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Portfolio Sales and Signaling

Spiros Bougheas
Tim Worrall

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Abstract

A common practice of banks has been to pool assets of different qualities and then sell a fraction of the newly created portfolios to investors. We extend the signaling model for single sales of risky assets to portfolio sales. We identify conditions under which signaling at the portfolio level dominates signaling at the single asset level. In particular, when banks have better information about loan types on their books, and some commitment power to sales, can profit by pooling assets whilst retaining a skin in the game.

JEL-Codes: D820, G210, G230.

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*Spiros Bougheas**
School of Economics
University of Nottingham
University Park
United Kingdom – Nottingham NG7 2RD
spiros.bougheas@nottingham.ac.uk

Tim Worrall
School of Economics
University of Edinburgh
31 Buccleuch Place
United Kingdom – Edinburgh EH8 9JT
tim.worrall@ed.ac.uk

*corresponding author

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1. Introduction

In the aftermath of the 2008 financial crisis we learned something more about securitization and about the selling and retention strategies that banks had employed. For example, it has been documented that many banks were keeping a high proportion of the securities that they were creating by pooling their assets on their own balance-sheets (see, for example, Acharya et al., 2009). As Gorton and Metrick (2013) have observed the loan pools have been homogeneous. In fact, homogeneity does not only apply to types of loans (e.g. credit cards, mortgages, etc.) but also to broad categories of risk (prime, sub-prime, etc.). However, within these broad categories of risk there was still a lot of variability in the risk level across individual assets within the pools (see, for example, Keys et al., 2010).

One possible explanation for the retention strategies of banks has been offered by DeMarzo and Duffie (1999). They argue that if banks have superior information about the risk of the securities that they attempt to sell to investors, then they can signal the risk of these securities by keeping a fraction on their books as 'skin in the game'.[2] Their model predicts that the fraction retained is inversely related to the risk of the securities. Empirical support for this prediction is offered by Chen et al. (2008) and Demiroglu and James (2012), who study the relationship between loan quality and the skin in the game. Both find that the skin in the game is inversely related to the risk of the securities retained. However, the theory applies to individual securities or maybe to portfolios as long as these portfolios are homogeneous. This raises the question of whether we can still account for the retention strategies of banks given that what they offer for sale are portfolios consisting of assets bearing various levels of risk.

In this paper we extend the DeMarzo and Duffie (1999) model to allow banks to choose between selling assets individually or as portfolios. Our model can account for both the pooling and the retention strategies of banks, as long as we introduce some type of commitment on the side of banks. We demonstrate that, if the bank can commit either to a menu of contracts or to sell its whole portfolio but without committing to the size of each pool offered for sale, portfolio sales can dominate single asset sales. More importantly, we show that the portfolios that banks sell are not composed of assets of the same quality. Some degree of commitment from banks is required for this result. However, we are not the first to argue that commitment is important to the understanding of securitization. Gorton and Souleles (2007) have argued that the willingness of banks to subsidize special purpose vehicles by buying back low

[2] This is a direct application of Leland and Pyle (1977) work on corporate financial structure.

quality assets can be interpreted as a form of commitment, known as 'implicit recourse'. In our model, the commitment of banks to a menu of contracts plays a similar role.[3]

We consider a bank that wishes to sell two assets (loans) to investors.[4] Each asset can be either high-quality (high probability of repayment) or low-quality. Returns on the two assets are assumed to be independently distributed. First, we replicate the main result in the signaling literature by showing that if the bank sells loans separately, the bank will signal the quality level of each asset by only keeping a fraction of the high quality asset on its books and will sell all low quality loans. Then, we consider an alternative selling strategy where the bank sells the two assets together as a portfolio. With two loans, and two assets, there are three possible portfolio types: both assets are high quality (H), one is high quality and the other is low quality (M), or both are low quality (L). In this case, the bank's optimal signaling strategy involves retaining a higher fraction of the portfolio when both assets are high quality than when one of the assets is low quality and retaining none of the portfolio if both assets are of low quality. In comparing the two selling strategies, we find that selling separately dominates selling as a portfolio. The intuition for this result is that in the latter case there is an extra type to be separated and signaling is costly.

Next, we allow the bank to engage in more complex strategies that may involve pooling two or more portfolio types. The advantage of pooling two portfolios is that it can reduce signaling costs. However, the disadvantage is that additional incentive constraints must be satisfied. We identify parameter values where such a mix of pooling portfolios is optimal.[5] One such strategy is for the bank to pool the two extreme portfolio types together, in which case it would keep as skin in the game a fraction of the pooled portfolio. If a bank would follow this strategy, then over time its portfolio of assets on its books would include similar fractions of both types of loans. The intuition for this result is as follows. Clearly the bank cannot do better than pooling the two types of assets together, which, given the competitive market for loans, has a payoff equal to the expected value of each loan. (In this case there is no difference between selling each asset separately or the two assets together.) However, this pooling strategy might not be credible because the bank might have an incentive to keep the high quality assets on its books. Then, the bank must use signaling. The bank's payoff is higher when it pools the two extreme portfolios relative to

[3] Gorton and Souleles (2007) also demonstrate using trigger strategies that in a repeated context such commitment can be credible (is self-enforcing).

[4] As in Shleifer and Vishny (2010) the bank makes profits by collecting fees when offering new loans.

[5] We also identify parameter values such that pure pooling and pure signaling equilibria are optimal.

its payoff when it sells the two assets separately because the skin in the game in the former case is lower. The reason is that the 'skin in the game' is increasing in the difference between the qualities of the assets that the bank is trying to sell. The difference in the expected payoffs of pooling the two extreme portfolios (two low-quality-risk assets and two high-quality assets) and the medium-risk (one low-quality asset and one high-quality asset) is relatively small and thus, the bank can signal the difference by retaining only a small fraction of the pooled portfolio. In contrast, the differences in expected payoffs between the high-quality asset and the low-quality asset is larger and requires a higher 'skin in the game'.[6]

Our paper is closely related to other work on signaling in the securitization literature that builds on the seminal work of Leland and Pyle (1977). This literature includes DeMarzo and Duffie (1999) and DeMarzo (2005) mentioned above.[7] These papers demonstrate that signaling can be further enhanced by tranching the payoffs and keeping on the books a fraction of the equity tranche.[8] Lastly, in Downing and Jaffee (2009) issuers do not have the option of retention and therefore the analysis is similar to that for a market for lemons.

It is interesting to compare the role of pooling of assets in the present paper from that in the pooling and tranching literature. Pooling and tranching refers to the pooling of assets (mortgages) whose returns are independently distributed and then creating new assets that vary in risk whose returns depend on the proportion of original assets that default. It is usually assumed that the assets in these pools have similar risk characteristics. But as we argued above, evidence suggests that this was not the case before the financial crisis. In the mechanism presented in this paper, portfolios are created by pooling together high quality (low risk) and low quality (high risk) assets. Separation through signaling is achieved at the portfolio level and thus the retained portfolio might be comprised of a mix of high-risk and low-risk assets. In any case, the two portfolio strategies are complementary and our analysis can be applied to

[6] The other strategy that combines pooling and signaling is for the bank to pool the mixed portfolio (one high-quality and one low quality asset) with the low quality portfolio together. Because the gap between the expected returns of the high quality portfolio and the pooled portfolio is lower than the gap between the returns of the high quality asset (which is the same as the return of the high quality portfolio) and the low quality asset, the bank has to offer a lower 'skin in the game' in the former case. Once more, this strategy is not always credible as the bank might have an incentive to keep the mixed portfolio on its books. When this strategy is credible the bank retains on its books only the high-quality asset.

[7] In contrast to these static models, Hartman-Glaser (2017) considers a dynamic environment where the issuer (bank) builds reputation over time. He shows that reputation mitigates the power of retention as a signaling device. For the optimality of more complex retention strategies in a dynamic moral hazard model, see Pagès (2013).

[8] These are examples of 'horizontal' slicing of payoffs in contrast to the earlier literature that focused on retention of a 'vertical' slice (see, for example, Gorton and Pennacchi, 1995).

the selling of tranches.[9]

We organize the paper as follows. In Section 2 we develop the model and derive some preliminary results. The main result of the paper is derived in Section 3. In order to keep the analysis simple in the main part of the paper we have assumed that there is only one round of sales. In Section 4 we relax this assumption and show how the analysis can be extended to multiple rounds. In section 5 we analyze an alternative form of commitment where the bank commits to the size of the portfolio that it will offer for sale. We offer some final comments in Section 6. Proofs not given in the text are relegated to the Appendix.

2. The Model

The economy lasts for one period and consists of banks and investors. All agents are risk-neutral. At the beginning of the period the bank offers loans of unit size to finance two risky projects. There are two types of loans. Loans of type $j \in \{h, l\}$ repay R with probability π_j and fail to repay anything with probability $1 - \pi_j$; where $\pi_h > \pi_l$. We assume that both types of loans have positive net present value; $\beta\pi_l R > 1$, where the discount factor β captures the time preference, common to all agents. Let θ denote the probability that a loan is of type h. Loan types are independently distributed. Let $\psi := \theta\pi_h + (1-\theta)\pi_l$ be the unconditional probability that a loan repays R. The unconditional probability is relevant both for investors, who do not know the loan type, and for banks when taking on new loans. With two loans there are three portfolio types. With probability θ^2 both loans are of type h (portfolio H), with probability $(1-\theta)^2$ both loans are of type l (portfolio L) and with probability $2\theta(1-\theta)$ one of the loans is of type h and the other loan is of type l (portfolio M). The probability θ and the size of the bank's portfolio are common knowledge to the bank and potential investors.

We assume that at the beginning of the period, before the bank knows the type of portfolio it has, the bank fully commits to a set of contracts for selling its portfolio to investors. We will show that full commitment maximizes the bank's *ex ante* payoff allowing it to use a pooling sales strategy. Commitment is required because such strategies will not necessarily maximize the bank's *ex post* payoff once it has learned its portfolio type. We will demonstrate that full commitment can explain why banks sell their

[9] For theoretical work on optimal retention strategies for tranches, see the screening models by Fender and Mitchell (2009) and Kiff and Kisser (2010) and the delegating monitoring model of Bougheas (2014). For related empirical evidence see Acharya and Schnabl (2009) and Jaffee et al. (2009).

assets as portfolios but do not necessarily pool together all asset types. After the announcement of sales contracts, the bank learns the types of its loans (and as a consequence the type of its portfolio).[10] This information is private. The bank can either keep the loans on its books or it can try to sell them to investors. If sold, the bank can finance new loans. The bank collects a fee f when it signs a new loan agreement. The role of the fee, as in Shleifer and Vishny (2010), is as a devise to generate trade between the bank and outside investors at the start of the trading period. The bank can also choose between selling the whole portfolio and single loan sales. When the bank keeps a loan on its books, it has to wait until the end of the period to receive a payoff. In contrast, when the bank sells the loan and uses the proceeds to make new loans it collects fees at the beginning of the period and purchases a new loan of unknown type. Define $\phi := f + \beta\psi R$ to be the expected discounted payoff a bank anticipates from selling a unit of loan: the arrangement fee from the new loan plus the expected discounted return from the new loan. By assumption $\phi > 1$. Potential investors, purchasers of loans, observe loans and the retention strategies of the bank but do not observe the loan, or portfolio, type. The market for loans is competitive.

