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Attention-driven demand for bonus contracts

Abstract

In many markets supply contracts include a series of small, regular payments made by consumers and a single, large bonus that consumers receive at some point during the contractual period. But, if for instance its production costs exceed its value to consumers, such a bonus creates inefficiencies. We offer a novel explanation for the frequent occurrence of bonus contracts, which builds on a model of attentional focusing. Our main result identifies market conditions under which bonus contracts should be observed: while a monopolist pays a bonus to consumers - if at all - only for low-value goods, firms standing in competition always - i.e., independent of the consumers' valuation - offer bonus contracts. Thus, competition does not eliminate but rather exacerbates inefficiencies arising from contracting with focused agents. Common contract schemes in markets for electricity, telephony, and bank accounts are consistent with our model, but cannot be reconciled with alternative approaches such as models on consumption smoothing, (quasi-)hyperbolic discounting, or switching costs.

JEL-Codes: D910, D180, D400, L100.

Keywords: attention, focusing, bonus contracts.

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1 Introduction

Supply contracts (e.g., for electricity, telephony, or banking services) typically include many payments, one of which often represents a bonus payment to consumers. More specifically, such *bonus contracts* involve a series of small, regular payments to be made by subscribers, and a single, large bonus (e.g., a monetary payment, or a premium such as a smartphone) that is paid to consumers at some point during the contractual period. As the eligibility for a bonus often needs to be verified and as transfers are to be put and tracked, each of these payments generates transaction costs, including quasi-fixed costs of hiring employees to do these tasks. Bonus payments, in addition, involve checks that need to be sent out and redeemed, imposing costs both on firms and consumers. Non-monetary bonuses may involve other inefficiencies, for instance, if the consumer values the bonus below its actual production costs. Thus, abandoning bonuses and reducing the number of transfers to be made by consumers may in general increase efficiency.[1] In this sense, the predominant use of bonus contracts appears puzzling through the lens of the classical model.

We offer a novel explanation for the frequent occurrence of bonus contracts that builds on a recent model of attentional focusing by Kőszegi and Szeidl (2013). Accordingly, consumers select an option which performs particularly well in those choice dimensions where the available alternatives differ a lot, while dimensions along which the available options are rather similar tend to be neglected in the decision-making process. In our setup, the choice dimensions correspond to the different payments specified in a contract. For illustrative reasons, suppose a consumer decides whether to sign some bonus contract for a certain good. Here, the large bonus payment attracts a great deal of attention as the difference between obtaining the bonus if the contract is signed and not getting the bonus otherwise is large. In contrast, regular fees (at least if sufficiently small) play only a minor role. The difference between paying one rate if the contract is signed and paying zero otherwise is relatively small, so that none of the regular payments attract much attention. Thus, the inclusion of a large bonus at the cost of slightly higher monthly payments can persuade a consumer to sign a contract which she might otherwise abandon.

In this paper, we derive a firm's optimal contract choice if consumers are focused thinkers.

[1] In particular situations, spreading payments over time can serve other purposes such as relaxing the budget or credit constraints, so that a contract with several regular payments is not inefficient per se.

Irrespective of the market structure, this contract exhibits two general features. On the one hand, payments to be made by consumers are equally dispersed over the contractual period in order to minimize the consumers' focus on costs. This contract feature is in line with anecdotal evidence on the attractiveness of installments.[2] On the other hand, the contract involves at most one bonus payment, and if this bonus is non-zero, it will always be maximal.[3] These features create a decision situation that is highly imbalanced with respect to the dispersion of the costs and benefits of the contract. In general, the more imbalanced a decision situation is, the stronger the distortion of a focused thinker's valuation for a good is, so that a consumer's willingness to pay for a subscription can be maximized by concentrating its benefits and equally dispersing its costs. This focusing mechanism has found strong support in a recent lab experiment by Dertwinkel-Kalt *et al.* (2017). In conclusion, firms should offer contracts that include (at most) a single, maximal bonus payment as well as dispersed and rather small regular payments.

