model I*)

MODEL I											
		T = 1000									
		MSE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	-7.21	-8.95	-10.40	-10.71	54.48	-7.27	-16.20	-19.56	-21.50	-22.75
	2	-8.00	-9.51	-11.24	-11.78	50.92	-10.44	-19.56	-23.04	-24.75	-25.93
	3	-6.72	-8.13	-9.77	-10.24	50.35	-8.12	-17.60	-21.24	-22.95	-24.22
	4	-5.79	-6.88	-8.22	-8.66	57.66	-4.91	-15.07	-18.43	-20.26	-21.68
	5	-4.52	-5.67	-6.69	-7.10	57.50	-1.26	-11.44	-15.38	-17.19	-18.61
	6	-3.91	-5.01	-5.80	-6.21	69.14	0.15	-9.10	-13.46	-15.37	-16.81
	7	-3.41	-4.44	-5.15	-5.53	78.46	3.14	-7.16	-11.42	-13.40	-14.83
	8	-2.80	-4.09	-4.64	-5.03	82.79	6.19	-4.89	-9.15	-10.97	-12.52
	9	-2.78	-4.08	-4.50	-4.85	79.37	7.62	-3.66	-7.87	-9.90	-11.42
	10	-2.85	-4.04	-4.47	-4.76	87.17	8.82	-2.53	-6.96	-9.04	-10.50
	25	-0.98	-1.43	-1.78	-1.93	127.07	20.61	6.76	1.32	-0.92	-2.57
	50	-0.50	-1.09	-1.03	-1.09	122.66	21.29	8.45	3.12	0.68	-1.07
	100	0.01	-0.02	-0.02	-0.03	117.87	21.97	9.34	4.10	1.88	0.29
		VARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	0.42	0.49	0.54	0.62	-0.67	-1.54	-1.29	-1.20	-1.11	-1.05
	2	0.90	1.11	1.24	1.39	-0.06	-1.95	-1.40	-1.15	-1.05	-0.94
	3	1.37	1.72	1.93	2.15	-0.50	-2.40	-1.29	-0.73	-0.53	-0.32
	4	1.74	2.22	2.49	2.75	1.17	-2.48	-0.86	0.02	0.55	0.91
	5	2.10	2.68	3.01	3.32	-3.67	-2.75	-0.09	1.29	2.01	2.59
	6	2.48	3.16	3.57	3.91	-1.47	-2.45	0.85	2.94	4.03	4.98
	7	2.88	3.68	4.17	4.56	1.26	-2.86	2.19	4.94	6.37	7.72
	8	3.31	4.24	4.82	5.25	-2.03	-3.09	3.38	7.31	9.31	10.90
	9	3.76	4.84	5.51	5.98	-7.59	-3.57	4.57	8.92	11.58	13.72
	10	4.24	5.47	6.26	6.78	-5.62	-4.18	5.25	11.04	14.13	17.01
	25	6.89	9.72	11.27	12.18	-26.83	-28.04	-11.91	-0.41	7.31	15.52
	50	3.56	5.12	5.77	6.21	-42.65	-39.03	-23.93	-12.30	-4.19	4.55
	100	0.43	0.54	0.61	0.64	-48.69	-45.23	-30.05	-18.22	-10.63	-2.38
		COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	-7.63	-9.44	-10.94	-11.33	55.14	-5.74	-14.91	-18.36	-20.39	-21.70
	2	-8.89	-10.62	-12.48	-13.17	50.97	-8.49	-18.17	-21.89	-23.70	-24.99
	3	-8.08	-9.84	-11.68	-12.37	50.82	-5.74	-16.32	-20.52	-22.43	-23.91
	4	-7.53	-9.09	-10.70	-11.40	56.47	-2.44	-14.22	-18.46	-20.81	-22.60
	5	-6.62	-8.34	-9.70	-10.41	61.16	1.49	-11.35	-16.67	-19.19	-21.21
	6	-6.38	-8.16	-9.36	-10.12	70.59	2.59	-9.95	-16.40	-19.41	-21.79
	7	-6.29	-8.11	-9.31	-10.08	77.19	6.00	-9.35	-16.37	-19.76	-22.55
	8	-6.10	-8.32	-9.45	-10.26	84.81	9.29	-8.28	-16.47	-20.28	-23.41
	9	-6.53	-8.91	-10.00	-10.83	86.95	11.19	-8.23	-16.79	-21.48	-25.14
	10	-7.08	-9.50	-10.72	-11.53	92.79	13.00	-7.77	-18.00	-23.17	-27.51
	25	-7.87	-11.15	-13.05	-14.11	153.89	48.64	18.67	1.73	-8.24	-18.09
	50	-4.05	-6.21	-6.80	-7.30	165.31	60.32	32.39	15.42	4.87	-5.62
	100	-0.41	-0.56	-0.63	-0.67	166.53	67.19	39.39	22.32	12.51	2.67

Table 3 continued: Gains from pooling of forecasts (*Model I*)

		MODEL II																			
		T = 25										T = 50									
		MSE (%)										MSE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-10.09	-12.80	-14.34	-15.28	73.91	-2.26	-13.07	-17.15	-18.83	-20.12	-7.34	-9.87	-11.19	-11.93	73.95	-1.65	-12.40	-16.44	-18.39	-19.78
	2	-10.78	-14.19	-15.95	-17.03	76.17	-3.08	-14.83	-18.60	-20.45	-21.74	-8.14	-11.77	-12.66	-13.72	79.39	-5.38	-15.51	-19.52	-21.40	-22.71
	3	-10.03	-12.91	-14.52	-15.60	103.36	0.98	-11.11	-15.30	-17.29	-18.58	-7.12	-10.71	-11.42	-12.28	95.93	-1.34	-12.71	-16.94	-18.83	-20.11
	4	-8.79	-11.23	-12.57	-13.61	134.07	7.05	-6.23	-11.13	-13.17	-14.66	-6.54	-9.61	-10.28	-11.02	93.71	3.57	-8.65	-13.12	-14.89	-16.32
	5	-7.66	-9.92	-11.03	-12.02	144.01	10.62	-3.09	-8.15	-9.95	-11.65	-6.01	-8.95	-9.52	-10.24	110.82	7.33	-5.96	-10.69	-12.58	-14.04
	6	-6.59	-8.58	-9.46	-10.43	191.95	15.69	-0.22	-5.18	-7.09	-8.87	-5.51	-8.06	-8.44	-9.09	134.80	10.47	-3.76	-8.37	-10.41	-11.85
	7	-6.03	-7.83	-8.53	-9.46	194.05	17.31	1.78	-3.55	-5.41	-7.16	-5.30	-7.49	-7.90	-8.50	125.56	11.30	-2.44	-7.33	-9.41	-10.85
	8	-5.29	-6.91	-7.48	-8.35	211.27	20.06	3.03	-2.14	-3.89	-5.67	-5.11	-6.90	-7.30	-7.89	157.24	14.50	-1.74	-6.34	-8.46	-9.92
	9	-4.93	-6.59	-7.07	-7.82	226.89	22.66	4.18	-1.33	-3.22	-4.94	-4.91	-6.33	-6.87	-7.41	161.52	14.64	-0.17	-5.44	-7.53	-9.00
	10	-4.31	-5.94	-6.44	-7.14	234.33	24.05	4.44	-0.52	-2.69	-4.28	-4.98	-6.11	-6.69	-7.17	174.75	15.93	0.95	-5.16	-7.18	-8.71
	25	-2.66	-3.17	-3.79	-4.21	286.16	18.78	2.37	-1.83	-3.85	-5.56	-1.97	-2.41	-2.95	-3.12	316.93	27.18	4.13	-2.03	-4.16	-5.56
	50	-1.37	-2.06	-2.80	-3.31	302.85	23.40	3.85	2.61	0.87	-1.73	-1.19	-1.17	-1.51	-1.69	333.97	27.86	5.25	-1.37	-3.40	-4.22
	100	-1.19	-2.42	-2.92	-3.46	346.64	18.67	3.37	5.05	7.54	1.27	-1.18	-1.83	-1.93	-2.05	248.16	27.73	5.53	-1.14	-2.78	-3.44
		VARIANCE (%)										VARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	0.47	0.55	0.60	0.67	4.41	0.03	0.35	0.52	0.59	0.65	0.39	0.53	0.57	0.59	2.73	-0.37	-0.09	0.07	0.15	0.21
	2	0.92	1.13	1.26	1.41	8.43	0.22	0.72	1.00	1.20	1.32	0.98	1.30	1.43	1.49	9.22	-0.30	0.31	0.67	0.87	1.02
	3	1.32	1.66	1.85	2.07	22.98	0.25	0.95	1.48	1.77	1.99	1.52	1.99	2.20	2.32	15.15	-0.22	0.96	1.61	1.96	2.22
	4	1.46	1.90	2.12	2.38	37.75	0.07	0.96	1.73	2.07	2.42	1.94	2.53	2.82	2.97	12.97	-0.35	1.47	2.64	3.05	3.50
	5	1.49	1.99	2.22	2.49	44.19	0.09	0.94	1.74	2.18	2.59	2.23	2.90	3.24	3.43	21.22	-0.37	1.92	3.22	3.94	4.51
	6	1.43	1.96	2.18	2.46	78.12	0.28	0.68	1.56	2.02	2.49	2.46	3.19	3.56	3.78	34.61	-0.76	2.01	3.61	4.58	5.29
	7	1.27	1.79	1.99	2.25	79.54	0.47	0.45	1.29	1.76	2.23	2.59	3.34	3.74	3.98	28.97	-1.23	1.88	3.89	4.94	5.75
	8	1.21	1.73	1.92	2.18	92.83	0.41	0.32	1.15	1.58	2.10	2.71	3.49	3.90	4.16	46.70	-1.80	1.81	4.00	5.15	6.11
	9	1.09	1.59	1.76	2.00	97.84	0.72	0.06	0.87	1.32	1.82	2.75	3.52	3.93	4.21	48.47	-2.19	1.55	4.04	5.33	6.40
	10	1.00	1.45	1.62	1.84	106.03	1.25	-0.01	0.68	1.15	1.61	2.74	3.48	3.90	4.19	54.57	-2.93	0.92	3.77	5.19	6.35
	25	0.52	0.69	0.83	0.94	158.82	1.55	0.37	0.59	0.88	1.26	1.46	1.78	2.08	2.25	132.81	-6.22	-2.03	1.58	3.31	4.64
	50	0.53	0.79	0.96	1.09	158.51	4.15	1.48	1.80	1.31	1.12	1.07	1.43	1.56	1.71	140.97	-7.27	-2.58	1.42	3.15	4.01
	100	0.51	0.80	1.00	1.13	194.61	2.09	1.69	4.48	4.97	2.28	0.99	1.36	1.48	1.61	80.74	-7.97	-2.56	1.61	2.97	3.53
		COVARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-10.57	-13.35	-14.95	-15.95	69.49	-2.30	-13.43	-17.68	-19.43	-20.78	-7.73	-10.40	-11.76	-12.51	71.20	-1.29	-12.32	-16.52	-18.55	-19.99
	2	-11.72	-15.32	-17.22	-18.45	67.72	-3.31	-15.56	-19.61	-21.66	-23.07	-9.12	-13.07	-14.08	-15.21	70.14	-5.10	-15.83	-20.21	-22.28	-23.74
	3	-11.35	-14.56	-16.38	-17.67	80.36	0.73	-12.05	-16.78	-19.07	-20.57	-8.63	-12.70	-13.62	-14.59	80.74	-1.13	-13.68	-18.55	-20.80	-22.34
	4	-10.26	-13.12	-14.69	-15.99	96.29	6.98	-7.19	-12.86	-15.24	-17.08	-8.47	-12.14	-13.09	-13.99	80.72	3.91	-10.12	-15.77	-17.95	-19.82
	5	-9.14	-11.90	-13.25	-14.52	99.79	10.52	-4.03	-9.88	-12.13	-14.23	-8.22	-11.83	-12.74	-13.66	89.56	7.69	-7.89	-13.91	-16.52	-18.55
	6	-8.01	-10.53	-11.63	-12.89	113.78	15.41	-0.88	-6.74	-9.10	-11.36	-7.96	-11.23	-12.00	-12.87	100.16	11.22	-5.77	-11.98	-14.99	-17.14
	7	-7.29	-9.61	-10.51	-11.70	114.47	16.85	1.34	-4.82	-7.16	-9.37	-7.87	-10.82	-11.62	-12.47	96.55	12.53	-4.32	-11.23	-14.35	-16.59
	8	-6.49	-8.63	-9.39	-10.52	118.40	19.66	2.72	-3.27	-5.45	-7.75	-7.81	-10.37	-11.19	-12.04	110.50	16.30	-3.55	-10.34	-13.61	-16.02
	9	-6.01	-8.17	-8.82	-9.81	129.03	21.95	4.13	-2.19	-4.52	-6.74	-7.65	-9.83	-10.79	-11.61	113.01	16.83	-1.72	-9.47	-12.85	-15.39
	10	-5.31	-7.39	-8.07	-8.98	128.26	22.81	4.46	-1.20	-3.83	-5.88	-7.71	-9.58	-10.58	-11.35	120.13	18.86	0.04	-8.93	-12.36	-15.05
	25	-3.18	-3.86	-4.61	-5.14	127.26	17.22	2.00	-2.42	-4.73	-6.82	-3.43	-4.19	-5.03	-5.36	184.03	33.40	6.16	-3.62	-7.47	-10.19
	50	-1.91	-2.85	-3.77	-4.40	144.27	19.24	2.37	0.81	-0.45	-2.85	-2.25	-2.59	-3.07	-3.40	192.91	35.13	7.84	-2.78	-6.54	-8.22
	100	-1.70	-3.23	-3.93	-4.59	151.96	16.57	1.67	0.57	2.56	-1.02	-2.16	-3.18	-3.41	-3.66	167.36	35.70	8.09	-2.74	-5.74	-6.96