Since investors do not observe the type of loan, there is a lemons problem. The maximum amount that investors are willing to pay for a loan is $\beta\pi_l R$. This is because they know if they offer to pay $\beta\psi R$, i.e., the expected loan payoff, then the bank will only sell l type loans. Since keeping a loan on its books is costly in terms of the fee forgone, the bank might be able to use a retention strategy, that is, keeping a fraction of a loan on its books as 'skin in the game', to signal the quality of the loan to investors. To keep things simple we have restricted our analysis to the case of loans of two quality grades and thus to portfolios of three quality grades. Nevertheless will will show that for any number of quality grades the 'skin in the game', d^q, for a given quality q, is given by

$$d^q = \frac{\phi\left(\frac{p_q}{p_{q^-}} - 1\right) + d^{q^-}(\phi - 1)}{\phi\left(\frac{p_q}{p_{q^-}}\right) - 1}$$

where q^- denotes the quality one level below q, and p_q and p_{q^-} denote the competitive prices of the corresponding assets.[11] Furthermore, when q^- is the lowest quality level then we have $d^{q^-} = 0$. Essentially,

[10] The lag between the announcement and the learning of types captures the period during which the bank learns the type of its portfolio (prime, sub-prime, etc.). The assumption that bank knows the type of loans on its books after purchase but does not know the loan type at the time of purchase is clearly an extreme one. It is however, meant to capture the idea that bank has better information after it has had loans on it books for some period.

[11] This equation will be explained fully in the next section.

the higher the quality of the loan portfolio, the lower is the cost of retaining part of the portfolio on the books and therefore banks are able to signal quality through a retention strategy.

We begin by considering the case of single loan sales and then we will analyze portfolio sales.

2.1. Single Loan Sales

In this subsection, we assume that investors know that the bank has two loans and can observe the fraction of each loan the bank retains on its books. The analysis here is standard, but it is useful to consider this case as a benchmark for the case of portfolio sales.

We first consider a separating equilibrium. Let d^j denote the fraction of a loan of type j that the bank keeps on its books. Let p_j denote the price of a loan of type j. The profit to the bank with a loan of type i of retaining a fraction d^j of a loan on its books is:

$$U_{ij} := d^j \beta \pi_i R + (1 - d^j) p_j \phi.$$

The first term is the bank's expected payoff from keeping on its books a fraction d^j of the loan that has a success probability of π_i. The second term is the sales revenues $(1 - d^j)p_j$ from the fraction of the loan not retained times the expected discounted payoff a bank anticipates from selling a unit of loan $\phi = f + \psi\beta R$: that is, the arrangement fee f from the new loan plus the expected discounted return from the new loan $\psi\beta R$. Here the expected repayment probability of new loans is ψ because at the point of purchase the bank is uncertain about the loan type.

The maximum price that an investor will pay for a loan of type j is $\beta\pi_j R$, that is, the loan's discounted expected payoff. Given that the market for loans is competitive, the price of loans will be bid up to this maximum value:

$$p_j = \beta \pi_j R \text{ for each } j. \tag{1}$$

Proposition 1. *Under a separating equilibrium of prices and retention strategies with single loan sales we have:*

$$\begin{aligned} p_l = \beta\pi_l R, \quad p_h = \beta\pi_h R, \\ d^l = 0, \quad d^h = \hat{d}^h := \frac{\phi\left(\frac{p_h}{p_l} - 1\right)}{\phi\left(\frac{p_h}{p_l}\right) - 1} = \frac{\phi\,(\pi_h - \pi_l)}{\phi\pi_h - \pi_l}. \end{aligned} \tag{2}$$

The expression for d^h in equation (2) is the skin in the game retained by the bank to signal that the loan is high quality. Since $\phi > 1$ and $\pi_h > \pi_l$, $d^h \in (0, 1)$. The intuition for the result is quite straightforward. Since $\phi > 1$ and the price received is given by (1), the bank will prefer, ceteris paribus, to sell a loan rather than retain it. However, there are six incentive constraints to ensure that a bank prefers the payoff when selling the loans according to the true portfolio type than the payoff it would have by selling the loans as any of the other two portfolio types. Despite there being six incentive constraints, it can be shown that the two relevant constraints are that the bank with portfolio H prefers not to sell it as a portfolio L and vice-versa. If these two constraints are satisfied, then so are all the others (see the appendix for the full proof of the proposition). This means that the analysis of the single loans case is identical to the case where banks have only one loan that can be of high or low type. First, it is clear that there is no advantage to have $d^l > 0$ because signaling is costly. Therefore, for a bank selling all of its low type loan $U_{ll} = p_l\phi$. Whereas if it retains a fraction d^h and receives a price $p_h = \beta\pi_h R$ for the loans sold, its expected payoff is $U_{lh} = d^h\beta\pi_l R + (1 - d^h)p_h\phi = d^h p_l + (1 - d^h)p_h\phi$, since the value of the loans retained is p_l. Incentive compatibility require $U_{ll} \geq U_{lh}$, and also $U_{hh} \geq U_{hl}$. Combining these two conditions gives

$$d^h p_h \geq \left(p_l - (1 - d^h)p_h\right)\phi \geq d^h p_l. \tag{3}$$

Equivalently,

$$\frac{(p_h - p_l)\phi}{p_h(\phi - 1)} \geq d_h \geq \frac{(p_h - p_l)\phi}{p_h\phi - p_l}.$$

It is clear that the most relevant constraint is that the bank should not wish to sell a low type loan as a high type. This is the second inequality in (3). Where it is satisfied as equality ($U_{ll} = U_{lh}$), the value of d^h is given by the expression in equation (2). An increase in ϕ reduces the required retention rate $\hat{d}^h$ because the cost of signaling is increased. A rise in the ratio of π_h/π_l however, has the opposite effect because it makes passing off low quality loans as high quality more tempting and therefore the required retention rate to signal high quality increases.

In principle, any prices satisfying (1) and d^h satisfying (3) can be supported as a separating equilibrium.[12] However, as is standard, the separating equilibrium of Proposition 1 is the Pareto-dominant separating equilibrium that also satisfies the intuitive criterion. To see this suppose that d^h satisfies (3)

[12] With investor beliefs that retention $d < d^h$ corresponds to a low quality loan and any $d \geq d^h$ comes from a high quality loan.

and $d^h > \hat{d}^h$. Low types would never choose such a d^h. Thus, investors believe that any such deviation to a lower d^h must come from a high type and therefore is rewarded with the high price p^h. Given that the bank's payoff decreases with d^h, the amount retained will be decreased until $d^h = \hat{d}^h$.

Next, we consider the possibility of pooling equilibria. This requires that the bank has some form of commitment to a selling strategy before it knows its portfolio type. For this section, we consider that the bank can commit to a menu of contracts conditional on sales taking place. In Section 5, we will consider an alternative where can only commit to a menu of contracts with each investor separately but now it also commits to sell all its loans.[13] In a pooling equilibrium, the bank will not keep any fraction of the loan on its books.[14] In this case, competition amongst investors means that in this case the price of loan sales is bid up to $\beta\psi R$. For this to be an equilibrium the bank must prefer to sell its loans rather than retaining them. For a loan of type l, the bank's expected payoff from sales is $\phi\beta\psi R$ compared with $\beta\pi_l R$ from retention. Since $\phi > 1$ and $\psi > \pi_l$, it follows that sales are always better than retention. On the other hand, if the bank sells a high quality loan to investors, its payoff will be $\phi\beta\psi R$ compared to $\beta\pi_h R$ from retention. Therefore, the bank would prefer to keep the high quality loan on its books rather than earning the pooling payoff if $\phi < \pi_h/\psi$. Thus, pooling cannot be an equilibrium when $\phi < \pi_h/\psi$ and the bank will sell the loans to investors individually using the 'skin in the game' as signal.

Next, we need to compare signaling and pooling when $\phi \geq \pi_h/\psi$. The bank's ex ante payoff from pooling, V_P, is given by the value of selling all loans at the price of $\beta\psi R$,

$$V_P = 2\phi\beta\psi R.$$

The bank's ex ante payoff from signaling when the loans are sold separately, V_S, is computed as the weighted average of the payoff to each of the three possible portfolio types. Letting $\rho_H := \theta^2$, $\rho_M := 2\theta(1-\theta)$ and $\rho_L := (1-\theta)^2$, the ex ante payoff from signaling is given by

$$V_S = \rho_H 2\beta\pi_h R \left(d^h + \phi\left(1 - d^h\right)\right) + \rho_M \left(\beta\pi_h R \left(d^h + \phi\left(1 - d^h\right)\right) + \beta\pi_l R\phi\right) + \rho_L 2\beta\pi_l R\phi \tag{4}$$

[13] This turns out to be a stronger form of commitment. We view the commitment as a short cut to modeling repeated interactions that might generate similar effects endogenously.

[14] Keeping a fraction of the pooled portfolio on its books is only beneficial if it is used as signaling.

