FENDING OFF ONE MEANS FENDING OFF ALL: EVOLUTIONARY STABILITY IN SUBMODULAR GAMES

WOLFGANG LEININGER

CESIFO WORKING PAPER NO. 1266
CATEGORY 9: INDUSTRIAL ORGANISATION
AUGUST 2004

CESifo *Working Paper No. 1266*

FENDING OFF ONE MEANS FENDING OFF ALL: EVOLUTIONARY STABILITY IN SUBMODULAR GAMES

Abstract

The implications of evolutionarily stable behavior in finite populations have recently been explored for a variety of aggregative games. This note proves an intimate relationship between submodularity and global evolutionary stability of strategies for these games, which -apart from being of independent interest - accounts for a number of results obtained in the recent literature: we show that any evolutionarily stable strategy (ESS) of a submodular aggregative game must also be globally stable. I.e. if one mutant cannot successfully invade a population, any number of mutants can even less do so.

JEL Code: C79, D72.

Wolfgang Leininger
Department of Economics
University of Dortmund
D – 44221 Dortmund
Germany
wolfgang.leininger@udo.edu

The author is grateful to Ana Ania and Carlos Alõs-Ferrer for comments on an earlier version.

0 Introduction

The implications of evolutionarily stable behavior in *finite* populations are the subject of a very active field of research in economics. Beginning with Schaffer (1989), whose analysis was generalized by Vega-Redondo (1997), economists looked beyond the popular perception of an evolutionarily stable strategy (ESS) as a refinement of Nash equilibrium. This perception implicitly assumed an *infinite* population of potential players for the game under consideration. The theory of ESS for finite populations provided by Schaffer(1988) shows, that it is precisely this infinite population assumption, which accounts for the general coincidence of ESS behavior and symmetric Nash equilibrium. Without this assumption the two solution concepts may propose different solutions to the game. These differences and the relation of the two solution concepts ESS and Nash Equilibrium in general have since been explored for a variety of games in several recent contributions (see Alōs-Ferrer and Ania (2004), Hehenkamp et al. (2004), Leininger (2002,2003), Possajennikov (2003) and Schenk-Hoppē (2000)). All these contributions analyse games, which belong to the class of submodular aggregative games. The present note is concerned with a general structural characteristic of this class of games, namely, that in these games stability from an evolutionary point of view always means *global* stability: if a small number of a mutant strategy cannot invade a given population, then neither can any larger number of the same mutant strategy do so. This somewhat counterintuitive conclusion is made precise by our Theorem in section 3. Section 1 presents the class of submodular aggregative games, while section 2 provides the definition of finite population ESS and its adaptation to the games under consideration. Section 4 applies our findings to an example from contest theory.

1 Submodular aggregative games

As in Alōs-Ferrer and Ania (2004) and Possajennikov (2003) we consider generalized aggregative games, which are symmetric. Symmetry means that all players have identical strategy space S$\subset \Re$ and payoff function Π. A game of n players is aggregative (Corchon,1994), if this function only depends on the player's own strategy s_i and an aggregate $g(s_1, ..., s_n) \in \Re$ of all the stategies employed. Other players behavior is only payoff-relevant for player i through its influence on the aggregate. We assume the aggregator function g to be monotonically increasing in all components.

Definition 1:

A (generalized) symmetric aggregative game is a game
$\Gamma = (N, S_1 \times ... \times S_n, \Pi_1(s_1, ..., s_n), ..., \Pi_n(s_1, ..., s_n))$ such that

i) $S_i = S \subseteq \Re \qquad for\ all\ i = 1, ..., n$

ii) $\Pi_i(s_1, ..., s_i, ..., s_n) = \Pi(s_i, s_{-i}) \qquad for\ i = 1, ..., n$

where $\quad \Pi : S^n \to \Re \quad$ with $\quad \Pi(s_i, s_{-i}) = \Pi(s_i, s'_{-i})$

for any permutation s'_{-i} of s_{-i}.

iii) there exist $\quad g : S^n \to \Re\ $, increasing in all components, and

$\bar{\Pi} : S \times \Re \to \Re$ such that $\quad \Pi(s_i, s_{-i}) = \bar{\Pi}(s_i, g(s_i, s_{-i}))$

for all $\quad (s_i, s_{-i}) \in S^n$ and $i \in \{1, ..., n\}$

The previous definition captures a large and economically important class of games (see e.g. Possajennikov, 2003, Alōs-Ferrer and Ania, 2004, and Topkis, 1998, chapter 4). It contains Cournot games, rent-seeking contests, public-good provision games, common resource extraction games, arms races and many others.

Aggregative n-player games reduce to the structure of a two-player game: for each single player it is only payoff- relevant what the value of the aggregate is, and not what the actions of other players are. This enables the straightforward definition of submodularity for these games.