Table 4: Gains from pooling of forecasts (*Model II*)

		T = 100										T = 200									
MODEL II																					
		MSE (%)										MSE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-7.31	-10.12	-11.03	-11.59	75.80	-2.44	-12.82	-16.55	-18.46	-19.83	-5.52	-8.09	-9.29	-9.84	64.99	-0.93	-11.81	-15.86	-17.61	-18.94
	2	-8.50	-11.56	-12.32	-13.14	60.75	-6.70	-16.42	-20.09	-21.87	-23.13	-6.71	-9.85	-11.12	-11.79	57.10	-6.10	-16.09	-20.26	-21.83	-23.08
	3	-7.93	-10.58	-11.33	-12.24	60.41	-4.22	-14.67	-18.72	-20.48	-21.88	-6.43	-9.20	-10.20	-11.01	60.37	-5.99	-15.39	-18.98	-20.80	-22.06
	4	-6.83	-9.38	-10.15	-10.81	74.15	-0.24	-11.43	-15.43	-17.31	-18.67	-5.92	-8.03	-9.20	-9.76	62.40	-2.35	-12.12	-16.33	-18.21	-19.53
	5	-6.07	-8.44	-9.11	-9.69	77.86	3.15	-8.55	-12.64	-14.55	-16.00	-5.41	-6.98	-8.13	-8.60	72.73	0.56	-9.54	-13.54	-15.38	-16.80
	6	-5.53	-7.58	-8.24	-8.76	88.14	6.34	-5.56	-10.03	-11.98	-13.49	-5.16	-6.64	-7.47	-7.94	79.12	3.54	-7.06	-11.24	-13.28	-14.72
	7	-5.18	-6.81	-7.44	-7.89	91.63	8.98	-3.41	-7.92	-9.90	-11.50	-4.41	-5.88	-6.62	-7.04	87.06	5.68	-5.22	-9.69	-11.58	-13.02
	8	-4.83	-5.95	-6.64	-7.01	113.45	11.38	-1.52	-6.39	-8.49	-10.06	-4.13	-5.42	-5.97	-6.36	91.93	7.55	-3.87	-8.01	-10.05	-11.56
	9	-4.49	-5.27	-6.02	-6.35	119.61	13.82	-0.66	-5.59	-7.61	-9.17	-3.82	-5.11	-5.51	-5.85	91.13	8.24	-2.47	-6.98	-9.11	-10.58
	10	-4.03	-4.59	-5.31	-5.71	111.16	14.20	0.23	-4.72	-6.80	-8.30	-3.46	-4.71	-5.07	-5.41	95.43	11.03	-1.45	-5.91	-7.85	-9.39
	25	-1.13	-1.40	-1.43	-1.62	225.36	30.45	7.76	0.69	-1.69	-3.49	-0.67	-0.85	-0.99	-1.09	191.83	27.91	8.42	2.68	0.06	-1.77
	50	-0.24	-0.26	-0.43	-0.45	223.18	30.41	10.46	3.30	0.50	-1.35	-0.24	-0.39	-0.32	-0.33	180.91	31.19	11.20	4.31	1.40	-0.40
	100	-0.46	-0.59	-0.62	-0.62	159.68	26.41	8.89	2.72	0.21	-1.37	-0.15	-0.21	-0.21	-0.21	135.84	24.19	9.69	3.85	1.45	-0.27
		VARIANCE (%)										VARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	0.49	0.49	0.65	0.67	2.48	-0.39	-0.03	0.12	0.21	0.26	0.49	0.54	0.56	0.62	0.65	-0.87	-0.61	-0.48	-0.41	-0.38
	2	1.03	1.15	1.45	1.50	2.63	-0.32	0.40	0.70	0.88	1.03	1.03	1.21	1.32	1.44	1.82	-1.13	-0.47	-0.12	0.00	0.13
	3	1.47	1.71	2.11	2.20	3.54	-0.24	1.01	1.65	1.95	2.23	1.49	1.79	2.00	2.16	3.22	-0.99	0.00	0.59	0.86	1.10
	4	1.79	2.14	2.59	2.72	6.57	-0.25	1.81	2.76	3.21	3.63	1.86	2.28	2.56	2.76	1.77	-1.02	0.72	1.70	2.16	2.48
	5	2.05	2.49	2.98	3.14	7.45	-0.06	2.62	3.96	4.54	5.15	2.12	2.65	3.00	3.23	5.04	-0.81	1.73	2.93	3.68	4.21
	6	2.31	2.84	3.36	3.55	8.83	-0.43	3.00	4.80	5.76	6.53	2.39	3.02	3.44	3.69	3.08	-0.85	2.38	4.43	5.38	6.27
	7	2.54	3.13	3.68	3.91	7.27	-1.10	3.09	5.22	6.57	7.52	2.66	3.39	3.88	4.15	7.22	-1.28	3.21	5.99	7.13	8.24
	8	2.72	3.36	3.93	4.19	17.18	-1.94	2.92	5.94	7.32	8.68	2.85	3.68	4.22	4.50	5.88	-2.21	3.59	6.64	8.39	9.83
	9	2.85	3.53	4.11	4.40	17.38	-3.11	2.65	6.22	8.02	9.55	3.04	3.95	4.55	4.84	2.03	-2.75	3.58	7.41	9.59	11.21
	10	2.95	3.64	4.24	4.54	12.87	-4.09	2.53	6.56	8.40	10.08	3.21	4.21	4.85	5.16	-1.06	-4.49	3.33	7.92	10.26	12.40
	25	1.70	2.10	2.42	2.58	45.92	-16.20	-7.60	-0.84	2.48	5.50	2.05	2.52	2.98	3.13	11.63	-25.83	-12.45	-4.16	0.94	5.62
	50	0.80	1.00	1.15	1.21	35.98	-20.35	-11.14	-4.04	-0.18	2.97	0.56	0.60	0.71	0.76	-4.69	-32.07	-18.95	-9.74	-3.94	0.78
	100	0.57	0.73	0.83	0.88	4.97	-20.30	-10.62	-3.98	-0.27	2.54	0.32	0.32	0.39	0.41	-19.92	-30.68	-17.98	-9.30	-4.24	0.20
		COVARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-7.78	-10.61	-11.68	-12.25	73.29	-2.06	-12.79	-16.67	-18.67	-20.09	-6.01	-8.63	-9.85	-10.46	64.33	-0.05	-11.20	-15.37	-17.19	-18.56
	2	-9.52	-12.71	-13.77	-14.63	58.10	-6.38	-16.82	-20.79	-22.74	-24.15	-7.75	-11.06	-12.45	-13.23	55.29	-4.96	-15.61	-20.13	-21.82	-23.19
	3	-9.39	-12.30	-13.44	-14.43	56.85	-3.98	-15.68	-20.37	-22.43	-24.10	-7.92	-10.99	-12.20	-13.17	57.14	-5.00	-15.39	-19.57	-21.66	-23.16
	4	-8.62	-11.53	-12.74	-13.52	67.54	0.00	-13.25	-18.18	-20.52	-22.30	-7.77	-10.31	-11.77	-12.52	60.61	-1.33	-12.85	-18.03	-20.37	-22.01
	5	-8.12	-10.95	-12.08	-12.82	70.38	3.21	-11.17	-16.60	-19.09	-21.15	-7.53	-9.63	-11.13	-11.82	67.67	1.37	-11.28	-16.48	-19.07	-21.02
	6	-7.84	-10.43	-11.60	-12.31	79.27	6.76	-8.57	-14.83	-17.74	-20.02	-7.55	-9.66	-10.92	-11.63	76.01	4.38	-9.45	-15.68	-18.67	-21.00
	7	-7.72	-9.95	-11.11	-11.80	84.33	10.07	-6.50	-13.14	-16.48	-19.02	-7.06	-9.28	-10.50	-11.19	79.80	6.94	-8.44	-15.70	-18.72	-21.27
	8	-7.54	-9.32	-10.57	-11.20	96.23	13.31	-4.45	-12.34	-15.81	-18.75	-6.98	-9.10	-10.20	-10.86	86.02	9.74	-7.47	-14.67	-18.45	-21.41
	9	-7.34	-8.80	-10.12	-10.75	102.19	16.93	-3.32	-11.81	-15.63	-18.72	-6.86	-9.07	-10.06	-10.70	89.08	10.98	-6.06	-14.40	-18.71	-21.80
	10	-6.98	-8.23	-9.54	-10.25	98.25	18.28	-2.30	-11.28	-15.20	-18.37	-6.67	-8.92	-9.93	-10.56	96.47	15.51	-4.78	-13.84	-18.11	-21.80
	25	-2.82	-3.50	-3.85	-4.20	179.41	46.65	15.36	1.53	-4.18	-8.99	-2.72	-3.37	-3.98	-4.22	180.17	53.73	20.87	6.83	-0.88	-7.39
	50	-1.05	-1.26	-1.57	-1.66	187.18	50.76	21.60	7.34	0.68	-4.32	-0.80	-0.99	-1.04	-1.08	185.59	63.26	30.15	14.04	5.34	-1.18
	100	-1.03	-1.32	-1.46	-1.50	154.68	46.70	19.51	6.69	0.48	-3.91	-0.47	-0.53	-0.60	-0.62	155.74	54.86	27.66	13.14	5.69	-0.48