Comparing the two payoffs we find that

$$V_P - V_S = 2\theta\pi_h\beta R\,(\phi - 1)\,d^h > 0.$$

This is not surprising given that signaling is costly. Thus, as long as pooling is feasible and the bank can commit to a selling strategy, the bank will pool to sell its loans. When pooling is not feasible the bank uses costly signaling. In summary:

Proposition 2. *Suppose that the bank sells each loan separately to investors. Then,*

1. *If* $\phi < \frac{\pi_h}{\psi}$, *then the bank will sell the loans using signaling.*
2. *If* $\phi \geq \frac{\pi_h}{\psi}$, *then the bank will sell the loans using pooling.*

2.2. Portfolio Sales

Now we allow the bank to bundle the two loans and sell them as a portfolio. By doing so the bank creates a new asset with an intermediate level of risk. The analysis of portfolio sales follows closely the one above for single loan sales. Let d^i denote the fraction of a portfolio of type i ($i = H, M, L$) that the bank keeps on its books. The maximum prices, that an investor will pay for portfolios of type H, M and L are equal to $2\beta\pi_h R$, $\beta(\pi_h + \pi_l)R$ and $2\beta\pi_l R$, respectively, which correspond to the expected payoffs of these portfolios. Let $\pi_m := (1/2)(\pi_h + \pi_l)$. In this section, we only consider signaling equilibria that achieve compete separation of types. The following proposition describes the results.[15]

Proposition 3. *Under a complete separating equilibrium of prices and retention strategies with portfolio loan sales we have:*

$$p_L = \beta\pi_l R, \quad p_M = \beta\pi_m R, \quad p_H = \beta\pi_H R, \quad d^L = 0,$$

$$d^M = \hat{d}^M := \frac{\phi\left(\frac{\pi_m}{\pi_l} - 1\right)}{\phi\left(\frac{\pi_m}{\pi_l}\right) - 1},$$

and

$$d^H = \hat{d}^H := \frac{\phi\left(\frac{\pi_h}{\pi_m} - 1\right) + d^M\,(\phi - 1)}{\phi\left(\frac{\pi_h}{\pi_m}\right) - 1},$$

where $\hat{d}^H > \hat{d}^M$.

[15] The complete proofs for this section are provided in a Supplementary Appendix.

With three types of loans the bank now needs two signals to separate them. Once more, as in DeMarzo and Duffie (1999), the 'skin in the game' is decreasing with asset quality. It is also easy to establish that Proposition 2 also applies to portfolio sales as well as sales on individual loans. We state formally as:

Proposition 4. *Suppose that the bank can either keep the portfolios on its books or sell them to investors. Then,*

1. *If* $\phi < \frac{\pi_h}{\psi}$*, then the bank will sell the portfolios to investors using signaling,*
2. *If* $\phi \geq \frac{\pi_h}{\psi}$*, then the bank will sell the portfolios to investors using pooling.*

As for the case of single loan sales, costly signaling is only used when pooling generates a lower expected payoff.

The next proposition compares compares Propositions 2 and 4 to determine whether the bank will sell the loans separately or as a portfolio (proof in the Appendix).

Proposition 5. *Suppose that the bank can sell the loans either separately or as a portfolio. Then, the bank will sell them separately to investors.*

When the bank sells the loans separately there are only two types to be separated. In contrast, when the bank sells them as a portfolio there are three portfolio types. Given that signaling is costly it is not surprising that the bank chooses to sell them separately.

3. Mixed Pooling and Signaling

In this section we consider sales strategies that have involve a mix of pooling portfolios and signaling. In particular, we consider that a bank pools two of the three potential portfolios. The advantage of this strategy is that the bank will only have to separate two portfolios, the mixed portfolio and the unmixed portfolio an will use only signal. There are three potential portfolios to mix: a pool of portfolios H and M (HM), a pool of portfolios H and L (HL) and a pool of portfolios M and L (ML).

To calculate the skin in the game required for each mix let

$$\pi_{ij} := \frac{\rho_i \pi_i + \rho_j \pi_j}{\rho_i + \rho_j}$$

denote the conditional probability of the successful outcome if the portfolio mix is ij where $i \in \{H, M\}$ and $j \in \{M, L\}$, $i \neq j$.[16] It follows straightforwardly that $\pi_{HM} > \pi_l$, $\pi_h > \pi_{ML}$ and $\pi_{HL} \gtreqless \pi_m$ as $\theta \gtreqless 1/2$.[17] The expected discounted value, per share, of the portfolio mix ij is therefore $\beta\pi_{ij}R$.

First, it is clear that the bank will only want to pool a mix of portfolios if it prefers to sell the pool rather than retaining the higher quality portfolio on its books. Thus, a necessary condition for the mixed portfolio ij to be sold is[18]

$$\phi\pi_{\mathsf{ij}} \geq \pi_{\mathsf{i}}.$$

It is also clear that bank will need to signal by retaining a fraction of the higher quality asset. That is, for the mixed portfolio HM, it should retain a fraction of the portfolio HM, for the mixed portfolio ML, it should retain a fraction of the high quality asset and for the mixed portfolio HL it should retain a fraction of HL or M depending on whether θ is greater or less than 1/2.[19] Letting d^{ij} denote the fraction retained on the bank's books, it can be shown that

$$\begin{aligned}
U_{HM} &= 2\left((\rho_H + \rho_M)\left(d^{HM} + \phi(1 - d^{HM})\right)\beta\pi_{HM}R + \rho_L\phi\beta\pi_l R\right), \\
U_{HL} &= \begin{cases} 2\left((\rho_H + \rho_L)\left(d^{HL} + \phi(1 - d^{HL})\right)\beta\pi_{HL}R + \rho_M\phi\beta\pi_m R\right) & \text{for } \theta \geqslant \frac{1}{2} \\ 2\left(\rho_M\left(d^{HL} + \phi(1 - d^{HL})\right)\beta\pi_m R + (\rho_H + \rho_L)\phi\beta\pi_{HL}R\right) & \text{for } \theta < \frac{1}{2}, \end{cases} \\
U_{ML} &= 2\left(\rho_H\left(d^{ML} + \phi(1 - d^{ML})\right)\beta\pi_h R + (\rho_M + \rho_L)\phi\beta\pi_{ML}R\right).
\end{aligned} \tag{5}$$

Also, by using exactly the same steps as for the case of single loan sales, it can be shown that the skin in the game is given by $d^{ij} = \hat{d}^{ij}$ where

$$\hat{d}^{HM} := \frac{\phi\left(\pi_{HM} - \pi_l\right)}{\phi\pi_{HM} - \pi_l}; \quad \hat{d}^{HL} := \begin{cases} \frac{\phi(\pi_{HL} - \pi_m)}{\phi\pi_{HL} - \pi_m} & \text{for } \theta \geqslant \frac{1}{2} \\ \frac{\phi(\pi_m - \pi_{HL})}{\phi\pi_m - \pi_{HL}} & \text{for } \theta < \frac{1}{2}; \end{cases} \quad \hat{d}^{ML} := \frac{\phi\left(\pi_h - \pi_{ML}\right)}{\phi\pi_h - \pi_{ML}}. \tag{6}$$

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[16] Note there is some abuse of notation here. If $i = H$, then π_i should be interpreted as π_h and so on.

[17] For $\theta \gtreqless 1/2$, $\psi \gtreqless \pi_m$ and $\pi_{HL} \gtreqless \psi$.

[18] It is *ex post* optimal for the bank to keep only a fraction of the portfolio in its books and use it to signal its type. However, in the beginning of the period, the bank maximized its *ex ante* expected payoff by committing to a set of selling contracts and this type of signaling strategy was not among the proposals.

[19] If $\theta > 1/2$, then the mix of portfolios H and L has more of the higher quality loans on average than portfolio M. The reverse is true for $\theta < 1/2$. If $\theta = 1/2$, then there is no need to signal because the mix of portfolios H and M is exactly equivalent to portfolio M.

3.1. *Feasible Sales Strategies of Mixed Pooling and Signaling*

When $\phi \geq \frac{\pi_h}{\psi}$, the bank will prefer to pool and sell the whole portfolio to the market. When $\phi < \frac{\pi_h}{\psi}$, the bank will use signaling and may sell the loans separately or may pool two of the portfolios. Firstly, we identify sales strategies that are feasible and then compare them in order to choose the strategies that maximize the bank's payoff.

Proposition 6. *[Feasible Mixed Pooling and Signaling Strategies]*

1. *A pooled mix of portfolios H and M is feasible when* $\frac{\pi_h}{\pi_{HM}} < \phi \leqslant \frac{\pi_h}{\psi}$.
2. *A pooled mix of portfolios H and L is feasible when* $\theta \geqslant \frac{1}{2}$ *and* $\frac{\pi_h}{\pi_{HL}} < \phi \leqslant \frac{\pi_h}{\psi}$.
3. *A pooled mix of portfolios M and L is feasible when* $\frac{\pi_m}{\pi_{ML}} < \phi \leqslant \frac{\pi_h}{\psi}$.

Proposition 6 identifies parameter restrictions such that mixed pooling and signaling strategies are feasible. In comparison with single asset sales, each of these new strategies must satisfy an additional incentive constraint. When the bank pools two of the portfolios together its payoff must be higher than what it could obtain by keeping the higher quality portfolio in its books.

Next, we compare the payoffs associated with these strategies to the corresponding payoffs of alternative strategies.

3.2. *Optimal Sales Strategies*

From Proposition 4 we know that for $\phi \leq \frac{\pi_h}{\psi}$ we need to compare single loan sales and sales that use mixed pooling and signaling. When the loans are sold individually the bank's payoff is given by V_S from equation (4). Comparing V_S with the utilities derived from mixed pooling and signaling and comparing the mixed pooling strategies HL and ML, we obtain the following result:

Proposition 7. *(Optimal Sales Strategies):*

1. *If* $\phi < \frac{\pi_m}{\pi_{ML}}$, *then the bank will sell the loans to investors separately using signaling,*
2. *If* $\frac{\pi_m}{\pi_{ML}} \leq \phi < \min\left\{\frac{\pi_h}{\psi}, \frac{\pi_h}{\pi_{HL}}\right\}$, *then the bank will pool portfolios M and L,*
3. *If* $\frac{\pi_h}{\pi_{HL}} \leq \phi < \frac{\pi_h}{\psi}$, *then the bank will choose either to pool portfolios H and L or pool portfolios M and L. For* $\theta \in (1/2, (\pi_h + \pi_l)/(\pi_h + 2\pi_l))$, *the bank will pool portfolios H and L. For larger values of* θ, *there is a critical* $\phi^c(\theta) \in (\pi_h/\pi_{HL}, \pi_h/\psi)$ *such that the bank chooses to pool portfolio H and L for* $\phi > \phi^c(\theta)$ *and chooses to pool portfolios M and L for* $\phi < \phi^c(\theta)$.