Definition 2:

i) A generalized aggregative game Γ is submodular (in individual strategy and the aggregate), if $\bar{\Pi}$ has decreasing differences; i.e. for all $s' \geq s$ and $g' \geq g$

$$\bar{\Pi}(s', g') - \bar{\Pi}(s, g') \leq \bar{\Pi}(s', g) - \bar{\Pi}(s, g)$$

- or equivalently -

$$\bar{\Pi}(s', g') - \bar{\Pi}(s', g) \leq \bar{\Pi}(s, g') - \bar{\Pi}(s, g)$$

ii) A generalized aggregative game Γ is strictly submodular (in individual strategy and aggregate), if $\bar{\Pi}$ has strictly decreasing differences; i.e. for all $s' > s$ and $g' > g$

$$\bar{\Pi}(s', g') - \bar{\Pi}(s, g') < \bar{\Pi}(s', g) - \bar{\Pi}(s, g)$$

Remark: It is well known (Topkis, 1998), that if $\bar{\Pi}$ is twice continuously differentiable, then $\bar{\Pi}$ has decreasing differences if and only if $\frac{\partial^2 \bar{\Pi}}{\partial g \partial s} \leq 0$ everywhere. Hence it is fairly simple to check in this case, whether a game Γ is submodular.

In submodular games the individual incentive to increase one's action (variable) *decreases* with increasing aggregate; hence individual action and the aggregate of (others) individual actions are strategic substitutes. This basic fact will be used to understand global stability of evolutionary equilibrium in these games. Since evolutionary stability in finite populations is closely linked to maximization of *relative* payoff (see the next section), we will read submodularity of the absolute payoff function $\bar{\Pi}$ of a player as a statement about his relative payoff function. For symmetric aggregative games the latter *is* a difference of $\bar{\Pi}$ in the sense of Definition 2. Decreasing differences of $\bar{\Pi}$ mean, that the relative payoff of a strategy s' against any smaller strategy s *decreases* with increasing values of aggregate g.

2 Evolutionarily stable strategies

We follow Schaffer (1988), who adapted the well-known notion of an evolutionarily stable strategy (ESS) due to Maynard Smith (1974) to finite populations of interacting agents.
A strategy is evolutionarily stable, if a whole population using that strategy cannot be invaded by a sufficiently small group of "mutants" using another strategy. Obviously, in the context of finite populations of n players the smallest meaningful number of mutants is one. This motivates the following definition: let the strategy s be adapted by all players i, $i = 1, ..., n$. A mutant strategy $s' \neq s$ can invade s, if the payoff for a single player using s' (against the (n-1) other players using s) is strictly higher than the payoff of a player using s (against (n-2) other players using s and the mutant using s'). A strategy s^* is then called evolutionarily stable, if it cannot be invaded by any other strategy.
Note that the comparison of payoffs implied by the above definition requires a *symmetric* game; i.e. a game satisfying i) and ii) of Definition 1. More formally,

Definition 3:

Let Γ be a symmetric game. A strategy s^* is an ESS, if for all $s \in S$

(*) $\Pi(s^*; s, s^*, ..., s^*) \geq \Pi(s; s^*, ..., s^*)$

s^* is a strict ESS, if the inequality is a strict one.

If Γ is also aggregative, then (*) reads

$$\bar{\Pi}(s^*; g(s, s^*, ..., s^*)) \geq \bar{\Pi}(s, g(s, s^*, ..., s^*)) \qquad for\ all\ s \in S$$

As observed by Schaffer (1988) this means, that an ESS is a strategy s^* with the property

$$s^* \in \ arg \max_{s' \in S} [\Pi(s'; s^*, ..., s^*) - \Pi(s^*; s', s^*, ..., s^*)]$$

Hence s^* corresponds to the Nash equilibrium of the game, in which players maximize *relative* (not absolute) payoff. This -in the usual way- can be found by looking for a fixed point of the best response correspondence, which in the case of symmetric games is just given by the above: it is the correspondence, that assigns to each strategy s the set of maximizers of the relative pay-off function (identical to all players), which results from all *other* players using s.