In a first step, we analyze a monopolistic market and show that a monopolist offers a bonus contract, if at all, only for low-value goods. If consumers already have a high valuation for the product, the payments to be made by consumers are relatively high, even absent a bonus payment. In this case, setting a bonus at the cost of increased regular payments cannot shift the consumer's attention solely toward the bonus, but draws attention also to the increased regular payments. Thus, offering a bonus contract does not pay off for high-value products.

In a second step, we consider a perfectly competitive market and show that, independent of the consumers' valuation for the product, competition forces firms to offer bonus contracts (at least in a symmetric equilibrium). If none of the firms pay a bonus, competition drives down regular payments to cost. Relative to these low regular payments the maximal bonus would attract much attention and each firm could obtain a competitive advantage by offering it. Thus, in any (symmetric) competitive equilibrium, consumers sign a bonus contract.

Our results mirror a practice that is common, among others, in markets for electricity, telephony, and bank accounts. As an illustration, consider the electricity retail market. Competition

[2]See, for instance, `https://smallbiztrends.com/2018/11/monthly-installments.html`, or `https://www.retailtouchpoints.com/topics/pos-payments-emv/54-of-shoppers-prefer-installment-payment-plans-over-free-shipping`, or `https://www.nickkolenda.com/psychological-pricing-strategies/#pricing-t7`, all accessed on February 11, 2019.

[3]As we discuss in more detail in Section 2, it seems reasonable to assume that bonus payments are bounded.

authorities in the European Union regard this market as split into two separate markets, one of which consists of *loyal consumers* who stay with their default provider, and the other one consists of *switching consumers* (see, for instance, Haucap *et al.*, 2013, pp. 282). This view is supported by recent empirical studies suggesting that a substantial share of consumers do not even consider *switching the provider* as a viable alternative, so that their default provider de facto serves as a monopolist for this group (e.g., Handel, 2013; Hortaçsu *et al.*, 2017). Since electricity is essential for running most devices, consumers can be assumed to have a high valuation. Our model predicts—as it is observed in practice—that electricity providers will offer their loyal consumers not a bonus contract, but a contract that involves only (relatively high) monthly fees. In contrast, firms fiercely compete for switching consumers, who are searching for the best deal in the market. As electricity is a homogeneous good, we predict that firms compete for consumers' limited attention by offering bonus contracts. That is indeed ongoing practice: on German price-comparison websites, for instance, virtually every power provider offers a large bonus payment instead of a reduction in regular fees in order to attract new customers.

Under standard assumptions, the common design of bonus contracts—that is, small regular payments uniformly dispersed over the contractual period and a single bonus paid at *some* point in time—is hard to reconcile with the classical model or established behavioral approaches such as (quasi-)hyperbolic discounting. According to the classical model, consumers should be indifferent between a bonus payment and a reduction of regular payments as long as the contract's net present value stays the same. As a consequence, inefficient bonuses should not occur in equilibrium. If consumers are (quasi-)hyperbolic discounters and therefore present-biased, it is suboptimal for a firm to pay a bonus at *some* point during the contractual period, since a present-biased agent prefers to obtain the bonus payment as soon as possible. Also, (quasi-)hyperbolic discounters would prefer a back-loaded instead of a uniform payment stream. In practice, however, the bonus is often paid at *some* point during the contractual period, and regular payments are small and constant (a thorough discussion is provided in Section 5.1).[4]

Our study adds to a growing body of theoretical and empirical research that has investigated and supported the importance of attentional focusing for economic choice. Accordingly, a decision maker automatically focuses on eye-catching choice features. These salient aspects of an

[4]See, for instance, `https://www.marktwaechter-energie.de/aerger-mit-energieversorgern/boni/`, accessed on October 1, 2018.