4. *If $\phi \geqslant \frac{\pi_h}{\psi}$, then the bank will sell the loans to investors using pooling.*

Proposition 6 has identified parameter values such that mixed pooling strategies are feasible. Proposition 7 shows which strategies are most profitable for different parameter values. When pooling all portfolios is not desirable, then either pooling the mix of portfolios H and L or the mix of portfolios M and L offers the highest expected payoff. The intuition is as follows. The mixed portfolio ML is better than selling loans separately because the skin in the game for the mixed portfolio is only required if both loans are of high quality and the skin in the game, d^{ML}, is less than the skin in the game, d^h, required to signal the high quality when loans are signalled separately. For $\theta < 1/2$, the mixed portfolio HL will not be used because it will be dominated by pooling all loans. For $\theta \geq 1/2$, the skin in the game required for the portfolio HL is less than the skin in the game required to signal the high quality asset and therefore, the mixed portfolio HL will dominate. It can further be shown that the alternative mixed portfolio HM is never used because it is always dominated by separate loan sales. In particular, the reduction in the cost of signaling the pooled portfolio relative to the cost of signaling the high-quality asset is not sufficient to compensate for the decline in the value of the pooled portfolio relative to the value of the high-quality asset. In comparing HL with ML, the mixed portfolio HL requires a skin in the game whenever both loans are of the same quality, whereas the mixed portfolio ML requires a skin in the game only when both loans are of high quality. However, the skin in the game required for the mixed portfolio HL is lower than for the mixed portfolio ML: $d^{HL} < d^{ML}$.[20] A larger value of ϕ decreases the skin in the game required. The effect of having the lower skin in the game can be shown to benefit the mixed portfolio HL relatively more than the mixed portfolio ML.[21]

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[20] For θ close to 1/2, the skin in the game required for the portfolio HL will be small whereas the skin in the game for ML remains non-negligible, and hence, the portfolio HL dominates. For θ close to 1, the difference in the skin in the game is smaller but the probability that both loans are low quality becomes smaller faster and again the portfolio HL dominates. For intermediate values of θ, whether HL dominates or not depends on the parameter configuration of ϕ, π_h and π_l.

[21] It is shown in the Appendix that a sufficient condition for HL to dominate ML is $\phi > \phi^c$ where

$$\phi^c = 1 + \frac{(\pi_h - \pi_l)\left(\sqrt{\pi_h(\pi_h + \pi_l)} - \pi_h\right)}{2\pi_h^2}.$$

It can be checked that $\phi^c < 2\pi_h/(\pi_h + \pi_l)$ and therefore, there is a non-empty set of parameter values $\theta \geq 1/2$ and $\pi_h > \pi_l$ such that $\phi > \phi^c$ and $\pi_h/\pi_{HL} \leq \phi < \pi_h/\psi$. It can be shown that $\phi^c \leq (10 + 7\sqrt{7})/27 \approx 1.0563059$. Since $\phi = f + \beta\psi R$ and $\beta\pi_l R > 1$, the fee required on sales for this condition to be satisfied is quite low.

4. Multiple Sales Rounds

In order to keep the exposition simple we have considered only one round of sales. In reality, banks keep recycling their assets by selling new loans and using the proceeds to offer new ones. Conceptually, it is straightforward to allow for multiple rounds. However, the complexity is increased considerably as the skin in the game can vary with each round.

In the main part of the paper we have assumed for simplicity of exposition that the original bank portfolio comprises of two loans. Given that there are two loan qualities there are three possible portfolios. When we consider multiple rounds we face two related problems: firstly, there is an 'integer problem' related to the number of new loans and, secondly, there are complications in deriving the composition of subsequent portfolios. We can avoid these problems by considering the case where the portfolio consists of a continuum of assets. Clearly, selling assets individually becomes tricky! Therefore, we concentrate on portfolio sales. Our intention here is to show how the analysis of the previous sections can, in principle, be extended to multiple sales rounds. With this in mind, we focus on the case where there is complete separation of three possible portfolio types.

Suppose that in each round the portfolio can only be one of the following three types: (a) all loans low quality (probability ρ_L), (b) all loans high quality (probability ρ_H) and (c) half the loans low quality and half the loans high quality (probability ρ_M). This has a close correspondence to the original model, but we restrict the portfolio types in a very arbitrary way. To simplify the exposition, we assume that there are two sub-periods. The multiple rounds of sales take place during the first sub-period, which is very short. (There is no discounting between rounds). At the end of the second sub-period all loans mature. The idea we try to capture is that the securitization process is very short relative to the duration of loan contracts (mortgages). Notice that after each round the bank faces exactly the same problem as in previous rounds (only the size of the portfolio changes) and therefore the 'skin in the game' will not vary.

As before, let d^i denote the fraction of a portfolio of type i ($i = H, M, L$) that the bank keeps on its books. Since markets are competitive markets, both sold and retained loans are priced at their expected value, Thus, the bank's payoff from the first round V_i^1 is given by the value of the retained loans plus the per unit fee f times the value of loans sold, that is,

$$V_i^1 = d^i p_i + f\left(1 - d^i\right) p_i.$$

Let

$$W := \rho_H p_H d^H + \rho_L p_L d^L + \rho_M p_M d^M,$$

and

$$Z := \rho_H \left(1 - d^H\right) p_H + \rho_L \left(1 - d^L\right) p_L + \rho_M \left(1 - d^M\right) p_M.$$

The term W is the expected value of loans to be retained in the next round and Z is the expected value of loans to be sold in the next round. We will restrict our attention to problems that satisfy the restriction $Z < 1$.[22] The bank's expected payoff from the second round can be shown to be given by

$$V_i^2 = (1 - d^i)p_i(W + fZ),$$

and the expected payoff from the third round is

$$V_i^3 = (1 - d^i)p_i(W + fZ)Z.$$

Then, by induction, we have, for $T \geq 2$

$$V_i^T = \left(1 - d^i\right) p_i \left(W + fZ\right) Z^{T-2}.$$

Adding the payoffs for all periods, we find that the bank's total expected payoff from portfolio i is equal to

$$\textstyle\sum_{t=0}^{\infty} \left(d^i p_i + \left(1 - d^i\right) p_i \left[f + \left(W + fZ\right) Z^t\right]\right) = d^i p_i + \left(1 - d^i\right) p_i \left(f + \frac{W + fZ}{1 - Z}\right).$$

Comparing the expression to the right of the inequality with the corresponding expression for single round sales, $d^i p_i + \left(1 - d^i\right) p_i(f + \beta\psi R)$, we find that the only difference is in the last term that includes the fees and retentions from all additional rounds. We can therefore, follow the same steps as those in the main section of the paper using these modified payoff functions. Conceptually, the problem is identical, however, technically is more complicated given that W and Z are functions of the d^is.

In the above example we have restricted our attention to complete separation strategies. By following the same steps as in the last section we can extend the analysis to portfolios. In principle, the method is

[22] An upper bound for Z is $\beta\psi R$, which is greater than one. Thus the assumption $Z < 1$ requires that the skin in the game is significant enough. If $Z \geq 1$, then the value of the bank's sales becomes infinite.

simple, however, the derivations can quickly become very complicated especially as the number of asset types, and consequently portfolio types, increase. But this observation might be the very answer to one of the puzzling questions that Gorton and Metrick (2013) have raised in relation to securitization; in their own words "The choice of loans to pool and sell to the SPV also remains a puzzle. Existing theories cannot address why securitized-loan pools are homogeneous – all credit cards or all prime mortgages, for example."

5. An Alternative Commitment Strategy

So far we have assumed that the bank can commit to a menu of contracts conditional on sales taking place. In this section, we demonstrate that pooling equilibria are also feasible when the bank can only commit to a menu of contracts with each investor separately but now it also commits to sell all its loans.[23] Thus, the bank can make credible agreements to individual investors about the contracts that it will use in future sales but cannot credibly commit to use the same contracts with other investors.

We are going to concentrate on the case of single sales as it is easier to analyze. It will become clear that a similar argument applies to the case of portfolio sales.

Thus far, we have assumed that when $\phi \geq \pi_h/\psi$ the bank can commit to offering pooling contracts to all investors. Now, we relax this assumption. That is, while an investor offered the pooling contact knows that by paying $\beta\psi R$ will be able to obtain one of the loans there is no way for the bank to convince the investor that it will make the same offer when selling the other loan.

If the bank were to allocate loans to investors randomly its inability to commit would not be a problem. In that case, investors would know that the expected probability of success of each loan would be equal to ψ and the loan has been priced accordingly. However, the bank might be able to increase its payoffs by committing to offer a pooling contract to only one of the investors while using the signaling mechanism to sell the other loan even if its expected payoff from pooling is higher than its expected payoff form signaling.

The way the bank can potentially increase its expected payoff is by allocating the loans non-randomly when its portfolio type is M. In particular, consider the case when it offers the type l loan to the investor to whom the bank has committed the pooling contract, while it sells the type h loan using signaling. Keep

[23] The bank might still have to keep a fraction of its portfolio on its books as 'skin in the game'.

in mind, that at the time of the sale, investors cannot observe loan types. While this action will keep the bank's expected payoff from the pooling contract the same, it would increase the bank's expected payoff from the loan that it sells using signaling. This payoff is now given by:

$$(\rho_H + \rho_M)\, \beta\pi_h R \left(d^h + \phi\left(1 - d^h\right)\right) + \rho_L \beta\pi_l R\phi$$

The benefit of this strategy is clear.[24] The bank will have no incentive to deviate to this strategy rather than seeling all its loans when

$$(\rho_H + \rho_M)\, \beta\pi_h R \left(d^h + \phi\left(1 - d^h\right)\right) + \rho_L \beta\pi_l R\phi \leq \beta\psi R\phi.$$

With d^h given in equation (2) and $\psi = \theta\pi_h + (1-\theta)\pi_l$, this inequality can be simplified to

$$\phi \geq \frac{(2-\theta)\,\pi_h - (1-\theta)\,\pi_l}{\pi_h}. \tag{7}$$

Recall from Proposition 2, that the pooling equilibrium exists when $\phi \geq \pi_h/\psi$. It is easy to check that

$$\frac{\pi_h}{\psi} \geq \frac{(2-\theta)\,\pi_h - (1-\theta)\,\pi_l}{\pi_h}$$

with equality only when $\theta = 1$. Thus, equation (7) shows that the commitment considered in this section is stronger than the commitment assumed in previous sections (there are some additional parameter values where pooling is feasible) because of the ability of the bank to sell its whole portfolio even though it does not commit to offer the same contract to all investors. Some commitment by banks is however, needed because if the bank were completely unable to make any commitments, then pooling would not be credible.

[24] If the bank were to allocate the loans randomly its expected payoff from signaling would be equal to

$$\theta\beta\pi_h R \left(d^h + \phi\left(1 - d^h\right)\right) + (1-\theta)\,\beta\pi_l R\phi.$$

6. Conclusion

We have extended the signaling model for single sales of risky assets to portfolio sales. We have identified conditions under which signaling at the portfolio level dominates signaling at the single asset level. In addition to contributing to the signaling literature we have offered an explanation for the portfolio sales and retention strategies that banks have commonly used. Our work has also identified some feasibility constraints on portfolio strategies thus providing a solution to the Gorton and Metrick (2013) puzzle discussed at the end of Section 4.