Table 4 continued: Gains from pooling of forecasts (*Model II*)

		MODEL II									
		T = 1000									
		MSE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	-4.55	-6.68	-7.80	-8.41	59.74	-4.16	-13.01	-16.71	-18.47	-19.81
	2	-5.96	-8.21	-9.38	-10.21	57.25	-8.10	-16.67	-20.57	-22.17	-23.45
	3	-5.45	-7.99	-8.91	-9.76	59.32	-5.86	-15.51	-18.88	-20.64	-21.95
	4	-4.94	-6.80	-7.58	-8.46	59.85	-1.82	-12.54	-16.20	-17.91	-19.30
	5	-4.25	-5.64	-6.39	-7.19	68.51	0.90	-9.42	-13.35	-15.14	-16.51
	6	-3.69	-4.95	-5.58	-6.29	75.27	2.74	-7.06	-11.31	-13.11	-14.54
	7	-3.36	-4.40	-4.95	-5.57	70.00	5.44	-4.13	-8.99	-10.96	-12.38
	8	-3.07	-4.16	-4.57	-5.10	79.61	7.20	-2.99	-7.22	-9.17	-10.60
	9	-3.03	-4.03	-4.31	-4.79	85.36	9.79	-2.01	-6.12	-8.20	-9.67
	10	-2.91	-3.79	-4.09	-4.44	76.04	8.77	-1.12	-5.63	-7.68	-9.15
	25	-0.33	-0.46	-0.60	-0.68	115.15	21.54	7.92	2.95	0.72	-0.95
	50	-0.06	-0.07	-0.03	-0.04	139.62	25.51	9.95	4.41	1.98	0.16
	100	-0.01	0.00	0.00	-0.01	108.73	21.57	8.99	4.16	1.94	0.29
		VARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	0.31	0.35	0.38	0.46	-0.59	-1.33	-1.09	-0.93	-0.86	-0.81
	2	0.83	1.03	1.14	1.29	0.94	-1.83	-1.19	-0.86	-0.71	-0.60
	3	1.27	1.63	1.82	2.01	1.88	-1.86	-0.56	-0.12	0.25	0.41
	4	1.66	2.18	2.43	2.67	0.31	-1.69	0.09	1.13	1.60	1.95
	5	2.01	2.66	2.97	3.24	1.03	-2.05	1.12	2.50	3.22	3.73
	6	2.35	3.14	3.51	3.81	-0.16	-2.00	2.15	4.30	5.09	6.06
	7	2.64	3.57	3.98	4.31	-4.90	-2.04	2.59	6.07	7.92	8.95
	8	2.91	3.96	4.42	4.77	-4.96	-1.63	4.15	8.03	10.04	11.65
	9	3.16	4.33	4.83	5.21	-4.58	-3.53	5.03	9.40	12.18	14.28
	10	3.41	4.70	5.24	5.64	-12.07	-4.11	4.82	10.41	13.70	16.47
	25	2.31	3.49	3.81	4.08	-42.73	-37.77	-21.93	-10.76	-3.37	5.06
	50	0.10	0.18	0.18	0.19	-44.96	-48.51	-31.72	-19.99	-12.14	-3.27
	100	0.03	0.03	0.03	0.03	-55.06	-45.73	-30.11	-18.82	-11.17	-2.71
		COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	-4.89	-7.06	-8.21	-8.89	60.31	-2.85	-11.93	-15.79	-17.63	-19.01
	2	-6.79	-9.25	-10.53	-11.51	56.31	-6.27	-15.48	-19.72	-21.46	-22.85
	3	-6.71	-9.61	-10.71	-11.76	57.45	-3.99	-14.93	-18.75	-20.88	-22.35
	4	-6.59	-8.96	-9.99	-11.11	59.56	-0.11	-12.61	-17.31	-19.49	-21.22
	5	-6.25	-8.29	-9.36	-10.42	67.49	2.96	-10.53	-15.84	-18.36	-20.22
	6	-6.02	-8.08	-9.08	-10.08	75.44	4.75	-9.20	-15.60	-18.19	-20.58
	7	-6.00	-7.96	-8.93	-9.87	74.90	7.49	-6.72	-15.06	-18.87	-21.32
	8	-5.98	-8.11	-8.98	-9.87	84.57	8.83	-7.13	-15.24	-19.21	-22.24
	9	-6.18	-8.35	-9.14	-9.98	89.95	13.33	-7.03	-15.51	-20.38	-23.94
	10	-6.30	-8.47	-9.31	-10.06	88.12	12.90	-5.92	-16.02	-21.36	-25.59
	25	-2.65	-3.95	-4.42	-4.77	157.83	59.29	29.83	13.70	4.08	-6.02
	50	-0.15	-0.25	-0.21	-0.23	184.52	74.01	41.66	24.40	14.12	3.44
	100	-0.04	-0.03	-0.04	-0.04	163.76	67.30	39.10	22.98	13.11	3.01

Table 4 continued: Gains from pooling of forecasts (*Model II*)