The Nash equilibrium of the game with relative payoff functions (i.e. ESS of the game with absolute payoff functions) need not correspond to a Nash equilibrium of the game with absolute payoffs; hence the well-known *refinement* property of ESS for infinite player populations does not hold in the context of finite player populations. E.g. Hehenkamp et al. (2004) show that for a certain class of submodular aggregative games, namely Tullock contests, ESS and Nash equilibrium always differ (and the former exists if and only if the latter exists). It always pays for a player to increase effort in an ESS beyond the Nash effort level, because such a (marginal) increase will lower an opponent's pay-off by *more* than it lowers his own. This form of spiteful behavior (Hamilton,1970) can even lead to *overdissipation* of the contested rent in an ESS.
An important strengthening of the ESS requirement -also due to Schaffer (1988)- demands additional stability against *several* identical mutants:

<u>Definition 4</u>:

An ESS s^* is called (strictly) M-stable, if a population of n players using s^* cannot be invaded by m mutants, who all use $s \neq s^*$, for $1 \leq m \leq M$ and $M \leq n-1$, i.e.

$$\Pi(s^*, s, \overbrace{...}^{m}, s, s^*, ..., s^*) \ \geq (>) \ \Pi(s, s, \overbrace{...}^{m-1}, s, s^*, ..., s^*)$$

$$for\ all\ s \in S\ and\ m \in \{1, ..., M\}$$

s^* is called (strictly) globally stable, if it is (strictly) (*n-1*)-stable.

Strictly globally stable ESS of finite population games have the important property of being the unique stable limit points of stochastic dynamic adjustment processes, which arise from imitation and experimentation behavior of players (see Schenk-Hoppē (2000) and Alōs-Ferrer and Ania (2004)). I.e. such strategies are the long-run outcome, if boundedly rational players behave in this way. Note, again, that the requirement from Definition 4 for aggregative games demands, that

$$\bar{\Pi}(s^*, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*)) \ \geq (>) \ \bar{\Pi}(s, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*))$$

$$for\ all\ s \in S\ and\ m \in \{1, ..., M\}$$

3 The Result

We now prove that for submodular aggregative games the notions of evolutionary stability (ESS) and global evolutionary stability (global ESS) *coincide*; i.e. in these games stability against invadibility by *one* mutant *implies* stability against invadibility by *any* number of mutants! This somewhat counterintuitive property follows from submodularity in these games: suppose a strategy s^* is an ESS; i.e. s^* is stable against a *single* mutant. If we now, in addition, require stability of s^* against invasion by two mutants, we extend the non-invadibility condition to a population, in which one incumbent strategy s^* has been *substituted* for an additional mutant strategy. A larger (smaller) mutant strategy, which *replaces* a incumbent strategy, will unambiguously increase (decrease) the aggregate of individual actions for all players. Submodularity then means that an incumbent strategy player's best response is to *decrease* (*increase*) his action. So the mutant strategy is always caught "wrong-footed" (i.e. as lying on the "wrong side" of the

incumbent strategy): it cannot do better than s^* with two copies, if it hasn't done so before with just a single copy.
More precisely, we prove

<u>Theorem</u>:

Let Γ be a (strictly) submodular generalized aggregative game.
Then any ESS of Γ is (strictly) globally stable.

<u>Proof</u>:

Let $s^* \in S$ be an ESS of Γ; i.e. it holds that

$$(1) \qquad \bar{\Pi}(s^*, g(s, s^*, ..., s^*)) \;\geq\; \bar{\Pi}(s, g(s, s^*, ..., s^*)) \qquad for\ all\ s \in S$$

(1) is our induction hypothesis (IH); we claim that (IH) implies global stability of s^*:

$$(2) \qquad \bar{\Pi}(s^*, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*)) \;\geq\; \bar{\Pi}(s, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*))$$

$$for\ all\ s \in S\ and\ all\ m = 1 \ldots, n-1$$

We will use aggregativeness and the decreasing differences-property of $\bar{\Pi}$ to infer that if (2) holds for $m \in \{1, ..., n-2\}$ than it must hold for $(m+1)$ as well. I.e. we have to show that

$$(C) \qquad \bar{\Pi}(s^*, g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)) \;-\; \bar{\Pi}(s, g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)) \geq 0$$

$$for\ all\ s \in S.$$

i) Suppose that $s^* < s$, then it follows that

$$g(s, \overbrace{...}^{m}, s, s^*, ..., s^*) < g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)$$

because of monotonicity of g.
Since $\bar{\Pi}$ has (strictly) decreasing differences this means, that

$$\bar{\Pi}(s, g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)) - \bar{\Pi}(s, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*))$$

$$\leq (<)\ \bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) - \bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*))$$

from this it follows that

$$\begin{aligned} 0 \leq (<)\ &\bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) - \bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) \\ &-\bar{\Pi}(s, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) + \bar{\Pi}(s, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) \\ \leq (<)\ &\bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) - \bar{\Pi}(s, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) \end{aligned}$$

since $\bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) \geq \bar{\Pi}(s, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*))$

because of our induction hypothesis (IH).
This proves (C) for $(m+1)$mutants s.

ii) Suppose now that $s^* > s$, then it follows that

$$g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*) > g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)$$

Since Π has (strictly) decreasing differences this means, that

$$\bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) - \bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*))$$

$$\leq (<)\ \bar{\Pi}(s, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) - \bar{\Pi}(s, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*))$$