We close by making an observation on the robustness of our results. In order to keep the analysis simple we have assumed that the bank holds only two uncorrelated assets (loans). This is purely for simplicity. The advantage of allowing the bank to sell portfolios of the two assets is that pooling allows the bank to create new quality grades - in our case typically the medium-quality portfolio. If there are more than two assets then this expands the possibilities pooling portfolios and our results will still hold. Equally, as long as the returns are not perfectly correlated, the motivation for pooling of portfolios remains and qualitatively the nature of our result will be unchanged.

7. Appendix

7.1. Proof of Proposition 1

Lemma 1 *Any solution that satisfies* IC2 and IC6 *will also satisfy* IC1*,* IC3*,* IC4 *and* IC5*.*

Proof There are six incentive compatibility constraints.

$$2\left(\beta d^h \pi_h R + \phi\left(1-d^h\right)p_h\right) \geq \beta d^h \pi_h R + \phi\left(1-d^h\right)p_h + \beta d^l \pi_h R + \phi\left(1-d^l\right)p_l \tag{IC1}$$

$$2\left(\beta d^h \pi_h R + \phi\left(1-d^h\right)p_h\right) \geq 2\left(\beta d^l \pi_h R + \phi\left(1-d^l\right)p_l\right) \tag{IC2}$$

$$\beta d^h \pi_h R + \phi\left(1-d^h\right)p_h + \beta d^l \pi_l R + \phi\left(1-d^l\right)p_l \geq \beta d^h \pi_h R + \beta d^h \pi_l R + 2\phi\left(1-d^h\right)p_h \tag{IC3}$$

$$\beta d^h \pi_h R + \phi\left(1-d^h\right)p_h + \beta d^l \pi_l R + \phi\left(1-d^l\right)p_l \geq \beta d^l \pi_h R + \beta d^l \pi_l R + 2\phi\left(1-d^l\right)p_l \tag{IC4}$$

$$2\left(\beta d^l \pi_l R + \phi\left(1-d^l\right)p_l\right) \geq \beta d^h \pi_l R + \phi\left(1-d^h\right)p_h + \beta d^l \pi_l R + \phi\left(1-d^l\right)p_l \tag{IC5}$$

$$2\left(\beta d^l \pi_l R + \phi\left(1-d^l\right)p_l\right) \geq \left(\beta d^h \pi_l R + \phi\left(1-d^h\right)p_h\right) \tag{IC6}$$

IC1 states that when the portfolio type is H the bank prefers to sell each loan as type h rather than one loan as type h and the other as type l. IC2 states that when the portfolio type is H the bank prefers to sell each loan as type h rather than selling each loan as type l. IC3 states that when the portfolio type is M the bank prefers to sell the type h loan as type h and the type l loan as type l rather than selling both loans as type h. IC4 states that when the portfolio type is M the bank prefers to sell the type h loan as type h and the type l loan as type l rather than selling both loans as type l. IC5 states that when the portfolio type is L the bank prefers to sell each loan as type l rather than one loan as type h and the other as type l. IC6 states that when the portfolio type is L the bank prefers to sell each loan as type l rather than selling each loan as type h. IC6 repeats condition IC from the text.

Comparing (IC1) and (IC2) it follows that if (IC2) is satisfied, so is (IC1). Comparing IC5 and IC6 it follows that if IC6 is satisfied so is IC5. Subtracting $\beta d^h \pi_h R + \phi\left(1-d^h\right) p_h$ from both sides of IC3 we obtain IC2. Subtracting $\beta d^l \pi_l R + \phi\left(1-d^l\right) p_l$ from both sides of IC4 we obtain IC6. QED

We can combine IC2 and IC6 to get

$$\beta\left(d^h - d^l\right) \pi_h R \geq \phi\left(\left(1-d^l\right) p_l - \left(1-d^h\right) p_h\ \right) \geq \beta\left(d^h - d^l\right) \pi_l R \tag{A.1}$$

Lemma 2 $p_h \geq p_l$.

Proof Given that bank's payoff is increasing in p_h and p_l in any signaling equilibrium at least one of the two constraints described by (1) must bind. This is because we can always increase both in such a way that leaves $\left(1-d^l\right) p_l - \left(1-d^h\right) p_h$ constant. If $p_h = \beta\pi_h R$ then the lemma is trivially satisfied. Suppose that $p_l = \beta\pi_l R$ and that $\beta\pi_l R > p_h$ and that (A1) is satisfied. Then set $p_h = \beta\pi_l R$ clearly increasing the bank's payoff. Given that $\beta\psi R > 1$ the second inequality is still satisfied. Increasing p_h also relaxes the first constraint and therefore we have a contradiction. QED

Lemma 3 $d^l = 0$.

Proof Given that $\pi_h R > \pi_l R$, (A1) implies that $d^h > d^l$. Further, notice that if a signaling equilibrium exists (2) implies that the bank's payoff will be decreasing in d^h and d^l. Suppose that the first constraint is not binding. Then decrease d^h and d^l by the same amount so that either $d^l = 0$ or the first constraint binds. Suppose that the second constraint is not binding. Then reduce d^h and d^l so that $\left(1-d^l\right) p_l - \left(1-d^h\right) p_h$ stays constant so that either $d^l = 0$ or the second constraint binds. Then the lemma follows from the fact that at least one of the constraints is not binding. QED

Lemma 3 and (A1) imply that

$$\beta d^h \pi_h R \geq \phi\left(p_l - \left(1 - d^h\right) p_h\ \right) \geq \beta d^h \pi_l R \tag{A.2}$$

Lemma 4 *$p_l = \beta \pi_l R$.*

Proof Suppose not. Increasing p_l relaxes the second constraint in (A2). Before we have argued that if $p_l < \beta \pi_l R$ then it must be the case that $p_h = \beta \pi_h R$. Suppose that the first constraint binds. Then increase p_l and decrease d^h so that the constraint remains binding. This is possible because reducing d^h relaxes the constraint and because (A2) implies that $d^h > 0$. We have a contradiction. QED

Lemma 5 *In a separating equilibrium the second constraint binds.*

$$\phi\left(p_l - \left(1 - d^h\right) p_h\ \right) = \beta d^h \pi_l R \tag{A.3}$$

Proof This follows from the fact that the payoff is increasing in p_h and decreasing in d^h and that reducing d^h relaxes the first constraint in (A2). QED

Lemma 6 *$p_h = \beta \pi_h R$.*

Proof Solving (A3) for p_h we get

$$p_h = \left(\frac{1 - \frac{1}{\phi} d^h}{1 - d^h}\right) \beta \pi_l R.$$

Changes in p_h and d^h affect the bank's payoff only when it sells a loan of type h. Substituting the above expression in that payoff we obtain $\beta d^h \pi_h R + \beta \pi_l R \left((\phi - 1) d^h\right)$ which is increasing in d^h. Then the lemma follows from $\mathrm{d}p_h/\mathrm{d}(d^h) > 0$. QED

Setting $p_h = \beta \pi_h R$ in (A3) and solving for d^h completes the proof of the proposition. QED

7.2. Proof of Proposition 5

For the case when $\phi < \pi_h/\psi$ we compare the two sales strategies for each portfolio type separately.

a) Type L portfolio.

The bank is indifferent between selling the loans separately or as a portfolio given that in both cases its payoff will be equal to $\phi \beta \pi_l R$.

b) Type M portfolio.

The bank's payoff from selling the loans separately is equal to

$$\beta d^h \pi_h R + \phi \left(1 - d^h\right) \beta \pi_h R + \phi \beta \pi_l R = -\left(\phi - 1\right) \pi_h R d^h + \phi \left(\pi_h + \pi_l\right) R$$

and its payoff from selling them as portfolio is equal to

$$\beta d^M \left(\pi_h + \pi_l\right) R + \phi \left(1 - d^M\right) \beta \left(\pi_h + \pi_l\right) R = -\left(\phi - 1\right) \left(\pi_h + \pi_l\right) R d^M + \phi \left(\pi_h + \pi_l\right) R.$$

Comparing the two payoffs we find that the bank will sell them separately if

$$\pi_h R d^h - \left(\pi_h + \pi_l\right) R d^M < 0.$$

Substituting the solution for d^h from (2) and the solution for d^M from the statement of Proposition 3, it follows that

$$\pi_h R d^h - \left(\pi_h + \pi_l\right) R d^M = -\frac{\phi \pi_l (\pi_h - \pi_l)^2}{(\phi \pi_h - \pi_l)\left((\phi \pi_h - \pi_l) + (\phi - 1)\pi_l\right)} < 0.$$

Therefore, the bank will sell separately the two assets.

c) Type H portfolio.

The bank's payoff from selling the loans separately is equal to

$$2\left(\beta d^h \pi_h R + \phi \left(1 - d^h\right) \beta \pi_h R\right) = 2\left(-\left(\phi - 1\right) \pi_h R d^h + \phi \pi_h R\right)$$

and its payoff from selling them as portfolio is equal to

$$2\left(\beta d^H \pi_h R + \phi \left(1 - d^H\right) \beta \pi_h R\right) = 2\left(-\left(\phi - 1\right) \pi_h R d^H + \phi \pi_h R\right)$$

Clearly, the bank will sell them separately if $d^H - d^h > 0$. After some simple algebraic manipulation:

$$d^H - d^h = (\phi - 1)\frac{\phi \pi_l (\pi_h - \pi_m)(\pi_m - \pi_l)}{(\phi \pi_h - \pi_m)(\phi \pi_h - \pi_l)(\phi \pi_m - \pi_l)},$$

which is positive because $\pi_h > \pi_m > \pi_l$ and $\phi > 1$. Therefore, the bank will sell the two assets separately using signaling.

Lastly, the proof of the second part of the proposition follows from Propositions 2 and 4. QED

7.3. Proof of Proposition 6

1. $\pi_{HM} > \psi$ which implies that $\pi_h/\pi_{HM} < \pi_h/\psi$.
2. $\pi_{HL} \geq \psi$ if and only if $\theta \geq 1/2$, so that $\pi_h/\pi_{HL} < \pi_h/\psi$ if and only if $\theta \geq 1/2$.
3. We need to compare π_m/π_{ML} and π_h/ψ. It can be shown that $\frac{\pi_m}{\pi_{ML}} - \frac{\pi_h}{\psi} = -\frac{(\pi_h - \pi_l)(1-\theta)\psi}{2\psi(\theta\pi_h + \pi_l)} < 0$.Thus, $\pi_m/\pi_{ML} < \pi_h/\psi$.