MODEL III																					
		T = 25										T = 50									
		MSE (%)										MSE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-6.37	-7.52	-8.92	-9.16	111.08	9.41	-3.99	-8.73	-10.59	-12.08	-3.77	-4.90	-5.72	-5.98	110.65	12.59	-0.93	-5.62	-7.71	-9.22
	2	-6.81	-8.09	-9.37	-9.71	114.90	8.57	-4.31	-9.36	-11.31	-12.80	-4.54	-5.61	-6.49	-6.86	115.88	10.09	-3.00	-7.61	-9.44	-11.00
	3	-5.99	-6.72	-7.97	-8.21	149.58	13.03	-1.94	-7.09	-9.29	-10.87	-4.71	-5.70	-6.40	-6.83	129.09	10.47	-2.82	-7.37	-9.33	-10.85
	4	-5.28	-5.67	-6.99	-7.23	165.73	16.41	0.71	-4.72	-7.05	-8.87	-3.84	-4.87	-5.64	-5.97	140.23	14.68	-0.19	-5.07	-7.16	-8.77
	5	-4.45	-4.93	-6.26	-6.40	233.07	23.02	2.80	-3.10	-5.64	-7.60	-3.16	-4.33	-5.07	-5.30	176.96	16.42	1.73	-3.65	-5.74	-7.38
	6	-3.89	-4.49	-5.56	-5.66	305.14	26.47	5.54	-1.15	-4.01	-6.13	-3.13	-4.44	-4.95	-5.18	165.74	17.73	2.54	-2.59	-5.02	-6.51
	7	-3.49	-4.34	-5.29	-5.36	292.77	29.60	5.60	-0.32	-3.35	-5.65	-2.12	-3.32	-3.97	-4.10	167.82	22.88	4.86	-0.66	-3.27	-4.92
	8	-3.20	-3.88	-4.64	-4.71	334.42	32.13	8.53	1.27	-1.82	-4.46	-2.24	-2.98	-3.68	-3.74	198.82	24.13	5.28	-0.28	-2.74	-4.46
	9	-3.03	-3.72	-4.42	-4.44	449.42	34.00	8.35	1.71	-1.16	-3.98	-2.22	-2.87	-3.54	-3.58	237.24	26.62	7.01	0.09	-2.38	-4.06
	10	-3.14	-3.82	-4.31	-4.34	423.01	35.48	10.32	2.47	-0.71	-3.67	-2.27	-2.94	-3.46	-3.51	268.58	29.10	8.37	0.67	-1.85	-3.73
	25	-2.04	-1.90	-2.40	-2.46	541.60	46.67	12.56	4.62	1.91	0.47	-1.01	-1.13	-1.29	-1.36	444.64	50.42	13.64	3.60	0.30	-1.67
	50	-1.91	-2.13	-2.48	-2.47	605.93	54.64	12.81	3.60	3.08	6.78	-0.67	-0.80	-0.90	-0.91	373.95	47.31	14.18	4.37	0.77	-1.20
	100	-1.65	-2.40	-2.85	-2.77	617.64	57.40	12.61	3.56	0.96	1.40	0.18	-0.16	-0.16	-0.26	337.75	39.56	12.50	4.27	0.83	-1.01
		VARIANCE (%)										VARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	0.52	0.61	0.68	0.73	11.75	-1.08	-0.38	0.05	0.27	0.43	0.44	0.57	0.74	0.77	6.11	-1.82	-0.75	-0.23	0.02	0.21
	2	1.03	1.24	1.41	1.48	17.82	-1.48	-0.45	0.31	0.61	0.88	0.97	1.30	1.61	1.70	12.34	-3.28	-1.38	-0.47	-0.05	0.40
	3	1.27	1.57	1.80	1.87	40.99	-1.77	-0.58	0.34	0.87	1.28	1.34	1.79	2.18	2.31	19.56	-4.18	-1.90	-0.57	-0.01	0.60
	4	1.35	1.70	1.96	2.02	47.29	-1.60	-0.59	0.49	1.11	1.67	1.53	2.03	2.46	2.61	27.09	-5.12	-2.19	-0.42	0.51	1.22
	5	1.37	1.75	2.03	2.08	90.88	-1.43	-0.91	0.43	1.16	1.85	1.70	2.24	2.69	2.86	44.39	-5.84	-2.40	-0.26	0.96	1.90
	6	1.32	1.69	1.98	2.01	143.14	-1.07	-1.27	0.22	1.09	1.87	1.74	2.28	2.73	2.90	38.48	-6.71	-2.83	-0.36	1.19	2.30
	7	1.20	1.56	1.82	1.84	130.00	-0.79	-1.14	-0.02	0.86	1.71	1.73	2.24	2.68	2.85	35.98	-7.25	-3.08	-0.30	1.60	2.84
	8	1.13	1.49	1.74	1.76	160.23	-0.69	-1.44	-0.31	0.63	1.61	1.67	2.16	2.58	2.73	50.71	-7.76	-3.17	-0.15	1.59	3.03
	9	1.07	1.43	1.67	1.67	250.59	-0.23	-1.49	-0.33	0.44	1.49	1.61	2.08	2.48	2.62	75.33	-8.41	-4.00	-0.34	1.54	3.02
	10	1.01	1.35	1.57	1.58	222.81	-0.25	-1.76	-0.65	0.25	1.33	1.54	1.97	2.35	2.49	93.20	-9.39	-4.83	-0.67	1.27	2.91
	25	0.61	0.78	0.92	0.92	313.68	2.95	-2.04	-0.72	-0.29	-0.09	0.75	0.89	1.06	1.11	199.44	-10.55	-7.82	-2.99	-0.42	1.47
	50	0.63	0.81	0.97	0.96	359.67	6.59	-1.59	-0.48	-0.10	0.10	0.54	0.65	0.76	0.80	151.26	-12.70	-9.02	-3.86	-0.98	1.01
	100	0.63	0.84	1.01	1.00	367.10	6.55	-1.33	0.14	0.05	-0.12	0.49	0.58	0.68	0.71	127.56	-12.77	-8.29	-3.75	-1.05	0.90
		COVARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-6.90	-8.14	-9.60	-9.90	99.32	10.49	-3.62	-8.78	-10.86	-12.51	-4.21	-5.48	-6.46	-6.76	104.53	14.41	-0.18	-5.39	-7.73	-9.43
	2	-7.83	-9.33	-10.77	-11.18	97.07	10.05	-3.86	-9.67	-11.91	-13.68	-5.51	-6.90	-8.10	-8.56	103.52	13.36	-1.62	-7.15	-9.39	-11.40
	3	-7.25	-8.28	-9.76	-10.07	108.56	14.81	-1.36	-7.43	-10.15	-12.15	-6.04	-7.48	-8.57	-9.13	109.49	14.63	-0.93	-6.81	-9.34	-11.46
	4	-6.62	-7.36	-8.94	-9.24	118.40	18.01	1.30	-5.20	-8.15	-10.54	-5.37	-6.90	-8.10	-8.58	113.09	19.78	1.99	-4.66	-7.68	-10.00
	5	-5.83	-6.68	-8.29	-8.48	142.14	24.44	3.72	-3.53	-6.80	-9.45	-4.85	-6.56	-7.76	-8.15	132.49	22.23	4.09	-3.41	-6.72	-9.29
	6	-5.21	-6.19	-7.54	-7.67	161.93	27.53	6.80	-1.37	-5.10	-8.00	-4.86	-6.71	-7.67	-8.07	127.20	24.42	5.34	-2.25	-6.23	-8.83
	7	-4.69	-5.90	-7.12	-7.21	162.70	30.37	6.73	-0.31	-4.22	-7.37	-3.84	-5.56	-6.65	-6.94	131.77	30.11	7.92	-0.39	-4.89	-7.77
	8	-4.34	-5.38	-6.39	-6.48	174.12	32.80	9.96	1.58	-2.45	-6.07	-3.91	-5.14	-6.26	-6.47	148.05	31.87	8.44	-0.15	-4.34	-7.50
	9	-4.11	-5.15	-6.09	-6.12	198.72	34.22	9.83	2.04	-1.61	-5.47	-3.83	-4.94	-6.01	-6.20	161.84	35.02	11.00	0.41	-3.93	-7.09
	10	-4.15	-5.17	-5.89	-5.92	200.12	35.72	12.07	3.12	-0.96	-5.00	-3.81	-4.91	-5.81	-5.99	175.32	38.48	13.19	1.33	-3.13	-6.65
	25	-2.66	-2.68	-3.32	-3.38	227.77	43.71	14.60	5.35	2.20	0.56	-1.76	-2.02	-2.34	-2.46	245.12	60.96	21.45	6.59	0.73	-3.13
	50	-2.54	-2.94	-3.45	-3.43	246.13	48.04	14.39	4.07	3.18	6.67	-1.21	-1.45	-1.66	-1.71	222.60	60.01	23.19	8.22	1.74	-2.21
	100	-2.29	-3.24	-3.87	-3.77	250.41	50.83	13.93	3.42	0.91	1.52	-0.31	-0.74	-0.85	-0.97	210.10	52.31	20.78	8.01	1.87	-1.92

Table 5: Gains from pooling of forecasts (*Model III*)

		MODEL III																			
		T = 100										T = 200									
		MSE (%)										MSE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-2.95	-3.70	-3.93	-4.33	93.96	10.54	-0.94	-5.62	-7.70	-9.26	-2.32	-3.60	-4.05	-4.23	94.31	11.87	0.01	-4.92	-6.96	-8.52
	2	-4.05	-4.91	-5.12	-5.66	101.79	7.69	-3.95	-8.37	-10.50	-11.95	-3.68	-4.94	-5.59	-5.76	86.27	6.80	-4.10	-8.44	-10.41	-11.89
	3	-3.61	-4.62	-4.80	-5.27	94.38	9.67	-2.35	-7.26	-9.21	-10.70	-3.77	-4.60	-5.21	-5.46	93.91	8.25	-3.60	-7.89	-10.02	-11.44
	4	-3.51	-4.36	-4.40	-4.80	109.74	11.87	-0.30	-4.93	-7.02	-8.64	-3.20	-3.77	-4.42	-4.65	94.78	9.18	-2.28	-6.73	-8.79	-10.29
	5	-2.79	-3.54	-3.64	-4.04	111.17	14.80	1.65	-3.72	-5.86	-7.46	-2.43	-2.88	-3.54	-3.68	95.39	12.68	0.19	-4.62	-6.61	-8.16
	6	-2.79	-3.26	-3.27	-3.63	127.37	15.65	3.04	-2.01	-4.26	-5.86	-1.97	-2.64	-3.04	-3.19	107.93	15.22	2.48	-2.51	-4.52	-6.07
	7	-2.16	-2.64	-2.59	-2.90	134.05	19.92	4.48	-0.48	-2.88	-4.57	-1.88	-2.59	-2.89	-3.04	117.32	15.91	3.55	-1.14	-3.20	-4.80
	8	-2.04	-2.37	-2.40	-2.63	143.16	20.71	5.86	0.03	-2.27	-4.00	-1.65	-2.14	-2.46	-2.56	118.36	18.82	5.25	0.35	-2.02	-3.57
	9	-2.10	-2.40	-2.51	-2.64	148.64	20.26	5.83	0.23	-1.99	-3.73	-1.56	-2.08	-2.36	-2.42	138.44	19.52	5.63	0.61	-1.57	-3.23
	10	-1.69	-2.07	-2.16	-2.26	148.86	22.93	6.06	0.63	-1.58	-3.35	-1.24	-1.67	-1.90	-1.98	127.02	19.12	6.42	1.45	-0.73	-2.40
	25	-0.23	-0.34	-0.37	-0.37	269.21	35.99	12.16	4.41	1.65	-0.33	0.10	0.00	-0.06	-0.04	207.57	31.04	11.38	4.74	1.96	0.20
	50	-0.04	-0.11	-0.16	-0.15	188.92	32.01	11.59	4.40	1.88	-0.11	-0.04	-0.04	-0.05	-0.04	139.20	24.80	10.01	4.25	1.86	0.12
	100	0.05	-0.06	0.01	-0.03	167.86	27.57	10.59	4.37	1.85	-0.01	-0.01	-0.04	-0.02	-0.03	136.18	24.22	9.43	4.14	1.85	0.13