Consequently,

$$\begin{aligned} 0 \leq (<)\ &\bar{\Pi}(s, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) - \bar{\Pi}(s, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) \\ &-\bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) + \bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) \\ \leq\ &\bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) - \bar{\Pi}(s, g(s, \overbrace{\cdots}^{m+1}, s, s^*, ..., s^*)) \end{aligned}$$

since $\bar{\Pi}(s, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) - \bar{\Pi}(s^*, g(s, \overbrace{\cdots}^{m}, s, s^*, ..., s^*)) \leq 0$ because of our induction hypothesis (IH). This again yields (C).
This proves the theorem. Our proof by induction generalizes Theorem 4 in Hehenkamp et al. (2004) from the context of Tullock contests to the entire class of submodular aggregative games.

Note, that our Theorem does not require an ESS to be a strict ESS to infer its global stability properties. An obvious Corollary, which directly follows from our method of proof, hence is

Corollary:

> Let Γ be a submodular generalized aggregative game.
> Then any strict ESS is strictly globally stable.

Our result is similar, yet different and of deeper nature, to that of Alōs-Ferrer and Ania (2004). They show, that it is submodularity of an aggregative game that accounts for the fact, that an aggregate-taking equilibrium must be a globally stable ESS. They use this fact to show existence of (globally stable) ESS via existence of an aggregate-taking equilibrium. Our result shows, that as far as global stability of an ESS is concerned this detour (via the notion of an aggregate-taking equilibrium) is neither necessary nor as general as our direct approach. E.g. for the example of rent-seeking contests presented below our Theorem implies existence of a globally stable ESS for parameter ranges, that imply non-existence of an aggregate-taking equilibrium.
The Theorem also gives a unifying account of results relating to the stochastic stability of finite population ESS. Vega-Redondo (1997) and -in more generality- Schenk-Hoppē (2000) show, that in Cournot games only ESS are selected as invariant states by certain explicitly dynamic and stochastic models of the evolutionary process. Alōs-Ferrer and Ania (2004) use a result of Ellison (2000) on stochastic stability to prove, that a strictly globally stable ESS (if it exists) is the unique stochastically stable state of such evolutionary process. We now add the insight, that Vega-Redondo (1997) and Schenk-Hoppē (2000) derive their results from games, whose ESS's are in fact strictly globally stable. Invariant states in these models can hence be easily computed by applying relative payoff maximization. Appendix 1 gives a slight generalization.

4 An example: evolutionary rent-seeking

Tullock (1980) modelled competition for a rent of value V as a contest between n players. All players expend effort (or income) to obtain the rent, but only the winner gets the entire rent while all expenditures are forfeited. If the contestants expend

$s = (s_1, ..., s_n),\ s_i \geq 0$, the probability of winning the rent for player i, i=1,...,n is given by

$$p_i(s_1, ..., s_n) = \frac{s_i^r}{\sum_{j=1}^n s_j^r} \qquad r > 0$$

and expected payoff for player i is given by

$$\Pi_i(s_1, ..., s_n) = p_i(s_1, ..., s_n) \cdot V - s_i = \frac{s_i^r}{\sum_{j=1}^n s_j^r} \cdot V - s_i$$

One can show that for $r \leq \frac{n}{n-1}$ a unique ESS $s^* = \frac{r}{n} \cdot V$ exists (Hehenkamp et al. (2004), Theorem 2).
Observe now that Tullock contests are aggregative games: define the aggregator g as

$$g(s_1, ..., s_n) = (\sum_{j=1}^n s_j^r)^{1/r}$$

to obtain

$$\Pi_i(s_1, ..., s_n) = \bar{\Pi}(s_i, g(s_1, ..., s_n)) = (\frac{s_i}{g(s_1, ..., s_n)})^r \cdot V - s_i$$

with $g(s_1, ..., s_n)$ strictly increasing in all its components.
Moreover, Tullock contests are strictly submodular (in individual strategy and the aggregate g) since

$$\frac{\partial^2 \bar{\Pi}}{\partial s_i \partial g} = -\frac{s_i^{r-1}}{g(s_1, ..., s_n)^{r+1}} \cdot r^2 \ < 0 \quad for\ all\ s_i > 0$$

According to our Theorem the unique ESS then in fact must be strictly globally stable. Note, that an aggregate-taking equilibrium only exists for $r < 1$ (Alōs-Ferrer and Ania, 2004). So, proving existence of a globally stable ESS via existence of an aggregate taking equilibrium for e.g. $n = 2$ only works, if $r < 1$, but a globally stable ESS exists for all $r \leq 2$. The direct proof of Hehenkamp et al. (2004), which does not refer to submodularity, combined with our Theorem then reveals the interplay between adding more and more mutants and the submodular payoff structure: Suppose m mutants have "invaded" a population of n players, who all used the ESS strategy s^*, with strategy $\bar{s}$. This means that m ESS-players using s^* have been replaced by m players using $\bar{s}$ in the n-player contest ($m < n$).