Then, the Proposition follows $\phi\pi_{ij} \geq \pi_i$, which is the necessary condition for the mixed portfolio ij to be sold. QED

7.4. Proof of Proposition 7

First, comparing V_S given in equation (4) with U_{HM} given in equation (5) and substituting the conditional probability π_{HM} we have

$$V_S - U_{HM} = (\phi - 1)\beta R \left((2\rho_H + \rho_M)\, \pi_h \left(d^{HM} - d^h \right) + \rho_M \pi_l d^{HM} \right). \tag{A.4}$$

It is easily checked that $d^h > d^{HM}$. This is intuitive because the skin in the game must be larger to signal a higher quality asset. In comparing the payoffs V_S and U_{HM} therefore, there are two effects. The mixed portfolio has the benefit of using a lower skin in the game whenever one or both loans are of high quality. Thus the first term in the brackets above, $(2\rho_H + \rho_M)\pi_h(d^{HM} - d^h)$, is negative. However, the mixed portfolio also has a cost if one of the loans is low quality, because the skin in the game d^{HM} is still required for the mixed pooling strategy where no skin in the game is required if loans are completely separated. It can be shown that the latter effect dominates. That is, $\rho_M \pi_l d^{HM} > (2\rho_H + \rho_M)\pi_h(d^h - d^{HM})$. Substituting for for d^{HM} from equation (6) and for d^h from equation (2) (and for the probability π_{HM} and π_m) into equation (A.4) gives

$$V_S - U_{HM} = (\phi - 1)\beta R \left(\frac{\phi(\pi_h - \pi_l)^2 \rho_M \pi_l (2\rho_H + \rho_M)}{(\phi\pi_h - \pi_l)\left((2\rho_H + \rho_M)(\phi\pi_h - \pi_l) + \rho_M \pi_l (\phi - 1) \right)} \right).$$

Since $\phi > 1$ and $\pi_h > \pi_l > 0$ and $\rho_M > 0$, it follows that $V_S > U_{HM}$. Thus, the mixed HM portfolio is always dominated by signaling.

Similarly, in comparing U_{ML} and V_S, the skin in the game for the mixed portfolio is only required if both loans are of high quality and the skin in the game d^{ML} is less than the skin in the game, d^h, required to signal the high quality when loans are signalled separately. Thus, we have

$$U_{ML} - V_S = (\phi - 1)\beta R \pi_h \left(2\rho_H (d^h - d^{ML}) + \rho_M d^h\right)$$

and since $d^h > d^{ML} > 0$ (which follows from $\pi_h > \pi_{ML} > \pi_l$), $\phi > 1$ we have $U_{ML} > V_S$ and the mixed portfolio ML will be preferred to signaling loans separately if it is feasible.

When $\theta < 1/2$, pooling all three portfolios dominates the mixed portfolio HL. Therefore in comparing the mixed portfolio HL with other strategies, we only need to consider the case $\theta \geq 1/2$. For $\theta \geq 1/2$, the mixed portfolio HL retains the skin in the game, d^{HL}, whenever both loans are of the high quality or both are of the low quality. The benefit of this strategy relative to the separating strategy is given by

$$\begin{aligned} U_{HL} - V_S &= (\phi - 1)\beta R \left(\pi_h \left(2\rho_H + \rho_M\right) d^h - \left(2\pi_h \rho_H + \pi_l \rho_L\right) d^{HL}\right) \\ &= (\phi - 1)\beta R \left(\pi_h \left(2\rho_H + \rho_M\right) \left(d^h - d^{HL}\right) + \left(\pi_h \rho_M - \pi_l \rho_L\right) d^{HL}\right). \end{aligned}$$

Given that $d^{HL} < d^h$ (which follows from the inequalities $\pi_h > \pi_{HL}$, $\pi_m > \pi_l$ and $\phi > 1$), and $\rho_M \geq \rho_L$ for $\theta > 1/2$ (and $\pi_h > \pi_l$), the above expression is always positive. Thus, the mixed portfolio HL will be preferred to signaling loans separately if it is feasible.

Finally, we compare the payoffs from the mixed portfolios HL and ML. The mixed portfolio HL requires a skin in the game, d^{HL}, whenever both loans are of the same quality. In contrast the mixed portfolio ML requires a skin in the game, d^{ML}, only when both loans are of high quality. However, the skin in the game required for the mixed portfolio HL is lower than for the mixed portfolio ML: $d^{HL} < d^{ML}$ (this follows from the inequalities $\pi_h > \pi_{HL}$, $\pi_m > \pi_{LM}$ and $\phi > 1$). Thus, we have

$$U_{HL} - U_{ML} = (\phi - 1)\beta R \left(2\pi_h \rho_H \left(d^{ML} - d^{HL}\right) - 2\pi_l \rho_L d^{HL}\right). \tag{A.5}$$

For $\theta = 1/2$, $d^{HL} = 0$, whereas $d^{ML} > 0$. Therefore for $\theta = 1/2$, the difference $U_{HL} - U_{ML} > 0$ and by continuity there is a $\tilde{\theta}$ such that for $\theta \in [1/2, \tilde{\theta})$ the difference is strictly positive. In the limit as $\theta \to 1$, $\lim_{\theta \to 1} d^{HL} > 0$ whereas $\lim_{\theta \to 1}(d^{ML} - d^{HL}) = 0$. Since $\rho_L \to 0$ as $\theta \to 1$, the difference $U_{HL} - U_{ML} \to 0$ as $\theta \to 1$. However, it can be checked that the term $\rho_H(d^{ML} - d^{HL})$ is declining in the limit whereas the term $\rho_L d^{HL}$ is nether increasing nor decreasing in the limit as $\theta \to 1$. Thus, by continuity, there is a range of θ, $(\hat{\theta}, 1)$ where the difference $U_{HL} - U_{ML} > 0$. We can conclude that there is a range of θ where the mixed HL portfolio dominates the ML portfolio. This range may be the interval $[1/2, 1)$ or there may be some values of θ interior to this interval where the ML portfolio dominates.

The skin in the game, for both mixed portfolios ML and HL, is decreasing in ϕ and $\lim_{\phi\to 1} d^{HL} \to 1$ and $\lim_{\phi\to 1} d^{ML} \to 1$ for $\theta \in (1/2, 1)$ ($d^{HL} = 0$ for $\theta = 1/2$). Thus, the term in brackets in equation (A.5) is decreasing in ϕ and is negative in the limit as $\phi \to 1$ for $\theta \in (1/2, 1)$. Therefore, it is possible to find a critical $\phi^c(\theta)$ such that the bracketed term is positive for $\phi > \phi^c(\theta)$. It can be shown that $\phi^c(\theta) < \pi_h/\pi_{HL}$ for $1/2 \leq \theta < (\pi_h + \pi_l)/(\pi_h + 2\pi_l)$ and that $\pi^c(\theta) \in (\pi_h/\pi_{HL}, \pi_h/\psi)$ for $1 > \theta > (\pi_h + \pi_l)/(\pi_h + 2\pi_l)$. Defining $\phi^c := \max_\theta \phi^c(\theta)$, it can be shown that

$$\phi^c = 1 + \frac{(\pi_h - \pi_l)\left(\sqrt{\pi_h(\pi_h + \pi_l)} - \pi_h\right)}{2\pi_h^2}.$$

It is clear that since $\pi_h > \pi_l$, $\phi^c \geq 1$ with equality only if $\pi_l = 0$. For $\phi > \phi^c$ the difference in (A.5) is positive and the mixed portfolio HL will dominate the mixed portfolio ML for any $\theta \in [1/2, 1)$. It is also easily checked that $\phi^c < 2\pi_h/(\pi_h + \pi_1)$. Since π_h/ψ and π_h/π_{HL} are decreasing in θ and $\pi_h/\psi > \pi_h/\pi_{HL}$ for $\theta > 1/2$ with $\pi_h/\psi = \pi_h/\pi_{HL} = 2\pi_h/(\pi_h + \pi_1)$ for $\theta = 1/2$, it follows that there exist values of θ, π_h and π_l such that $\phi > \phi^c$ and $\pi_h/\pi_{HL} \leq \phi < \pi_h/\psi$.

To complete the proof we note that the inequalities $\pi_h > \pi_m$ and $\pi_{HL} < \pi_{ML}$ imply that $\frac{\pi_m}{\pi_{ML}} < \frac{\pi_h}{\pi_{HL}}$. Hence, considering each of the statements of the proposition in turn:

1. Follows from $\pi_m/\pi_{ML} < \pi_h/\pi_{HL}$ and the the necessary conditions $\phi\pi_{ML} \geq \pi_m$ and $\phi\pi_{HL} \geq \pi_h$, that neither the mixed portfolio ML nor the mixed portfolio HL will be sold for $\phi < \pi_m/\pi_{ML}$. Neither will the mixed portfolio HM be sold because, as we have shown above, it is dominated by signaling of separate loan sales, $V_S > U_{HM}$.
2. For $\theta < 1/2$, $\pi_h/\psi < \pi_h/\pi_{HL}$ and therefore the condition $\phi\pi_{HL} \geq \pi_h$ for the mixed portfolio HL to be sold is not satisfied. The mixed portfolio ML may be sold because $\phi \geq \pi_m/\pi_{ML}$ and since $U_{ML} > V_S$, this portfolio dominates separate loan sales, which in turn dominates the mixed portfolio HM.
3. Since $\pi_m/\pi_{ML} < \pi_h/\pi_{HL} \leq \phi$, the necessary conditions for the sale of the mixed portfolios ML and HL are both satisfied. Since $\phi < \pi_h/\psi$, pooling does not dominate these mixed portfolios. Since $U_{ML} > V_S$ and $U_{HL} > V_S$, both mixed portfolios are better than signaling loans separately. The comparison of the two mixed portfolios for $\theta \geq 1/2$ depends on the sign of $U_{HL} - U_{ML}$, which as shown above is positive for θ near to one and θ near to 1/2 and is such that for $\phi > \phi^c$, the mixed

portfolio HL dominates for all $\theta \in [1/2, 1)$.

4. Follows from the domination of the pooling strategy when $\phi \geq \pi_h/\psi$.

QED

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8. Supplementary Appendix (not intended for publication)

8.1. Portfolio Sales

Let $2p_i$ equal the price that the bank is willing to sell a portfolio of type i (that is p_i denotes half the portfolio price). Investor participation requires that

$$p_H \leqslant \beta \pi_h R,\ p_M \leqslant \beta \pi_m R \equiv \beta \pi_M R \text{ and } p_L \leqslant \beta \pi_l R \tag{S1}$$

Suppose that the bank's portfolio type is type H. Under a signaling equilibrium, the bank's expected payoff from the sale of its portfolio is equal to $2\left(\beta d^H \pi_h R + \phi\left(1 - d^H\right) p_H\right)$. The interpretation is similar to that for the case for single loan sales. Similar arguments show that the bank's expected payoff when its portfolio is type M is equal to $2\left(\beta d^M \pi_m R \ + \phi\left(1-d^M\right) p_M\right)$and its expected payoff when its portfolio is type L is equal to $2\left(\beta d^L \pi_l R + \phi\left(1-d^L\right) p_L\ \right)$.