		VARIANCE (%)										VARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	0.29	0.44	0.51	0.49	1.40	-2.56	-1.60	-1.02	-0.78	-0.61	0.12	0.27	0.32	0.35	1.48	-2.82	-1.85	-1.34	-1.06	-0.87
	2	0.88	1.23	1.38	1.40	6.40	-4.57	-2.93	-2.01	-1.45	-1.13	0.52	0.86	1.01	1.10	-1.77	-5.25	-3.68	-2.69	-2.24	-1.88
	3	1.35	1.83	2.03	2.09	2.73	-5.89	-3.08	-1.79	-1.14	-0.66	0.95	1.45	1.69	1.84	0.68	-7.71	-4.79	-3.52	-2.67	-2.22
	4	1.71	2.31	2.53	2.62	8.04	-6.65	-3.09	-1.20	-0.32	0.59	1.33	1.97	2.30	2.49	-1.70	-10.25	-6.71	-4.91	-3.79	-2.99
	5	1.99	2.67	2.90	3.02	7.50	-8.02	-3.98	-1.41	0.03	1.07	1.62	2.37	2.77	3.00	-6.95	-12.30	-7.79	-4.95	-4.06	-2.92
	6	2.17	2.91	3.16	3.29	10.31	-9.25	-4.15	-1.10	0.55	1.81	1.84	2.66	3.12	3.36	-4.51	-14.25	-7.86	-4.69	-3.16	-1.81
	7	2.21	2.98	3.22	3.36	8.66	-11.68	-4.94	-1.37	0.65	2.55	1.99	2.85	3.35	3.60	-4.27	-15.06	-8.04	-4.21	-2.18	-0.63
	8	2.18	2.94	3.17	3.32	11.14	-12.67	-5.81	-1.02	1.64	3.59	2.01	2.89	3.39	3.64	-8.56	-17.01	-8.57	-3.56	-0.63	1.52
	9	2.12	2.87	3.09	3.23	9.86	-13.15	-5.84	-0.94	1.60	3.96	2.00	2.86	3.36	3.61	-0.88	-18.69	-9.26	-3.72	-0.82	1.83
	10	2.04	2.76	2.98	3.11	9.54	-14.71	-6.39	-1.13	1.50	4.09	1.93	2.77	3.25	3.49	-11.35	-19.36	-10.17	-4.08	-0.60	2.25
	25	0.57	0.78	0.85	0.88	58.88	-25.14	-14.55	-6.56	-2.57	0.92	0.25	0.45	0.50	0.54	9.32	-33.75	-20.17	-10.88	-5.05	-0.28
	50	0.24	0.32	0.34	0.36	14.58	-24.84	-14.48	-6.93	-3.27	0.22	0.04	0.10	0.10	0.11	-22.21	-32.09	-19.92	-11.24	-6.11	-1.35
	100	0.20	0.26	0.28	0.29	7.29	-23.92	-13.72	-7.05	-3.30	0.09	0.04	0.09	0.09	0.10	-23.57	-31.46	-18.82	-10.49	-5.57	-0.90

		COVARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-3.24	-4.13	-4.44	-4.82	92.55	13.12	0.68	-4.58	-6.90	-8.63	-2.45	-3.86	-4.37	-4.58	92.82	14.70	1.85	-3.58	-5.90	-7.65
	2	-4.93	-6.13	-6.50	-7.06	95.36	12.26	-1.01	-6.35	-9.04	-10.81	-4.20	-5.80	-6.59	-6.86	88.03	12.05	-0.42	-5.75	-8.17	-10.01
	3	-4.96	-6.45	-6.83	-7.35	91.63	15.57	0.75	-5.45	-8.04	-10.02	-4.72	-6.05	-6.90	-7.30	93.22	15.96	1.19	-4.37	-7.36	-9.23
	4	-5.23	-6.66	-6.93	-7.42	101.67	18.52	2.80	-3.72	-6.68	-9.21	-4.54	-5.74	-6.72	-7.15	96.46	19.43	4.42	-1.82	-5.01	-7.30
	5	-4.77	-6.20	-6.55	-7.07	103.64	22.83	5.64	-2.30	-5.88	-8.51	-4.06	-5.25	-6.32	-6.68	102.34	24.97	7.98	0.32	-2.56	-5.24
	6	-4.96	-6.17	-6.43	-6.92	117.03	24.91	7.20	-0.91	-4.80	-7.66	-3.82	-5.30	-6.17	-6.55	112.42	29.47	10.34	2.18	-1.37	-4.26
	7	-4.37	-5.61	-5.80	-6.26	125.36	31.60	9.43	0.89	-3.52	-7.11	-3.88	-5.44	-6.24	-6.64	121.58	30.96	11.58	3.06	-1.03	-4.18
	8	-4.23	-5.32	-5.57	-5.94	131.98	33.37	11.67	1.05	-3.91	-7.59	-3.67	-5.02	-5.85	-6.20	126.90	35.82	13.82	3.91	-1.39	-5.10
	9	-4.22	-5.27	-5.60	-5.87	138.73	33.40	11.66	1.17	-3.59	-7.69	-3.55	-4.94	-5.72	-6.03	139.32	38.22	14.89	4.33	-0.75	-5.06
	10	-3.73	-4.83	-5.14	-5.36	139.27	37.64	12.44	1.75	-3.09	-7.45	-3.17	-4.44	-5.16	-5.47	138.36	38.48	16.59	5.52	-0.13	-4.65
	25	-0.80	-1.12	-1.22	-1.25	210.28	61.12	26.71	10.97	4.23	-1.25	-0.15	-0.44	-0.56	-0.58	198.24	64.79	31.55	15.62	7.01	0.47
	50	-0.28	-0.43	-0.51	-0.52	174.29	56.84	26.07	11.32	5.14	-0.33	-0.08	-0.13	-0.15	-0.14	161.39	56.88	29.93	15.49	7.97	1.47
	100	-0.15	-0.32	-0.26	-0.32	160.53	51.48	24.31	11.42	5.15	-0.10	-0.05	-0.13	-0.11	-0.13	159.73	55.68	28.24	14.63	7.43	1.03

Table 5 continued: Gains from pooling of forecasts (*Model III*)

MODEL III											
		T = 1000									
		MSE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	-1.81	-2.89	-3.30	-3.47	88.22	9.61	-1.26	-5.57	-7.75	-9.25
	2	-2.15	-3.59	-3.83	-4.19	74.30	6.85	-3.91	-8.13	-10.03	-11.54
	3	-2.51	-3.95	-4.40	-4.52	79.92	8.82	-2.99	-7.07	-9.20	-10.68
	4	-2.53	-3.79	-4.07	-4.23	77.26	9.92	-1.26	-5.84	-7.90	-9.41
	5	-2.54	-3.56	-3.78	-3.95	90.14	11.24	-0.40	-4.91	-6.82	-8.36
	6	-2.21	-3.04	-3.31	-3.47	97.86	13.55	1.68	-2.91	-5.12	-6.65
	7	-1.85	-2.56	-2.89	-3.05	90.96	14.95	3.38	-1.53	-3.81	-5.28
	8	-1.43	-2.03	-2.44	-2.56	105.66	17.45	4.43	-0.35	-2.31	-3.95
	9	-1.12	-1.46	-1.91	-2.01	93.60	17.53	5.73	0.79	-1.41	-2.99
	10	-1.12	-1.30	-1.71	-1.78	108.35	17.82	6.42	1.42	-0.83	-2.45
	25	-0.14	-0.14	-0.13	-0.16	141.97	23.52	9.54	4.12	1.71	0.06
	50	0.01	0.01	0.01	0.01	114.10	22.11	9.23	4.21	2.02	0.30
	100	0.00	0.00	0.00	0.00	109.17	20.58	9.14	4.08	1.96	0.29
		VARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	0.44	0.54	0.62	0.67	-0.15	-2.87	-1.93	-1.40	-1.17	-0.98
	2	1.00	1.29	1.47	1.57	-3.50	-5.58	-3.89	-2.93	-2.65	-2.29
	3	1.49	1.97	2.24	2.38	-5.63	-7.85	-4.74	-3.86	-3.21	-2.59
	4	1.92	2.57	2.93	3.11	-9.44	-9.67	-6.16	-4.32	-3.40	-2.69
	5	2.25	3.06	3.48	3.69	-10.62	-12.10	-6.84	-4.86	-3.61	-2.54
	6	2.55	3.51	3.98	4.23	-12.92	-14.19	-8.76	-4.54	-2.77	-1.31
	7	2.76	3.81	4.33	4.60	-19.02	-15.76	-8.59	-3.76	-1.06	0.53
	8	2.90	4.03	4.58	4.86	-18.23	-17.65	-7.97	-1.96	0.70	3.43
	9	3.02	4.22	4.79	5.10	-27.89	-20.02	-9.05	-2.19	2.17	5.36
	10	3.06	4.30	4.87	5.19	-28.21	-23.45	-11.34	-2.55	2.40	6.31
	25	0.53	0.74	0.81	0.88	-42.36	-47.00	-30.66	-18.69	-10.61	-1.99
	50	0.01	0.02	0.02	0.02	-53.88	-46.87	-31.16	-19.58	-12.21	-3.30
	100	0.00	0.01	0.01	0.01	-55.07	-45.51	-30.71	-19.22	-11.67	-2.88
		COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5
h	1	-2.26	-3.44	-3.92	-4.14	88.35	12.48	0.67	-4.18	-6.58	-8.27
	2	-3.15	-4.88	-5.30	-5.75	77.78	12.42	-0.03	-5.21	-7.39	-9.25
	3	-4.00	-5.92	-6.64	-6.90	85.53	16.65	1.75	-3.22	-5.99	-8.10
	4	-4.45	-6.36	-7.00	-7.34	86.68	19.59	4.90	-1.52	-4.51	-6.71
	5	-4.79	-6.62	-7.25	-7.65	100.74	23.34	6.43	-0.06	-3.21	-5.82
	6	-4.76	-6.54	-7.29	-7.70	110.76	27.73	10.44	1.63	-2.35	-5.34
	7	-4.60	-6.37	-7.22	-7.65	109.96	30.71	11.98	2.23	-2.75	-5.81
	8	-4.32	-6.06	-7.01	-7.43	123.86	35.09	12.40	1.61	-3.01	-7.37
	9	-4.13	-5.68	-6.70	-7.11	121.46	37.54	14.77	2.98	-3.57	-8.35
	10	-4.17	-5.60	-6.57	-6.97	136.53	41.27	17.76	3.98	-3.23	-8.76
	25	-0.68	-0.88	-0.94	-1.03	184.29	70.52	40.20	22.81	12.32	2.05
	50	0.01	0.00	-0.01	-0.01	167.94	68.98	40.38	23.79	14.23	3.61
	100	0.00	-0.01	-0.01	-0.01	164.21	66.08	39.85	23.30	13.63	3.17