The relative payoff function of one of the m mutants, Π_M^m, is then given by

$$\Pi_M^m = \bar{\Pi}(\bar{s}, g(\bar{s}, ..., \bar{s}, s^*, ..., s^*)) - \bar{\Pi}(s^*, g(\bar{s}, ..., \bar{s}, s^*, ..., s^*))$$
$$= \frac{\bar{s}^r - s^{*^r}}{g(\bar{s},..., \bar{s}, s^*, ..., s^*)^r} \cdot V - (\bar{s} - s^*) = \frac{\bar{s}^r - s^{*^r}}{[m \cdot \bar{s}^r + (n-m)s^{*^r}]} \cdot V - (\bar{s} - s^*)$$

Substituting yet another s^*-player for a mutant $\bar{s}$ then simply amounts to a *change* of the aggregate variable g in the relative and absolute payoff functions of all players, mutants and ESS-strategists alike: if $\bar{s} > s^*$ holds, g will increase to $\bar{g}$; likewise, if $\bar{s} < s^*$, then g will decrease to $\bar{\bar{g}}$.

Since $\bar{\Pi}$ is strictly submodular and hence has strictly decreasing differences (in individual strategy and aggregate) this means that

i) $\bar{\Pi}(\bar{s}, \bar{g}) - \bar{\Pi}(s^*, \bar{g}) < \bar{\Pi}(\bar{s}, g) - \bar{\Pi}(s^*, g) \qquad if\ \bar{s} > s^*$

and so the relative payoff of a mutant *decreases*, if the aggregate changes from g to $\bar{g} > g$. Consequently, if this relative payoff has been negative before (with just m mutants), it will be negative as well after the introduction of an additional mutant;

ii) $\bar{\Pi}(s^*, g) - \bar{\Pi}(\bar{s}, g) < \bar{\Pi}(s^*, \bar{\bar{g}}) - \bar{\Pi}(\bar{s}, \bar{\bar{g}}) \qquad if\ \bar{s} < s^*$

and so the relative payoff of an ESS-strategist *increases*, if the aggregate changes from g to $\bar{\bar{g}} < g$. If it has been positive before (with just m mutants), it will also be positive after the introduction of an additional mutant. Equivalently, the mutants relative payoff decreases as before.

Hehenkamp et al. (2004) heuristically explained this result by treating m -the number of mutants- as a continuous variable and pointing to the fact, that

$$\frac{\partial \Pi_M^m}{\partial m} = -\left[\frac{\bar{s}^r - s^{*^r}}{g(\bar{s}, ..., \bar{s}, s^*, ..., s^*)^r}\right]^2 \cdot V < 0 \qquad for\ all\ \bar{s} \neq s^*$$

We now see that the property of submodularity accounts for the minus sign in this expression exclusively. Consequently, submodularity ensures that stability against 'small' invasions is synonymous with stability against *any* invasion.

References

[1] Alōs-Ferrer, C. and A. Ania, 2004, The evolutionary stability of perfectly competitive behavior, Economic Theory, forthcoming

[2] Corchon, L. 1994, Comparative statics for aggregative games, Mathematical Social Science, 28, 151-165

[3] Ellison, G., 2000, Basins of attraction, long-run stochastic stability, and the speed of step-by-step evolution, Review of Economic Studies, 67, 17-45

[4] Hamilton, W., 1970, Selfish and spiteful behavior in an evolutionary model, Nature 228, 1218-1220

[5] Hehenkamp, B., Leininger, W., and A. Possajennikov, 2004, Evolutionary equilibrium in Tullock contests: Spite and overdissipation, European Journal of Political Economy, forthcoming

[6] Leininger, W., 2002, Contests over public goods: Evolutionarily stability and the free-rider problem, CESifo Working Paper No. 713, Munich

[7] Leininger, W., 2003, On evolutionarily stable behavior in contests, Economics of Governance, 4, 177-186

[8] Maynard Smith, J., 1974, Evolution and the theory of games, Cambridge University Press

[9] Possajennikov, A., 2003, Evolutionary foundations of aggregate-taking behavior, Economic Theory, 21, 921-928

[10] Schaffer, M., 1988, Evolutionarily stable strategies for a finite population and a variable contest size, Journal of Theoretical Biology, 132, 469-478

[11] Schaffer, M., 1989, Are Profit-maximizers the best survivers?, Journal of Economic Behavior and Organization, 12, 29-45

[12] Schenk-Hoppē, R., 2000, The evolution of Walrasian behavior in oligopolies, Journal of Mathematical Economics, 33, 35-55