The bank will prefer to sell a fraction of a type i portfolio to investors rather than keeping it on its books if the following condition is satisfied:

$$\beta d^i \pi_i R + \phi\left(1-d^i\right) p_i \geq \beta \pi_i R$$

or

$$\phi p_i \geq \beta \pi_i R \tag{S2}$$

where $\pi_H = \pi_h$, $\pi_M = \frac{\pi_h+\pi_l}{2}$ and $\pi_L = \pi_l$.

For signaling to be effective the following incentive compatibility constraints must also be satisfied:

$$\beta d^H \pi_h R + \phi\left(1-d^H\right) p_H \geq \beta d^M \pi_h R + \phi\left(1-d^M\right) p_M \tag{SIC1}$$

$$\beta d^H \pi_h R + \phi\left(1-d^H\right) p_H \geq \beta d^L \pi_h R + \phi\left(1-d^L\right) p_L \tag{SIC2}$$

$$\beta d^M \pi_m R + \phi\left(1-d^M\right) p_M \geq \beta d^H \pi_m R + \phi\left(1-d^H\right) p_H \tag{SIC3}$$

$$\beta d^M \pi_m R + \phi\left(1-d^M\right) p_M \geq \beta d^L \pi_m R + \phi\left(1-d^L\right) p_L \tag{SIC4}$$

$$\beta d^L \pi_l R + \phi\left(1-d^L\right) p_L \geq \beta d^M \pi_l R + \phi\left(1-d^M\right) p_M \tag{SIC5}$$

$$\beta d^L \pi_l R + \phi\left(1-d^L\right) p_L \geq \beta d^H \pi_l R + \phi\left(1-d^H\right) p_H \tag{SIC6}$$

Each of the above expressions is equal to half the expected payoff of the corresponding portfolio. SIC1 states that when the portfolio type is H the bank prefers to sell it as type H rather than selling it as type M. SIC2 states that when the portfolio type is H the bank prefers to sell it as type H rather than selling it as type L. SIC3 states that when the portfolio type is M the bank prefers to sell it as type M rather than selling it as type H. SIc4 states that when the portfolio type is M the bank prefers to sell it as type M rather than selling it as type L. SIC15 states that when the portfolio type is L the bank prefers to sell it as type L rather than selling it as type M. SIC6 states that when the portfolio type is L the bank prefers to sell it as type L rather than selling it as type H.

The constraints can be written as:

$$\beta\pi_h R\left(d^H - d^M\right) \geq \phi\left(\left(1-d^M\right)p_M - \left(1-d^H\right)p_H\right) \tag{SIC1*}$$

$$\beta\pi_h R\left(d^H - d^L\right) \geq \phi\left(\left(1-d^L\right)p_L - \left(1-d^H\right)p_H\right) \tag{SIC2*}$$

$$\beta\pi_m R\left(d^H - d^M\right) \leqslant \phi\left(\left(1-d^M\right)p_M - \left(1-d^H\right)p_H\right) \tag{SIC3*}$$

$$\beta\pi_m R\left(d^M - d^L\right) \geq \phi\left(\left(1-d^L\right)p_L - \left(1-d^M\right)p_M\right) \tag{SIC4*}$$

$$\beta\pi_l R\left(d^M - d^L\right) \leqslant \phi\left(\left(1-d^L\right)p_L - \left(1-d^M\right)p_M\right) \tag{SIC5*}$$

$$\beta\pi_l R\left(d^H - d^L\right) \leqslant \phi\left(\left(1-d^L\right)p_L - \left(1-d^H\right)p_H\right) \tag{SIC6*}$$

We can now prove the two propositions.

8.1.1. Proof of Proposition 3

Lemma S1 $d^H > d^M > d^L$.

Proof The first inequality follows from SIC1* and SIC3*. The second inequality follows from SIC4* and SIC5*. Notice that SIC2* and SIC6* also imply that $d^H > d^L$. QED

Lemma S2 *Any solution that satisfies S*IC1*, SIC3*, SIC4* *and S*IC5* *will also satisfy S*IC2* and SIC6*.

Proof

$$\beta\pi_h R\left(d^H - d^L\right) \geq \beta\pi_h R\left(d^H - d^M\right) + \beta\pi_m R\left(d^M - d^L\right) \geq$$

$$\phi\left(\left(1-d^M\right)p_M - \left(1-d^H\right)p_H + \left(1-d^L\right)p_L - \left(1-d^M\right)p_M\right) =$$

$$\phi\left(\left(1-d^L\right)p_L - \left(1-d^H\right)p_H\right)$$

The second weak inequality follows from adding SIC1* and SIC4*.

$$\beta\pi_l R\left(d^H - d^L\right) \leqslant \ \beta\pi_m R\left(d^H - d^M\right) + \beta\pi_l R\left(d^M - d^L\right) \leqslant$$

$$\phi\left(\left(1-d^M\right)p_M - \left(1-d^H\right)p_H + \left(1-d^L\right)p_L - \left(1-d^M\right)p_M\right) =$$

$$\phi\left(\left(1-d^{L}\right)p_{L}-\left(1-d^{H}\right)p_{H}\right)$$

The second weak inequality follows from adding SIC3* and SIC5*. QED

We can combine SIC1* and SIC3* to get

$$\beta\pi_{h}R\left(d^{H}-d^{M}\right)\geq\phi\left(\left(1-d^{M}\right)p_{M}-\left(1-d^{H}\right)p_{H}\right)\geq\beta\pi_{m}R\left(d^{H}-d^{M}\right) \tag{S3}$$

We can combine IC10* and IC12* to get

$$\beta\pi_{m}R\left(d^{M}-d^{L}\right)\geq\phi\left(\left(1-d^{L}\right)p_{L}-\left(1-d^{M}\right)p_{M}\right)\geq\beta\pi_{l}R\left(d^{M}-d^{L}\right) \tag{S4}$$

Lemma S3 $p_L \leqslant p_M \leqslant p_H$.

Proof

a) Suppose that $p_L > p_M$. At least one of the following is true: the first constraint in (S4) binds or $p_L = \beta\pi_l R$.

We first show that in both cases the fist constraint in (S3) must bind. Suppose that $p_L = \beta\pi_l R$. Then $p_M < \beta\pi_l R$ implies that the second constraint in (S4) does not bind (given that it does not bind for $p_L = p_M = \beta\pi_l R$). Given that bank's payoff is increasing in p_M the first constraint in (S3) must bind. Next, suppose that $p_L < \beta\pi_l R$. Then the first constraint in (S4) binds which implies that the second constraint does not bind and, as before, it must be the case that the first constraint in (S3) binds.

Decrease d^M and p_M so that the bank's payoff $\beta d^M \pi_m R + \phi\left(1-d^M\right)p_M$ remains constant. Notice that lemma S1 implies that $d^M > 0$ and that if $p_M = 0$ the first constraint in (S3) is not satisfied. Totally differentiating and rearranging we find that the changes must satisfy $\frac{\mathrm{d}p_M}{\mathrm{d}(d^M)} = \frac{\phi p_M - \beta\pi_m R}{\phi(1-d^M)}$ where the numerator must be positive for the bank to be willing to sell a portfolio of type M. The change does not affect (S4) but relaxes the first constraint in (S3). Therefore, we have a contradiction.

b) Suppose that $p_M > p_H$. The inequality $p_H < \beta\pi_h R$ implies that the second constraint in (S3) binds. It must also be true that $p_M < \beta\pi_m R$ (given it does not bind for $p_H = p_M = \beta\pi_m R$). But then it follows that the second constraint in (S4) must bind (if not increase p_M, thus, raising the bank's payoff). Increase d^M and p_M so that the second constraint still binds. But given that $\pi_m R > \pi_l R$ the change relaxes the second constraint in (S3) and also increases the bank's payoff. Therefore, we have a contradiction. QED

Lemma S4 $d^L = 0$.

Proof Suppose that the first constraint in (S4) does not bind. Then decrease d^M and d^L by the same amount so that either $d^L = 0$ or the first constraint binds. Suppose that the second constraint is not binding. Then reduce d^M and d^L so that $\left(1 - d^L\right) p_L - \left(1 - d^M\right) p_M$ stays constant so that either $d^L = 0$ or the second constraint binds. Then, as long as the changes have not violated the constraints in (S3), the lemma follows from the fact that at least one of the inequalities is not binding. If one of the constraints in (S3) is violated then decrease d^H either by the same amount as d^M when the first constraint is the one that binds or decrease d^H so that to keep $\left(1 - d^M\right) p_M - \left(1 - d^H\right) p_H$ constant if the second constraint is the one that binds. QED

Lemma S5 *In a signaling equilibrium the second constraint in* (S3) *and the second constraint in* (S4) *bind. Further,* $p_L = \beta \pi_l R$.

Proof Suppose that the second constraint in (S3) does not bind. Then we have $p_H = \beta \pi_h R$. But then the constraint can be relaxed by decreasing d^H and thus increasing the bank's payoff. We have a contradiction. Next, suppose that the second constraint in (S4) does not bind. Then, it must be the case that the first constraint in (S3) binds. If $p_M < \beta \pi_m R$ then increase p_M till either the second constraint binds or $p_M = \beta \pi_m R$. (This is feasible because the first constraint in (S3) does not bind.) Thus, we have a contradiction. In contrast, if $p_M = \beta \pi_m R$ decrease d^M thus relaxing the constraint. We also have a contradiction. Given that the second constraint in (S4) binds we have $p_L = \beta \pi_l R$. QED

Then, a signaling equilibrium must satisfy (S2) and the following constraints:

$$\phi\left(\left(1 - d^M\right) p_M - \left(1 - d^H\right) p_H\right) = \beta \pi_m R \left(d^H - d^M\right) \tag{S5}$$

and

$$\phi\left(\beta \pi_l R - \left(1 - d^M\right) p_M\right) = \beta \pi_l R d^M \tag{S6}$$

Lemma S6 $p_M = \beta \pi_m R$, $p_H = \beta \pi_h R$.