Table 5 continued: Gains from pooling of forecasts (*Model III*)

7 Conclusion

This paper employs a Monte Carlo study based on a standard DSGE model to quantify the gains from pooling of forecasts in the absence of any accidental effects frequently found in empirical applications. Given strict lab conditions, we identify the virtues of pooling that are independent of additional external effects on the DGP and thus realizable in almost any forecast situation encountered in practice.

Built on simulated data sets, we specify parsimonious VAR models to derive h–step ahead predictions of output. The single forecasts are pooled using standard weighting schemes and the resulting forecast errors are decomposed into bias, variance and covariance part. Using Monte Carlo techniques, we identify settings where pooling yields a higher gain in forecast accuracy and show how different combination schemes work under certain economic structures. Additionally, we show how the number of pooled forecasts effects the performance and analyze the size of the gains as the forecast horizon and the estimation samples vary.

Our results show, that the gain in forecast accuracy increases with the number of forecasts pooled. However, confirming the conclusions of Armstrong (2001), we find that the combination of only a small number of 4 predictions is sufficient to achieve most of the possible gain. Most notably, the decline in MSE by pooling is comparable to the decline achieved by eliminating estimation uncertainty. This gains practical relevance, as one usually has to deal with a rather limited number of observations yielding a relative high estimation uncertainty when forecasting macroeconomic aggregates. Regarding the structure of the underlying DGP, the largest benefits are achieved for rigid economies that are harder to forecast in general. Interestingly, the benefits reach a maximum for the 2–steps ahead predictions and decrease with a growing forecast horizon. However, in finite estimation samples, they remain significant even for very long horizons reflecting the estimation uncertainty regarding the unconditional mean of the single VAR processes. Our analysis shows that – under lab conditions – pooling leads to a substantial reduction of MSE of up to 20%. Decomposing MSE, the results show that the gains due to pooling mainly reflect a better forecast performance with respect to the covariance part whereas the variance part of MSE increases. Our results indicate that the estimation of optimized weights built on the covariance matrix of the forecast errors yields a substantial improvement only in the absence of corresponding sample uncertainty. In contrast, for reasonable sizes of the underlying optimization window the estimation uncertainty dominates the positive effects resulting in larger forecast errors. This leads to the conclusion that optimal pooling can be discarded for most practical applications.

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Appendix

A Quantification of the gain from pooling of forecasts

The expected error variance $E(\sigma_m^2)$ of an equally weighted average of a set of m randomly selected single forecasts can be derived as follows:

$$\begin{aligned}
E(\sigma_m^2) &= E[var(e_m)] \\
&= var(E[e_m]) \\
&= var(\tfrac{1}{m}\textstyle\sum_{i=1}^{m} e_i) \\
&= \tfrac{1}{m^2} var(\textstyle\sum_{i=1}^{m} e_i) \\
&= \tfrac{1}{m^2}(\textstyle\sum_{i=1}^{m} var(e_i) + 2\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} cov(e_i, e_j)) \\
&= \tfrac{1}{m}\overline{\sigma}^2 + \tfrac{2}{m^2}\textstyle\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} cov(e_i, e_j) \\
&= \tfrac{1}{m}\overline{\sigma}^2 + \tfrac{2}{m^2}\tfrac{m(m-1)}{2}\frac{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} cov(e_i,e_j)}{\frac{m(m-1)}{2}} \\
&= \tfrac{1}{m}\overline{\sigma}^2 + \tfrac{m^2-m}{m^2}\overline{s} \\
&= \tfrac{1}{m}\overline{\sigma}^2 + \tfrac{m-1}{m}\overline{s}
\end{aligned}$$

B Decomposition of the MSE

The mean squared error (MSE) can be derived as follows:

$$\begin{aligned}
MSE_{t+h} &= E[e_{t+h}^2] \\
&= (E[e_{t+h}])^2 + var(e_{t+h}) \\
&= (E[\hat{y}_{t+h} - y_{t+h}])^2 + var(\hat{y}_{t+h} - y_{t+h}) \\
&= (E[\hat{y}_{t+h} - y_{t+h}])^2 + \sigma_{\hat{y}_{t+h}}^2 + \sigma_{y_{t+h}}^2 - 2\rho\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}} \\
&= (E[\hat{y}_{t+h}] - E[y_{t+h}])^2 + \sigma_{\hat{y}_{t+h}}^2 + \sigma_{y_{t+h}}^2 - 2\rho\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}} \\
&= (E[\hat{y}_{t+h}] - E[y_{t+h}])^2 + +(\sigma_{\hat{y}_{t+h}} - \sigma_{y_{t+h}})^2 - 2\rho\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}} + 2\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}} \\
&= (E[\hat{y}_{t+h}] - E[y_{t+h}])^2 + (\sigma_{\hat{y}_{t+h}} - \sigma_{y_{t+h}})^2 + 2(1-\rho)\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}}
\end{aligned}$$

C Impulse Response Functions

In the following, we describe the characteristics of the six observable variables (r, y, n, w, π_w, π) of the economy.