[13] Topkis, D., 1998, Supermodularity and complementarity, Princeton University Press, Princeton, NJ

[14] Tullock, G., 1980, Efficient rent-seeking, in: Buchanan, J., Tollison, R. and G. Tullock (eds.), Towards a theory of the rent-seeking society, Texas, A&M University Press, 3-15

[15] Vega-Redondo, F., 1997, The evolution of Walrasian behavior, Econometrica, 65, 375-384

Appendix 1:

We have read the property of decreasing differences of the pay-off function $\bar{\Pi}$ as a property of the *relative* pay-off function $\bar{\Pi}(s', g) - \bar{\Pi}(s, g)$ to see its natural relation to the property of an ESS, which has to maximize this relative pay-off. As stated this is a cardinal property of $\bar{\Pi}$. A well-known ordinal version of it is the socalled single-crossing property, SCP.

<u>Definition 5</u>:

A generalized aggregative game Γ is quasi-submodular, if $\bar{\Pi}$ satisfies the single-crossing property (SCP); i.e. for all $s' \geq s$ and $g' \geq g$

$$\bar{\Pi}(s', g) - \bar{\Pi}(s, g) \leq 0 \;\; \textit{implies} \;\; \bar{\Pi}(s', g') - \bar{\Pi}(s, g') \leq 0$$

If all inequalities are strict ones, the game is strictly quasi-submodular.

<u>Remark</u>: If a game is submodular; i.e. has decreasing differences, then it is also quasi-submodular; i.e. satisfies SCP:
if for all $s' \geq s$ and $g' \geq g$ we have that

$$\bar{\Pi}(s', g') - \bar{\Pi}(s, g') \leq \bar{\Pi}(s', g) - \bar{\Pi}(s, g)$$

then the latter difference being non-positive, of course, implies that the former must be likewise. Clearly, decreasing differences of $\bar{\Pi}$ imply SCP of $\bar{\Pi}$.

Our Theorem also holds under the weaker assumption of SCP for $\bar{\Pi}$:

<u>Theorem</u>(ordinal version):

Let Γ be a (strictly) quasi-submodular generalized aggregative game.
Then any ESS of Γ is (strictly) globally stable.

<u>Proof</u>: Let $s^* \in S$ be an ESS of Γ; i.e. it holds that

$$\bar{\Pi}(s^*, g(s, s^*, ..., s^*)) \geq \bar{\Pi}(s, g(s, s^*, ..., s^*)) \qquad \textit{for all } s \in S.$$

The introduction hypothesis (IH) is that the same holds for m mutants. That is,

$$\bar{\Pi}(s^*, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*)) \geq \bar{\Pi}(s, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*)) \qquad \textit{for all } s \in S.$$

We now want to show that the inequality will also hold for m+1 mutants by invoking aggregativeness and (SCP).

i) Suppose $s^* < s$, then $g(s, \overbrace{...}^{m}, s, s^*, ..., s^*) < g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)$. By (IH)

$$\bar{\Pi}(s, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*)) - \bar{\Pi}(s^*, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*)) \leq 0.$$

Since $\bar{\Pi}$ satisfies (SCP), we have that

$$\bar{\Pi}(s, g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)) - \bar{\Pi}(s^*, g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)) \leq 0.$$

ii) Suppose now that $s^* > s$, then $g(s, \overbrace{...}^{m}, s, s^*, ..., s^*) > g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)$. By contradiction suppose that

$$\bar{\Pi}(s^*, g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)) - \bar{\Pi}(s, g(s, \overbrace{...}^{m+1}, s, s^*, ..., s^*)) < 0.$$

Since $\bar{\Pi}$ satisfies (SCP), we have that

$$\bar{\Pi}(s^*, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*)) - \bar{\Pi}(s, g(s, \overbrace{...}^{m}, s, s^*, ..., s^*)) < 0,$$

which contradicts (IH).

Analogously, it follows that if (SCP) holds strictly, then an ESS is strictly globally stable. $\square$

Analogously, the Corollary generalizes:

<u>Corollary</u>(ordinal version):

Let Γ be a quasi-submodular generalized aggregative game.
Then any strict ESS is strictly globally stable.

Appendix 2:

If one defines supermodularity of a generalized aggregative game Γ as $\bar{\Pi}$ having *increasing* differences; i.e. for all $s' \geq s$ and $g' \geq g$

$$\bar{\Pi}(s', g') - \bar{\Pi}(s, g') \geq \bar{\Pi}(s', g) - \Pi(s, g),$$

and quasi-supermodularity as the property, for all $s' > s$ and $g' > g$,

$$\bar{\Pi}(s', g) - \bar{\Pi}(s, g) \geq 0 \;\; \textit{implies} \;\; \bar{\Pi}(s', g') - \bar{\Pi}(s, g') \geq 0$$

then the following implication can be derived. Again, quasi-supermodularity is implied by supermodularity; i.e. increasing differences of $\bar{\Pi}$.