Proof

a) Solve (S5) for p_H to get

$$p_H = \frac{1-d^M}{1-d^H} p_M + \frac{\beta}{\phi}\pi_m R \frac{d^H - d^M}{1-d^H}.$$

Changes in p_H and d^H affect the bank's payoff only when it sells a portfolio of type H. Substituting the above expression in that payoff we obtain

$$\beta d^H \pi_h R + \phi\left(\left(1-d^M\right) p_M + \frac{\beta}{\phi}\pi_m R\left(d^H - d^M\right)\right)$$

which is increasing in d^H. Then the first part of the lemma follows from $\frac{\mathrm{d}p_H}{\mathrm{d}(d^H)} > 0$.

b) Solve (S6) for p_M to get

$$p_M = \beta \pi_l R\left(\frac{1}{1-d^M} + \frac{1}{\phi}\frac{d^M}{1-d^M}\right).$$

Changes in p_M and d^M affect the bank's payoff only when it sells a portfolio of type M. Substituting the above expression in that payoff we obtain

$$\beta d^M \pi_m R + \phi\beta\pi_l R\left(1 + \frac{1}{\phi}d^M\right)$$

which is increasing in d^M. Then the second part of the lemma follows from $\frac{\mathrm{d}p_M}{\mathrm{d}(d^M)} > 0$. QED

To complete the proof of the proposition substitute the results of Lemma S6 in (S5) and (S6). Solve (S6) for d^M. Then substitute the latter solution in (S5) and solve for d^H.

$$d^H = \frac{\phi\left(\frac{\pi_h}{\pi_m} - 1\right) + d^M\left(\phi - 1\right)}{\phi\frac{\pi_h}{\pi_m} - 1}$$

After substituting the solution for d^M in the above expression and subtract the denominator from the numerator we find that the difference is equal to $d^M - 1 < 0$ and therefore $d^H < 1$. Lastly, $d^H - d^M = \frac{\phi\left(\frac{\pi_h}{\pi_m}-1\right)\left(1-d^M\right)}{\phi\frac{\pi_h}{\pi_m}-1} > 0$. QED.

8.1.2. Proof of Proposition 4

We consider the possibility of pooling equilibria. If a pooling equilibrium exists then the bank will not keep any fraction of the portfolio on its books. The maximum price that investors would be willing to pay for a portfolio (assuming that the bank is willing to sell all types of portfolios) is equal to $2\beta\psi R$. If the bank keeps a type H portfolio on its books its payoff will be equal to $2\beta\pi_h R$. If the bank sells the portfolio to investors its payoff will be $2\beta\phi\psi R$. Then the bank will prefer to keep the portfolio on its books if $\phi < \frac{\pi_h}{\psi}$. Clearly, if the bank is willing to sell at type H portfolio will also be willing to sell portfolios of types M and L. The above argument together with Proposition 3 and (S2) imply that if $\phi < \frac{\pi_h}{\psi}$ then the bank will sell the portfolio to investors using the 'skin in the game' as signal.

Next, we need to compare signaling and pooling when $\phi \geq \frac{\pi_h}{\psi}$. The bank's payoff from pooling is equal to[25]

$$W_P = V_P = 2\phi\beta\psi R = 2\phi\beta\left(\theta\pi_h + (1-\theta)\pi_l\right)R.$$

The bank's payoff from signaling when the loans are sold together as a portfolio is equal to

$$W_S = \theta^2 2\beta\pi_h R\left(d^H + \phi\left(1-d^H\right)\right) +$$

$$2\theta(1-\theta)\beta(\pi_h+\pi_l)R\left(d^M + \phi\left(1-d^M\right)\right) + (1-\theta)^2 2\beta\pi_l R\phi.$$

$$W_S - V_P = -2\theta\beta\pi_h R(\phi-1)d^H - \theta(1-\theta)2(\phi-1)d^M < 0.$$

QED.

8.2. Further Details of Proof of Proposition 7

In comparing the separate loans and portfolio HM we have (repeating (A.4))

$$V_S - U_{HM} = (\phi-1)\beta R\left((2\rho_H+\rho_M)\pi_h\left(d^{HM}-d^h\right) + \rho_M\pi_l d^{HM}\right).$$

Substituting for for d^{HM} from equation (6) and for d^h from equation (2) gives

$$V_S - U_{HM} = (\phi-1)\beta R\left(\frac{\phi(\pi_{HM}-\pi_l)\pi_l\rho_M}{(\phi\pi_{HM}-\pi_l)} - \pi_h\frac{\phi(\pi_h-\pi_l)}{(\phi\pi_h-\pi_l)} + \frac{\phi(\pi_{HM}-\pi_l)}{(\phi\pi_{HM}-\pi_l)}(2\rho_H+\rho_M)\right).$$

Substituting for the conditional probabilities π_{ij} and $\pi_m = (1/2)(\pi_h+\pi_l)$ gives

$$V_S - U_{HM} = (\phi-1)\beta R\left(\frac{\phi(\pi_h-\pi_l)^2\rho_M\pi_l(2\rho_H+\rho_M)}{(\phi\pi_h-\pi_l)\left((2\rho_H+\rho_M)(\phi\pi_h-\pi_l)+\rho_M\pi_l(\phi-1)\right)}\right),$$

[25] The bank's payoff from pooling does not depend on whether the loans are sold separately or as a portfolio.

which is equation (A.5).

In comparing U_{HM} and U_{ML} we have

$$U_{HL} - U_{ML} = (\phi - 1)\beta R \left(2\pi_h \rho_H \left(d^{ML} - d^{HL}\right) - 2\pi_l \rho_L d^{HL}\right).$$

Substituting for d^{ML} and d^{HL} gives

$$U_{HL} - U_{ML} = (\phi - 1)\beta R \pi_l \rho_L d^{HL} \left(\left(\frac{\pi_h \rho_H}{\pi_l \rho_L}\right)\left(\frac{d^{ML} - d^{HL}}{d^{HL}}\right) - 1\right).$$

The sign of $U_{HL} - U_{ML}$ depends on the sign of the bracketed term. Substituting for d^{HL} and d^{ML} gives

$$\left(\frac{\pi_h \rho_H}{\pi_l \rho_L}\right)\left(\frac{d^{ML} - d^{HL}}{d^{HL}}\right) - 1 = \frac{\pi_h \rho_H (\phi - 1)(\pi_h \pi_m - \pi_{HL}\pi_{ML})}{\pi_l \rho_L (\pi_{HL} - \pi_m)(\phi \pi_h - \pi_{ML})} - 1.$$

Differentiating this term with respect to ϕ gives the derivative

$$\frac{\pi_h \rho_H (\pi_h - \pi_{ML})(\pi_h \pi_m - \pi_{HL}\pi_{ML})}{\pi_l \rho_L (\pi_{HL} - \pi_m)(\phi \pi_h - \pi_{ML})^2}.$$

This is positive because $\pi_h > \pi_{ML}$, $\pi_{HL} > \pi_m$ and $\pi_h \pi_m > \pi_{HL}\pi_{ML}$. Hence, there will a critical value of ϕ, ϕ^c such that $U_{HL} \gtreqless U_{ML}$ and $\phi \gtreqless \phi^c$. This critical value of ϕ depends on parameters and in particular depends on θ because ρ_i is a function of θ. Hence, we write $\phi^c(\theta)$. Solving

$$\frac{\pi_h \rho_H (\phi - 1)(\pi_h \pi_m - \pi_{HL}\pi_{ML})}{\pi_l \rho_L (\pi_{HL} - \pi_m)(\phi \pi_h - \pi_{ML})} - 1 = 0$$

gives

$$\phi^c(\theta) = 1 + \frac{\pi_l (\pi_{HL} - \pi_m)(\pi_h - \pi_{ML})\rho_L}{\pi_h \left(\pi_h \pi_m - \pi_{HL}\pi_{ML}\right)\rho_H + \pi_l \left(\pi_m - \pi_{ML}\right)\rho_L} = 1 + \frac{\pi_l (\pi_h - \pi_l)(1 - \theta)(2\theta - 1)}{\pi_h \left(\pi_h \theta^2 + \pi_l (1 - \theta)^2\right)}$$

where the second equality follows from substituting for the the conditional probabilities and for the probabilities ρ_i. It follows that $\phi^c(\theta) > 1$ for $\theta \in (1/2, 1)$ and $\phi^c(1/2) = \phi^c(1) = 1$. We are interested in $\pi_h/\pi_{HL} \leq \phi < \pi_h/\psi$. We have

$$\frac{\pi_h}{\psi} - \phi^c(\theta) = \frac{\pi_h}{\pi_h \theta + \pi_l (1 - \theta)} - \phi^c(\theta) = \frac{(\pi_h - \pi_l)(1 - \theta)(\pi_h (\pi_h - \pi_l)\theta^2 + \pi_l (1 - \theta)(\pi_h - \pi_l (2\theta - 1))}{\pi_h \left(\pi_h \theta + \pi_l (1 - \theta)\right)\left(\pi_h \theta^2 + \pi_l (1 - \theta)^2\right)}.$$

Since terms on the LHS are positive for $\theta \in [1/2, 1]$, we have $\phi^c(\theta) < \pi_h/\psi$ for $\theta \in [1/2, 1)$. Equally,

$$\phi^c(\theta) - \frac{\pi_h}{\pi_{HL}} = \phi^c(\theta) - \frac{\theta^2 + (1-\theta)^2}{\pi_h\theta^2 + \pi_l(1-\theta)^2} = \frac{(\pi_h - \pi_l)(1-\theta)\left(\pi_l(2\theta - 1) - \pi_h(1-\theta)\right)}{\pi_h\theta^2 + \pi_l(1-\theta)^2}.$$

The above term has the same sign as the sign of $\pi_l(2\theta - 1) - \pi_h(1-\theta)$. Thus, we have

$$\phi^c(\theta) - \frac{\pi_h}{\pi_{HL}} \gtreqless 0 \quad \text{as} \quad \theta \gtreqless \frac{\pi_h + \pi_l}{\pi_h + 2\pi_l}.$$

It is checked that $2/3 < (\pi_h + \pi_l)/(\pi_h + 2\pi_l) \leq 1$ with the second weak inequality holding as equality only if $\pi_l = 1$. It is possible to find the θ^* that maximizes $\phi^c(\theta)$. Solving gives

$$\theta^* = \frac{(\pi_h - \pi_l) + \sqrt{\pi_h(\pi_h + \pi_l)}}{3\pi_h - \pi_l}.$$

Substituting into $\phi^c(\theta)$ gives

$$\phi^c := \phi^c(\theta^*) = 1 + \frac{(\pi_h - \pi_l)\left(\sqrt{\pi_h(\pi_h + \pi_l)} - \pi_h\right)}{2\pi_h^2}.$$

The maximum value of ϕ^c occurs when $\pi_l = (1/9)(2\sqrt{7} - 1)\pi_h \approx 0.476834\pi_h$. Hence, substituting into the the formula for $\phi^c(\theta^*)$ gives

$$\phi^c \leq \frac{1}{27}\left(10 + 7\sqrt{7}\right) \approx 1.0563059.$$