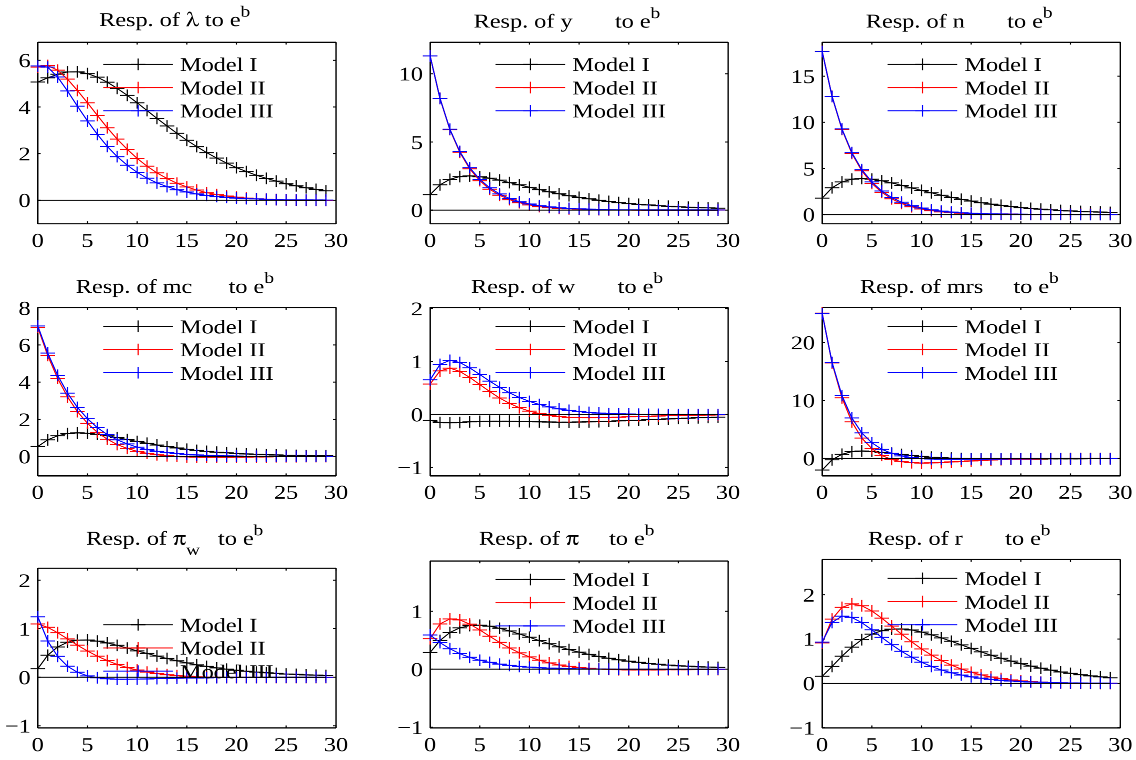

Figure 5: Response to a taste shock

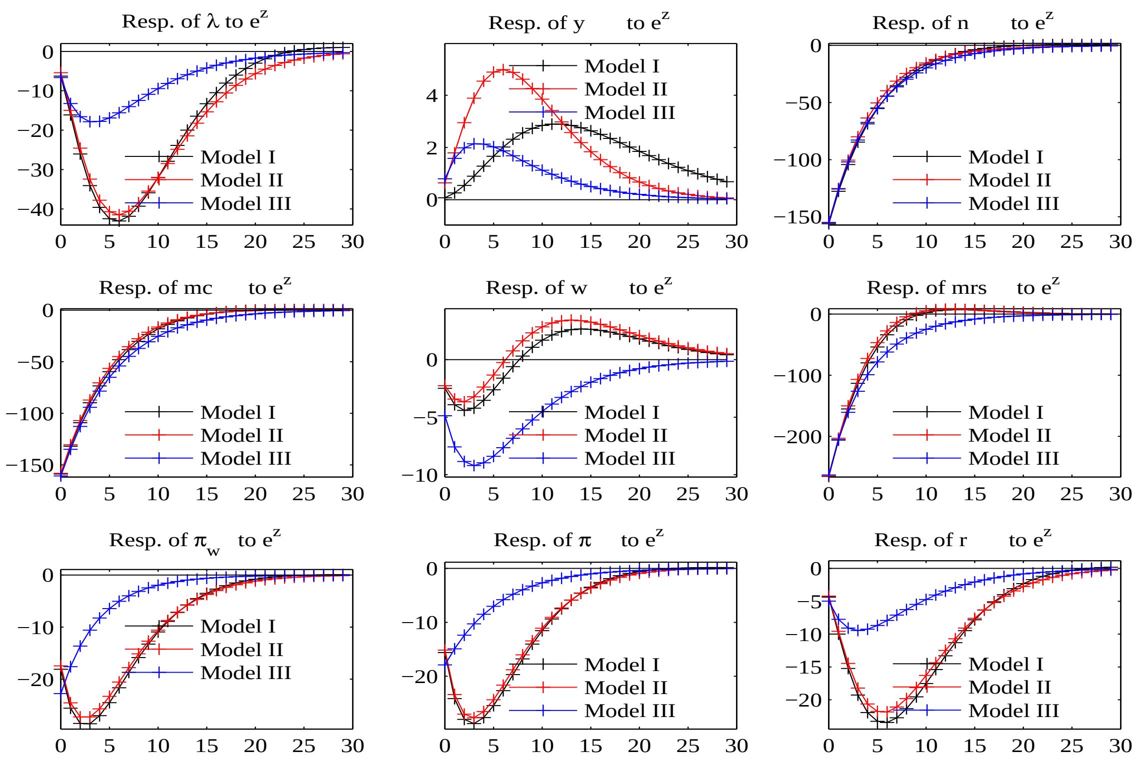

Figure 6: Response to a productivity shock

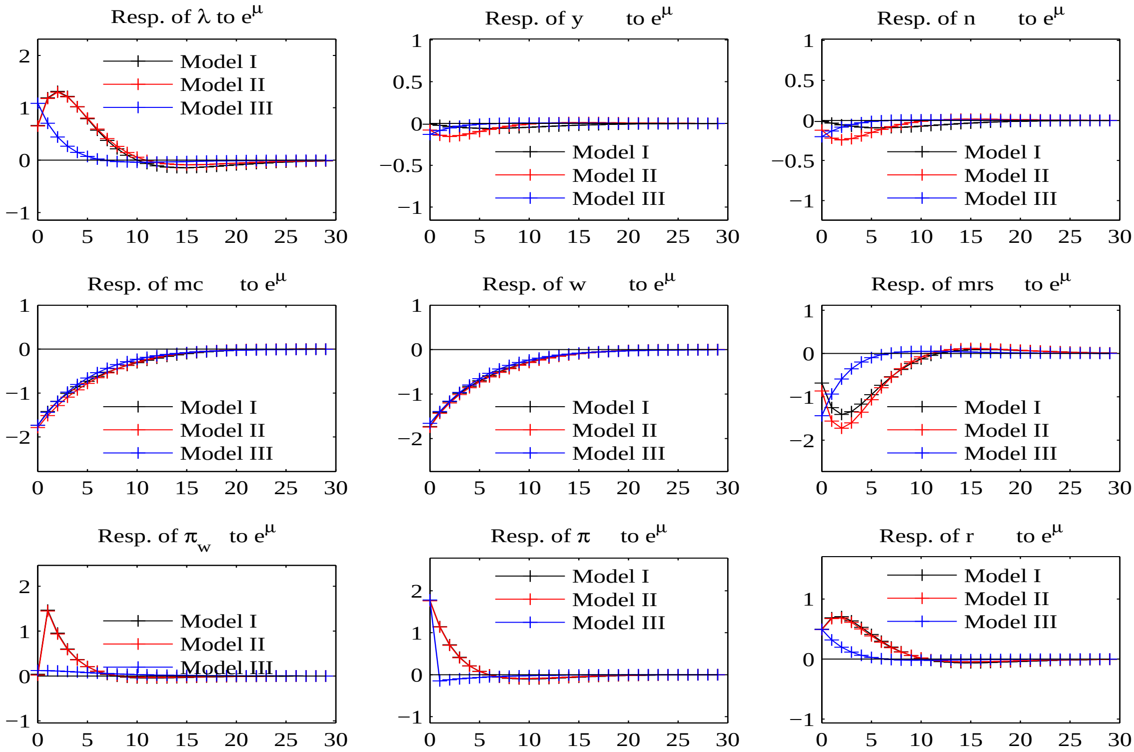

Figure 7: Response to a markup shock

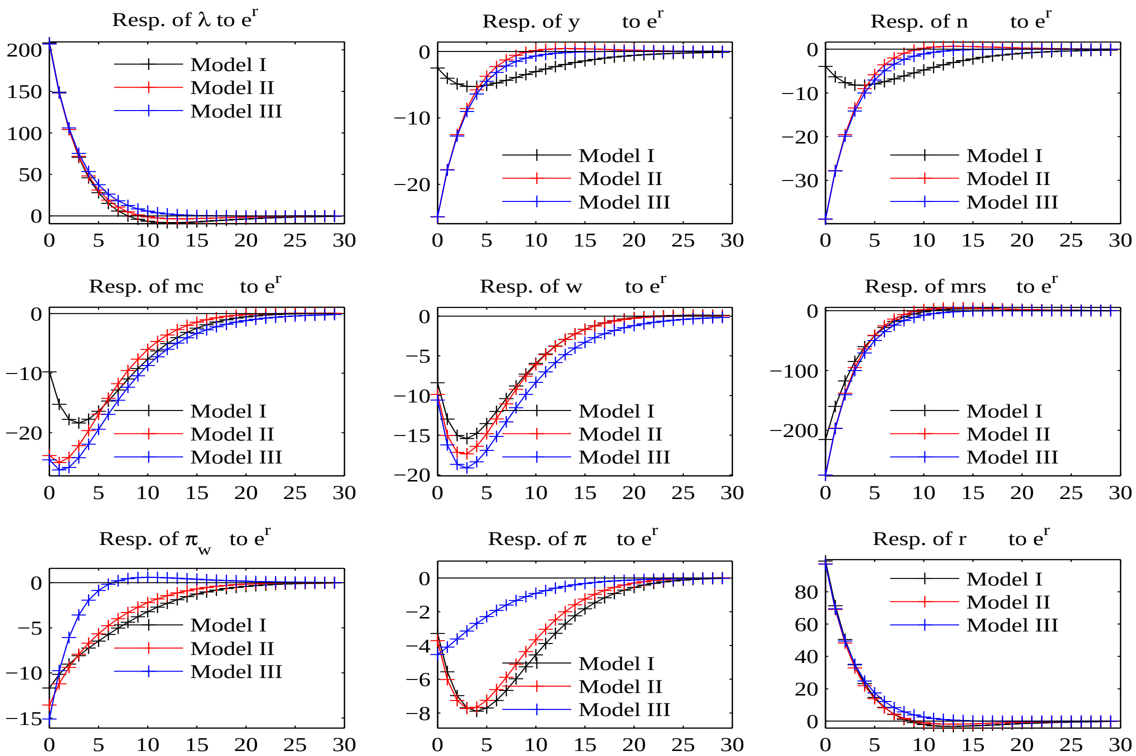

Figure 8: Response to a monetary policy shock

D Characteristics of the Data

Figure 9 to figure 11 show 100 observations for the observable variables for each model.

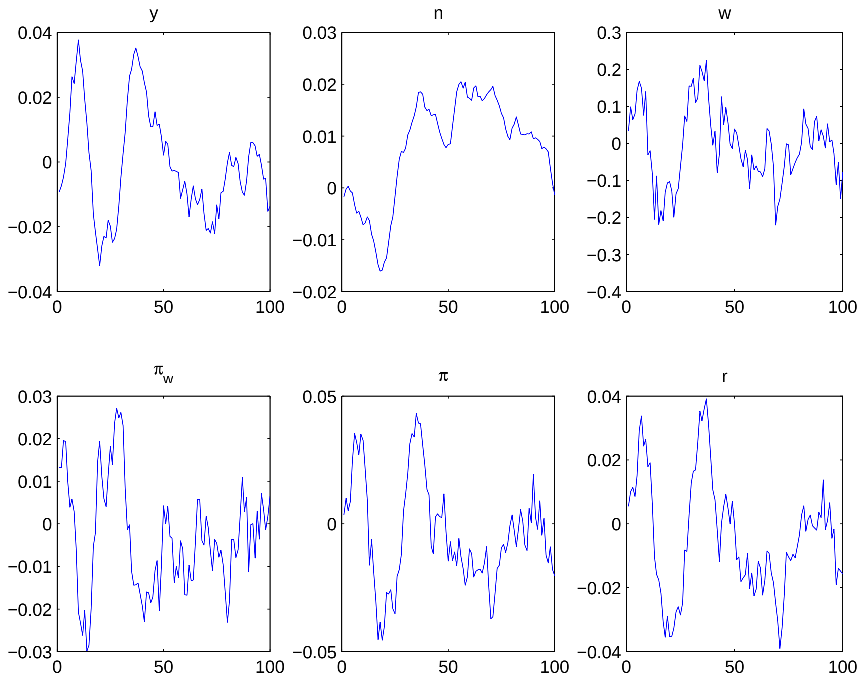

Figure 9: 100 observations of each variable (*Model I*)