<u>Lemma</u>:

> Let Γ be a (quasi-)supermodular generalized aggregative game. Then it holds, that if a strategy $s \in S$ is *not* m-stable, then it is *not* $(m+1)$-stable. Equivalently, if $s \in S$ is $(m+1)$-stable, then it is m-stable.

The Lemma shows, that the "intuitive" inclusion, namely that if $(m+1)$ mutants cannot invade a population, then neither can only m mutants, is, in fact, tied to the property of supermodularity, which expresses *complementarity* of s and g.
Our Theorem identifies submodularity - and hence *substitutability* between s and g - as the source of the "unintuitive" reverse inclusion, namely that if m mutants cannot invade, then neither can $(m+1)$ mutants.

CESifo Working Paper Series

(for full list see www.cesifo.de)

1204 Elmer Sterken, The Role of the IFO Business Climate Indicator and Asset Prices in German Monetary Policy, May 2004

1205 Jan Jacobs and Jan-Egbert Sturm, Do Ifo Indicators Help Explain Revisions in German Industrial Production?, May 2004

1206 Ulrich Woitek, Real Wages and Business Cycle Asymmetries, May 2004

1207 Burkhard Heer and Alfred Maußner, Computation of Business Cycle Models: A Comparison of Numerical Methods, June 2004

1208 Costas Hadjiyiannis, Panos Hatzipanayotou, and Michael S. Michael, Pollution and Capital Tax Competition within a Regional Block, June 2004

1209 Stephan Klasen and Thorsten Nestmann, Population, Population Density, and Technological Change, June 2004

1210 Wolfgang Ochel, Welfare Time Limits in the United States – Experiences with a New Welfare-to-Work Approach, June 2004

1211 Luis H. R. Alvarez and Erkki Koskela, Taxation and Rotation Age under Stochastic Forest Stand Value, June 2004

1212 Bernard M. S. van Praag, The Connexion Between Old and New Approaches to Financial Satisfaction, June 2004

1213 Hendrik Hakenes and Martin Peitz, Selling Reputation When Going out of Business, June 2004

1214 Heikki Oksanen, Public Pensions in the National Accounts and Public Finance Targets, June 2004

1215 Ernst Fehr, Alexander Klein, and Klaus M. Schmidt, Contracts, Fairness, and Incentives, June 2004

1216 Amihai Glazer, Vesa Kanniainen, and Panu Poutvaara, Initial Luck, Status-Seeking and Snowballs Lead to Corporate Success and Failure, June 2004

1217 Bum J. Kim and Harris Schlesinger, Adverse Selection in an Insurance Market with Government-Guaranteed Subsistence Levels, June 2004

1218 Armin Falk, Charitable Giving as a Gift Exchange – Evidence from a Field Experiment, June 2004

1219 Rainer Niemann, Asymmetric Taxation and Cross-Border Investment Decisions, June 2004

1220 Christian Holzner, Volker Meier, and Martin Werding, Time Limits on Welfare Use under Involuntary Unemployment, June 2004

1221 Michiel Evers, Ruud A. de Mooij, and Herman R. J. Vollebergh, Tax Competition under Minimum Rates: The Case of European Diesel Excises, June 2004

1222 S. Brock Blomberg and Gregory D. Hess, How Much Does Violence Tax Trade?, June 2004

1223 Josse Delfgaauw and Robert Dur, Incentives and Workers' Motivation in the Public Sector, June 2004

1224 Paul De Grauwe and Cláudia Costa Storti, The Effects of Monetary Policy: A Meta-Analysis, June 2004

1225 Volker Grossmann, How to Promote R&D-based Growth? Public Education Expenditure on Scientists and Engineers versus R&D Subsidies, June 2004

1226 Bart Cockx and Jean Ries, The Exhaustion of Unemployment Benefits in Belgium. Does it Enhance the Probability of Employment?, June 2004

1227 Bertil Holmlund, Sickness Absence and Search Unemployment, June 2004

1228 Klaas J. Beniers and Robert Dur, Politicians' Motivation, Political Culture, and Electoral Competition, June 2004

1229 M. Hashem Pesaran, General Diagnostic Tests for Cross Section Dependence in Panels, July 2004

1230 Wladimir Raymond, Pierre Mohnen, Franz Palm, and Sybrand Schim van der Loeff, An Empirically-Based Taxonomy of Dutch Manufacturing: Innovation Policy Implications, July 2004