option obtain an over-proportionate weight in the decision-making process, while less prominent attributes tend to be neglected. A key implication of attentional focusing is *a bias toward concentration* (Kőszegi and Szeidl, 2013) whereby a decision maker pays disproportionately more attention to concentrated rather than dispersed outcomes; which has been supported by recent lab evidence (Dertwinkel-Kalt *et al.*, 2017). Attentional focusing further provides a unified account for puzzling behavior in a wide range of domains, such as consumer choice (e.g., the attraction effect and the efficacy of misleading sales, see Bordalo *et al.*, 2013b), choice under risk (e.g., the fourfold pattern of risk attitudes, the Allais paradox, and preference reversals, see Bordalo *et al.*, 2012), and financial decision making (e.g., the equity premium puzzle and skewness preferences, see Bordalo *et al.*, 2013a; Dertwinkel-Kalt and Köster, 2018). Applied to industrial organization, attentional focusing can explain, for instance, why drastic (minor) innovations yield decommoditized (commoditized) markets (Bordalo *et al.*, 2016). We apply attentional focusing in order to understand how firms design contracts to attract focused thinkers.

2 Model

Suppose there are L firms offering a homogeneous product at zero production costs, and a unit mass of homogeneous consumers who value the good at $v \geq 0$ and purchase at most one unit.

Contract Space. Each firm $k \in \{1, \dots, L\}$ can offer an $M + N$-part tariff that consists of

(i) $M \geq 1$ bonus payments $b_1^k, \dots, b_M^k \geq 0$ to be paid to consumers, and

(ii) $N \geq 2$ regular payments $p_1^k, \dots, p_N^k \geq 0$ to be made by consumers.

While we interpret the regular payments by consumers, p_i^k, as payments to be made at different points in time, we stay agnostic in regard to the timing of different bonus payments. As we will show in the next section, the assumption of a fixed number of bonus payments is without loss of generality. In contrast, without imposing further restrictions, a fixed number of payments to be made by consumers entails a loss. But, on the one hand, it seems plausible to assume that consumers aggregate payments they have to make for a specific good within a short time period, so that firms may not be able to increase the perceived number of payments beyond a certain threshold.[5] And, on the other hand, if each additional payment to be made by consumers is accompanied by an increasing transaction cost (i.e., transaction costs incurred by consumers are a convex function of the number of regular payments), an "optimal" number of regular payments exists and N could be understood as being optimally chosen by the firms. We discuss implications of this interpretation in the next section when analyzing the robustness of our results.

We also limit the maximum bonus that firms can pay. In other words, we impose a floor on the total price a firm could charge (see Heidhues and Kőszegi, 2018, for a broader discussion).

Assumption 1. *The sum of bonus payments is bounded from above by some* $\bar{b} > 0$.

Since even large firms face financial constraints, in practice firms cannot afford very large bonus payments. More importantly, a very large bonus may create incentives for the consumers to betray the firm and to not fulfill the contract. Finally, offering too large bonus payments might make consumers suspicious in that they believe something fishy to be going on. In this sense, setting a bonus beyond some level $\bar{b}$ may never pay off for a firm.

Timing of the Game. In a first stage, each firm $k \in \{1, \dots, L\}$ chooses a contract

$$\boldsymbol{c}^k := (v, b_1^k, \dots, b_M^k, p_1^k, \dots, p_N^k).$$

In a second stage, consumers decide whether and from which firm to buy the product. Formally, each consumer chooses a contract from the set

$$\mathcal{C} := \{\boldsymbol{c}^k \mid 0 \leq k \leq L\},$$

where $\boldsymbol{c}^0 := (0, \dots, 0) \in \mathbb{R}^{M+N+1}$ refers to the outside option of not buying the product.

For simplicity, firms and consumers adopt the same discount factor which may be determined by the market interest rate. Throughout our analysis we assume that all payments refer to

[5] In an experimental study, Dertwinkel-Kalt *et al.* (2017) find that subjects regard payments as separate that are dispersed over several weeks, but aggregate payments that are split within a day. In the context of supply contracts, for instance, it feels natural to assume that consumers aggregate all payments they have to cover with one salary. Then, there is no reason for firms to disperse payments between two paydays as this raises transaction costs, but does not affect a consumer's valuation for the contract. Supportive of this, in Europe where salaries are typically paid monthly, most supply contracts (i.e., mobile or electricity contracts) also involve monthly payments. In the US, where salaries are often paid weekly, supply contracts also often involve weekly payments.

present values (i.e., real instead of nominal sums). While this assumption is not crucial for our qualitative insights, it allows us to abstract from discounting.