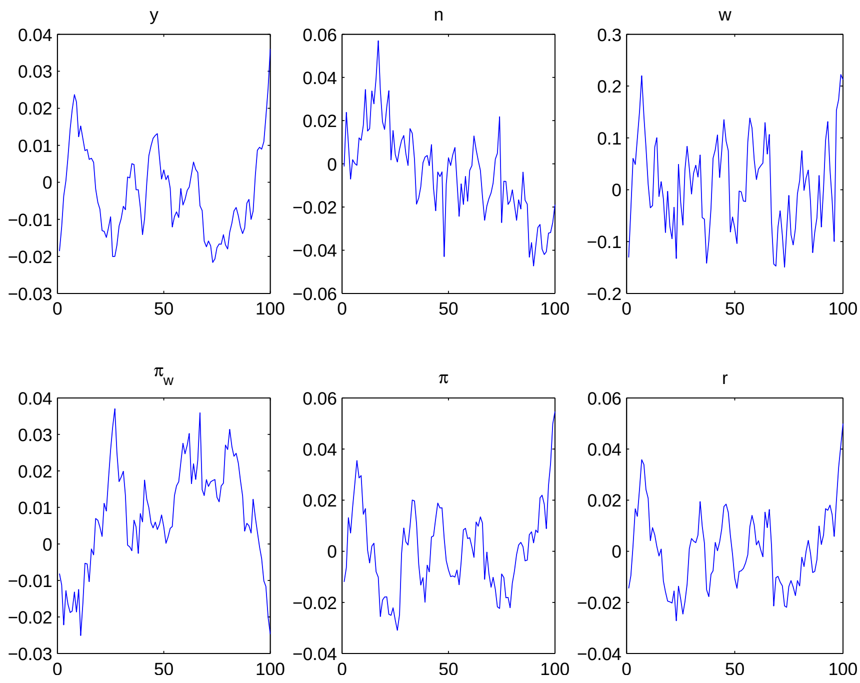

Figure 10: 100 observations of each variable (*Model II*)

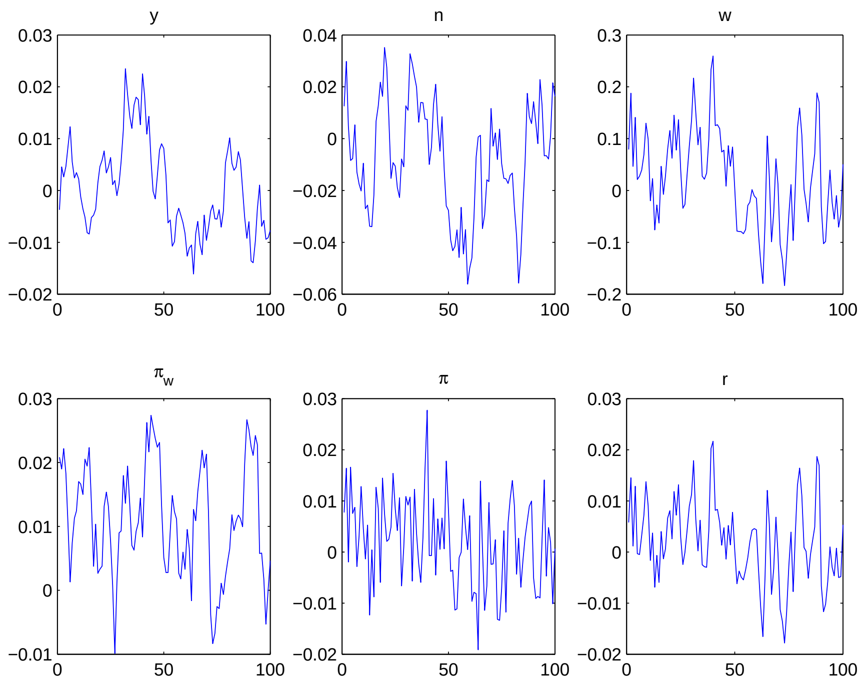

Figure 11: 100 observations of each variable (*Model III*)

Based on the full sample of $N = 10.000$ observations, figures 12 to 14 report the means of the corresponding auto–correlation functions (ACF) and partial auto–correlation functions (PACF) up to 20 lags each.

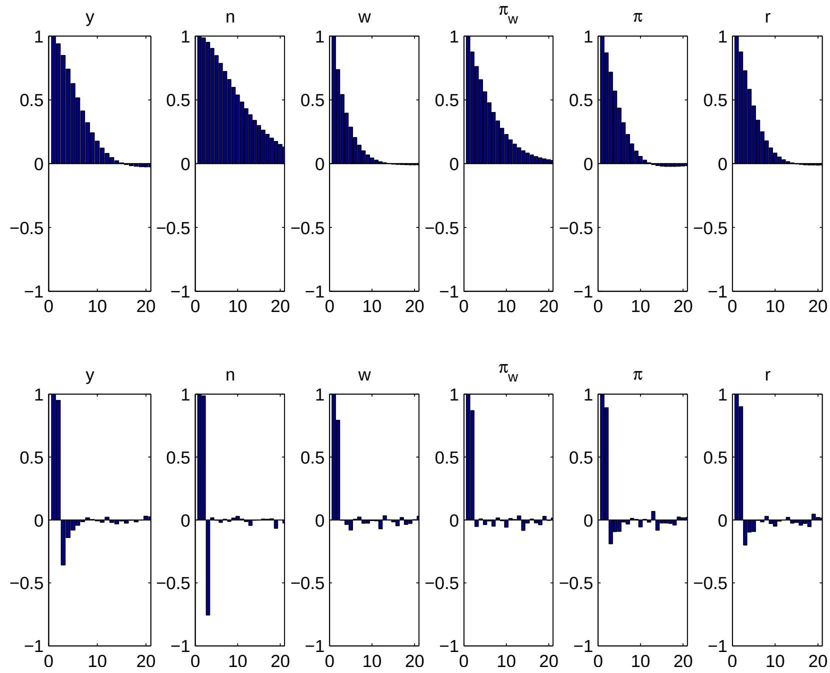

Figure 12: Mean of ACFs and PACFs (*Model I*)

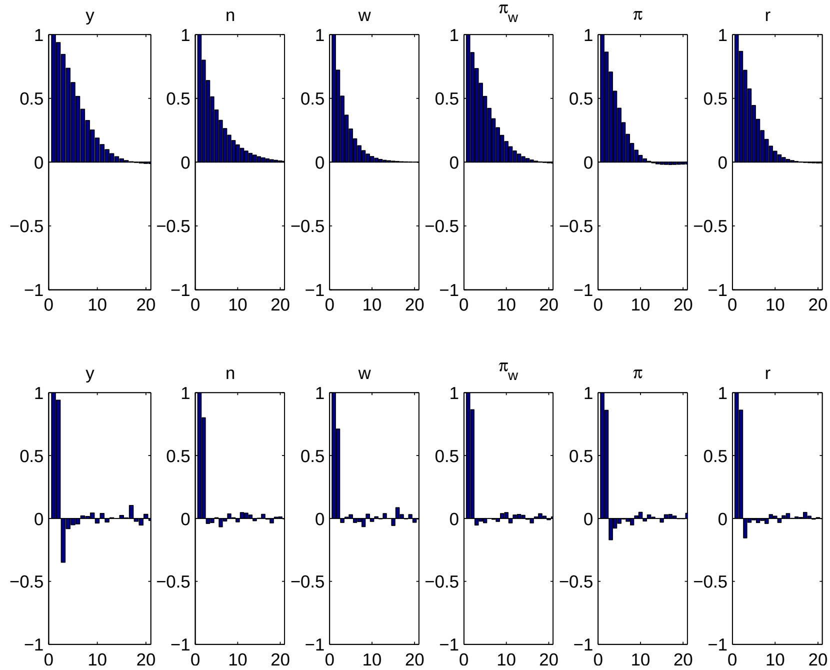

Figure 13: Mean of ACFs and PACFs (*Model II*)

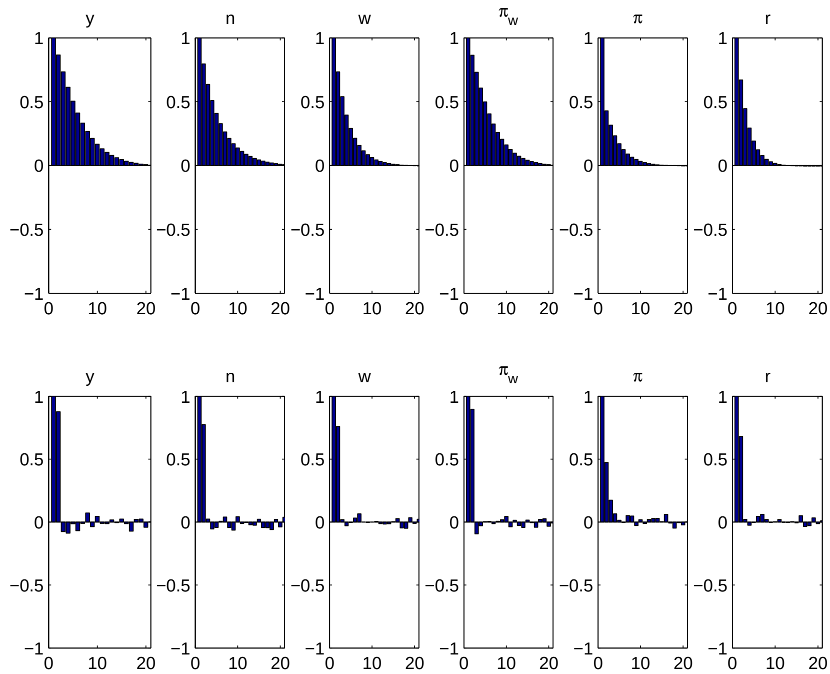

Figure 14: Mean of ACFs and PACFs (*Model III*)

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