1231 Stefan Homburg, A New Approach to Optimal Commodity Taxation, July 2004

1232 Lorenzo Cappellari and Stephen P. Jenkins, Modelling Low Pay Transition Probabilities, Accounting for Panel Attrition, Non-Response, and Initial Conditions, July 2004

1233 Cheng Hsiao and M. Hashem Pesaran, Random Coefficient Panel Data Models, July 2004

1234 Frederick van der Ploeg, The Welfare State, Redistribution and the Economy, Reciprocal Altruism, Consumer Rivalry and Second Best, July 2004

1235 Thomas Fuchs and Ludger Woessmann, What Accounts for International Differences in Student Performance? A Re-Examination Using PISA Data, July 2004

1236 Pascalis Raimondos-Møller and Alan D. Woodland, Measuring Tax Efficiency: A Tax Optimality Index, July 2004

1237 M. Hashem Pesaran, Davide Pettenuzzo, and Allan Timmermann, Forecasting Time Series Subject to Multiple Structural Breaks, July 2004

1238 Panu Poutvaara and Andreas Wagener, The Invisible Hand Plays Dice: Eventualities in Religious Markets, July 2004

1239 Eckhard Janeba, Moral Federalism, July 2004

1240 Robert S. Chirinko, Steven M. Fazzari, and Andrew P. Meyer, That Elusive Elasticity: A Long-Panel Approach to Estimating the Capital-Labor Substitution Elasticity, July 2004

1241 Hans Jarle Kind, Karen Helene Midelfart, Guttorm Schjelderup, Corporate Tax Systems, Multinational Enterprises, and Economic Integration, July 2004

1242 Vankatesh Bala and Ngo Van Long, International Trade and Cultural Diversity: A Model of Preference Selection, July 2004

1243 Wolfgang Eggert and Alfons J. Weichenrieder, On the Economics of Bottle Deposits, July 2004

1244 Sören Blomquist and Vidar Christiansen, Taxation and Heterogeneous Preferences, July 2004

1245 Rafael Lalive and Alois Stutzer, Approval of Equal Rights and Gender Differences in Well-Being, July 2004

1246 Paolo M. Panteghini, Wide vs. Narrow Tax Bases under Optimal Investment Timing, July 2004

1247 Marika Karanassou, Hector Sala, and Dennis J. Snower, Unemployment in the European Union: Institutions, Prices, and Growth, July 2004

1248 Engin Dalgic and Ngo Van Long, Corrupt Local Government as Resource Farmers: The Helping Hand and the Grabbing Hand, July 2004

1249 Francesco Giavazzi and Guido Tabellini, Economic and Political Liberalizations, July 2004

1250 Yin-Wong Cheung and Jude Yuen, An Output Perspective on a Northeast Asia Currency Union, August 2004

1251 Ralf Elsas, Frank Heinemann, and Marcel Tyrell, Multiple but Asymmetric Bank Financing: The Case of Relationship Lending, August 2004

1252 Steinar Holden, Wage Formation under Low Inflation, August 2004

1253 Ngo Van Long and Gerhard Sorger, Insecure Property Rights and Growth: The Roles of Appropriation Costs, Wealth Effects, and Heterogeneity, August 2004

1254 Klaus Wälde and Pia Weiß, International Competition, Slim Firms and Wage Inequality, August 2004

1255 Jeremy S. S. Edwards and Alfons J. Weichenrieder, How Weak is the Weakest-Link Principle? On the Measurement of Firm Owners' Control Rights, August 2004

1256 Guido Tabellini, The Role of the State in Economic Development, August 2004

1257 François Larmande and Jean-Pierre Ponssard, EVA and the Controllability-congruence Trade-off: An Empirical Investigation, August 2004

1258 Vesa Kanniainen and Jenni Pääkkönen, Anonymous Money, Moral Sentiments and Welfare, August 2004

1259 Panu Poutvaara and Andreas Wagener, Why is the Public Sector More Labor-Intensive? A Distortionary Tax Argument, August 2004

1260 Lars P. Feld and Stefan Voigt, Making Judges Independent – Some Proposals Regarding the Judiciary, August 2004

1261 Joop Hartog, Hans van Ophem, and Simona Maria Bajdechi, How Risky is Investment in Human Capital?, August 2004

1262 Thomas Eichner and Rüdiger Pethig, Efficient Nonanthropocentric Nature Protection, August 2004

1263 David-Jan Jansen and Jakob de Haan, Look Who's Talking: ECB Communication during the First Years of EMU, August 2004

1264 David F. Bradford, The X Tax in the World Economy, August 2004

1265 Hans-Werner Sinn, Migration, Social Standards and Replacement Incomes. How to Protect Low-income Workers in the Industrialized Countries against the Forces of Globalization and Market Integration, August 2004

1266 Wolfgang Leininger, Fending off one Means Fending off all: Evolutionary Stability in Submodular Games, August 2004