A Firm's Problem. Each firm k designs a contract $\boldsymbol{c}^k$ in order to maximize her profits,

$$\pi_k(\boldsymbol{c}^k, \boldsymbol{c}^{-k}) := D_k \cdot \left(\sum_{i=1}^{N} p_i^k - \sum_{j=1}^{M} \left[\, b_j^k \; + \; \mathbb{1}_{\mathbb{R}_{>0}}(b_j^k) \cdot \kappa \, \right] \right),$$

where $D_k = D_k\big(\boldsymbol{c}^k, \boldsymbol{c}^{-k}\big)$ corresponds to the share of consumers choosing the contract offered by firm k from the set $\mathcal{C}$, where $\mathbb{1}_{\mathbb{R}_{>0}}$ is the indicator function on the interval of positive, real numbers, and where $\kappa > 0$ are per-customer transaction costs for each additional bonus payment. We discuss below why we regard it as a plausible assumption that bonuses cause an inefficiency.

A Consumer's Problem. We assume that consumers are *focused thinkers* (Kőszegi and Szeidl, 2013, henceforth: KS). Focused thinkers put an excessive weight on the salient choice dimension(s) of a contract, while they partly neglect less prominent attributes. Following KS, we assume that payments at different points in time as well as a good's quality (or its value to consumers) correspond to different choice dimensions.[6] Moreover, we assume that consumers also perceive the different bonus payments as distinct attributes.[7] Altogether, we assume that the N regular payments to be made by consumers, the M bonus payments offered by the firms, and the consumption value of the product all represent distinct choice dimensions.

Given these assumptions, a focused thinker chooses a contract from the choice set $\mathcal{C}$ in order to maximize her *focus-weighted utility* given by

$$U\big(\boldsymbol{c}^k \big| \, \mathcal{C}\big) := \begin{cases} g(\Delta^v)v - \sum_{i=1}^{N} g(\Delta_i^p)p_i^k + \sum_{j=1}^{M} g(\Delta_j^b)b_j^k & \text{if } k > 0, \\ 0 & \text{if } k = 0, \end{cases}$$

[6] While KS do not analyze a model of industrial organization, they point out that it is plausible to assume that in such models quality represents a choice dimension that is distinct from the price dimension(s); also the related model by Bordalo *et al.* (2013b) adopts the assumption that quality constitutes a separate choice dimension.

[7] This assumption is particularly plausible if some of the bonus payments refer to non-monetary premiums such as a smartphone or an other gadget while others refer to monetary payments. In addition, it is straightforward to show that our results would not change if consumers did not perceive the different bonus payments as distinct attributes, but aggregated them into a single bonus attribute.

whereby the weights on the different choice dimensions are determined by a *focusing function* $g : \mathbb{R}_+ \to \mathbb{R}_+$. According to KS, the weight on price component i, $g(\Delta_i^p)$, depends on the range of attainable utility along this choice dimension denoted as

$$\Delta_i^p := \max_{0 \leq k \leq L} p_i^k - \min_{0 \leq k \leq L} p_i^k = \max_{0 < k \leq L} p_i^k,$$

where the equality follows from the fact that the outside option does not involve any non-zero regular payments. Analogously, the weight on bonus payment j depends on the range of attainable utility along this bonus attribute, which we denote as Δ_j^b, and the weight on the product's consumption value depends on the utility range in this choice dimension, Δ^v, which is spanned by v in the case that the consumer buys and 0 in the case that she does not buy.

Following KS, we assume that the weight assigned to a certain attribute increases in the utility range along this choice dimension that is attainable given $\mathcal{C}$. This captures the intuition that large contrasts are particularly salient (see, e.g., Schkade and Kahneman, 1998), so that choice dimensions along which the available options differ a lot attract a great deal of attention.

Assumption 2 (Contrast Effect)**.** *The focusing function g is strictly increasing with $g' > 0$.*

In addition, we assume that the contrast effect is sufficiently strong.

Assumption 3. *The function $h : \mathbb{R}_+ \to \mathbb{R}_+$ with $h(x) := g(x)x$ is convex.*

Notice that Assumption 3 is not very restrictive as it admits for convex, linear, and mildly concave focusing functions. In fact, it is violated only for strongly concave focusing functions.[8]

Contractual Inefficiencies. We assume that for each non-zero bonus payment, a firm bears per-customer transaction costs $\kappa > 0$.[9] In the case of a monetary bonus payment, these costs

[8]More formally, Assumption 3 holds if and only if $-\frac{g''(x)x}{g'(x)} < 2$ for any $x \in \mathbb{R}_+$, which is satisfied for any convex or linear focusing function and for concave focusing functions with a first derivative that is not too elastic. In particular, any increasing power function $g(x) = x^\alpha$, $\alpha > 0$, satisfies Assumption 3.

[9]For certain types of bonuses, it might be more plausible to assume a cost function that continuously increases in the size of the bonus, but most examples of bonuses that we discuss below have in common that the inefficiency is independent of the size of the bonus. By constraining the maximal bonus, we consider the limit case of an infinitely convex cost function. While this assumption substantially simplifies our analysis (e.g., we do not have to deal with implicitly defined interior solutions), economic intuition suggests that our results do not hinge on this cost specification, but should hold under the assumption of a continuous and convex cost function.

could come from issuing, sending, and tracking checks. More generally, the inefficiency could arise due to administrative processes related to bonus schemes: in order to receive the bonus payment consumers often have to make an inquiry, which implies costs both for the consumer and the firm that needs employees verifying and handling these inquiries. In the case of a non-monetary bonus, the inefficiency could come from an imperfect match between the bonus and the consumer's preferences in the sense that the costs to produce the bonus exceed the consumer's valuation for it. In this case the inefficiency could be exacerbated due to consumers opting for a new bonus product, such as a smartphone, inefficiently often: as consumers typically do not sell their old smartphones, the inefficiency increases by the remaining value of the old and now unused smartphone.[10]

In order to allow firms to increase a consumer's focus-weighted utility using a bonus payment and to break even at the same time, we assume that the costs of paying a bonus are not too large relative to the maximum bonus itself, that is,

$$\frac{\kappa}{\bar{b}} < \frac{g(\bar{b})}{g(\bar{b}/N + c/N)} - 1. \tag{1}$$

While this assumption allows firms to benefit from using bonus contracts, it is not very restrictive and becomes weaker for larger values of N or $\bar{b}$, respectively. Suppose, for instance, that the number of regular payments is $N = 24$, that the maximum bonus is $\bar{b} = 120$, and that the focusing function is the identity function. In this case, the costs of making a bonus payment could be much larger than the maximum bonus itself without violating (1).

3 Equilibrium Analysis

In this section, we first analyze under which conditions a monopolist offers a bonus contract. Second, we derive equilibrium contracts in a perfectly competitive market. Third, we discuss the robustness of our findings. All missing proofs can be found in Appendix A.

[10]See, for instance, https://www.umweltbundesamt.de/sites/default/files/medien/378/publikationen/texte_10_2015_einfluss_der_nutzungsdauer_von_produkten_auf_ihre_umwelt_obsoleszenz_17.3.2015.pdf or https://www.beuc.eu/documents/files/FC/durablegoods/articles/0913_Stiftung_Warentest_Germany.pdf or https://www.faz.net/aktuell/finanzen/meine-finanzen/geld-ausgeben/nachrichten/neue-smartphones-oft-schlechter-als-aeltere-geraete-13726913.html, all accessed on February 11, 2019.

3.1 Monopolistic Market

Suppose that a single firm monopolizes the market (i.e., $L = 1$). For brevity, we drop the index k in this subsection. Then, the monopolist's maximization problem is given by

$$\max_{\boldsymbol{c}} \pi(\boldsymbol{c}) \quad \text{subject to} \quad \sum_{i=1}^{N} g(p_i)p_i \leq g(v)v + \sum_{j=1}^{M} g(b_j)b_j, \text{ and } \sum_{j=1}^{M} b_j \leq \bar{b},$$

and the optimal contract offer is characterized in the following lemma.

Lemma 1. *A contract* $\boldsymbol{c} = (v, b_1, \ldots, b_M, p_1, \ldots, p_N)$ *maximizes the monopolist's profit only if*

(i) the payments made by consumers are spread equally across periods, that is, $p_1 = \ldots = p_N$*,*

(ii) if bonus payment(s) are made, the bonus is maximal, that is, $\sum_{j=1}^{M} b_j = \bar{b}$*, and*

(iii) the contract involves at most a single bonus payment, that is, if a bonus payment is made, then $b_j = \bar{b}$ *for some* $j \in \{1, \ldots, M\}$ *and* $b_i = 0$ *for any* $i \neq j$*.*

Since the monopolist can fully extract the consumers' willingness to pay, he offers a contract that maximizes focus-weighted utility conditional on extracting it. According to the contrast effect a focused thinker's attention is directed to particularly large payments, so that the monopolist can minimize the consumers' perceived costs by dispersing the regular payments uniformly over the entire contractual period. More formally, suppose that one of the regular payments was larger than the others and, without loss of generality, let $p_1 > p_i$ for all $i \in \{2, \ldots, N\}$. Then, since $g(p_i)p_i$ is convex (Assumption 3), decreasing p_1 by ϵ and increasing each of the other payments by $\epsilon/(N-1)$ lowers the perceived costs of the contract, while keeping revenue constant. As a result, a necessary condition for maximizing the consumers' willingness to pay conditional on extracting a fixed revenue (and therefore to maximize the monopolist's profit) is that all payments to be made by consumers are of equal size. In contrast, if the monopolist chooses to pay a bonus, it should attract as much attention as possible, which is achieved by setting the maximal bonus, $\bar{b}$, and concentrating it into a single payment.

Yet the monopolist will not always choose a bonus contract. A bonus will be offered if and only if the following *two* conditions are satisfied: (i) the consumers' valuation for the good is sufficiently low, and (ii) the inefficiency that arises from a bonus payment is sufficiently small.

Proposition 1. *There exists a threshold value* $\hat{\kappa} > 0$ *and, for any* $\kappa < \hat{\kappa}$*, a threshold value* $\hat{v}(\kappa) > 0$ *such that the monopolist offers a bonus contract if and only if* $\kappa < \hat{\kappa}$ *as well as* $v < \hat{v}(\kappa)$*. In addition, the threshold value* $\hat{v}$ *monotonically decreases in* κ *on* $(0, \hat{\kappa})$*.*

Even if the costs for paying a bonus are low, the monopolist offers a bonus only if the consumers' valuation for the product is sufficiently low as well. This follows from the fact that only if the consumers' valuation and therefore the regular payments are sufficiently low, the monopolist can increase its relatively small margin by setting a bonus that grabs attention. If the valuation is high, consumers are already willing to accept relatively high regular payments, even absent a bonus. Then, the focus on the bonus—although it is maximal—cannot outweigh the consumers' focus on the even higher regular payments that are necessary to make a bonus contract profitable. Thus, the monopolist cannot benefit from offering a bonus payment.

In order to put the preceding result into perspective, we consider an example.

Example 1. *Suppose that the focusing function is linear with* $g(x) = x$*. Then, we obtain a threshold value* $\hat{\kappa} = (\sqrt{N} - 1)\bar{b}$ *and, for any* $\kappa < \hat{\kappa}$*, a threshold value* $\hat{v}(\kappa) = \frac{(N-1)\bar{b}^2 - (2\bar{b}+\kappa)\kappa}{2\sqrt{N}(\bar{b}+\kappa)}$*.*

Since $\hat{v}(\kappa)$ strictly decreases with the inefficiency arising from bonus payments, Example 1 further suggests that the monopolist will offer a bonus contract only if the regular payments he would charge when not paying a bonus, lie strictly below $\frac{\bar{b}}{2}\frac{(N-1)}{N}$. Suppose, for instance, that the number of regular payments is $N = 24$, and that the maximum bonus is $\bar{b} = 120$. If the consumers' valuation is high enough, so that the monopolist would already charge regular payments $p > 57.5$ when not paying a bonus, then offering a bonus contract would not increase his profits. And with a concave focusing function—such as $g(x) = \sqrt{x}$, which is consistent with experimental evidence by Dertwinkel-Kalt *et al.* (2017)—regular payments when not paying a bonus would have to be even lower to make a bonus contract profitable.

3.2 Competitive Market

Suppose that there are at least two firms trying to attract customers. As the product is homogeneous, firms fiercely compete for consumer attention and, as we will see below, bonus contracts play an even larger role than in a monopolistic market, despite the inefficiencies they produce. The (symmetric) equilibria of the game are characterized in the following proposition.

Proposition 2. *If* $L = 2$*, an equilibrium exists and any equilibrium has the following properties:*

(i) the market is covered and firms earn zero profits,

(ii) payments to be made by consumers are spread equally across periods (i.e., $p_1^k = \ldots = p_N^k$), and both firms charge the exact same regular payments (i.e., $p_i^1 = p_i^2$), and

(iii) both firms offer the maximum bonus (i.e., $\sum_{j=1}^{M} b_j = \bar{b}$) using a single bonus payment.

If $L \geq 3$, a symmetric equilibrium exists and any symmetric equilibrium satisfies (i) – (iii).[11]

As in the monopoly case, the payments to be made by consumers are equally spread over the contractual period. Given that the remaining firms offer the equilibrium contract, no firm can benefit from unilaterally decreasing some payments and increasing some others. In doing so, the firm would induce consumers to focus more on increased payments, that is, exactly on those choice dimensions along which it offers a worse deal compared to the competitors. At the same time, the focus-weight attached to the decreased payment does not change, as it is determined by the other firms' high regular payments. To sum up, a price hike attracts more attention than the corresponding price cuts, so that the firm cannot benefit from such a contract adjustment.[12]

In contrast to the monopoly case, competing firms always offer a bonus contract (at least in the symmetric equilibrium), irrespective of the consumers' valuation for the product or service. For the sake of contradiction, suppose that firms do not offer a bonus in a symmetric equilibrium. Since firms must earn zero profits, the regular payments consumers make would have to be zero. But then, any firm could benefit from offering a single bonus $\bar{b}$ and increasing each regular payment to $\frac{\kappa+\bar{b}}{N} + \epsilon$ for some sufficiently small $\epsilon > 0$, since Assumption 2 together with Eq. (1) ensure that—given zero regular payments offered by the other firms—consumers focus more on the bonus payment than on the increase in regular payments. We have already discussed

[11]Note that for $L \geq 3$ also asymmetric equilibria exist, where at least two firms offer $(v, 0, \ldots, 0)$ and serve the market, while at least one firm offers a contract with regular payments exceeding the maximum bonus payment. Importantly, these asymmetric equilibria are neither robust to assuming that clearly dominated options do not affect a consumer's attention allocation nor to assuming that firms want to maximize demand for a given profit level. In this sense, we would argue that the symmetric equilibria delineated above are the only plausible equilibria.

[12]Notably, the idea of avoiding high and therefore attention-grabbing prices is also relevant in the model by de Clippel *et al.* (2014) where firms compete for consumers' inattention (to the own price) by making price components non-salient. Here, each firm avoids charging a sum that exceeds the other firms' payments as this would attract a great deal of attention, thereby deterring consumers from signing the respective contract.

in the monopoly case that a bonus attracts most attention if it is concentrated into a single payment. In addition, raising the bonus to the maximal level increases the focus on the own contract's advantage—the large bonus—by more than it increases the focus on the also higher regular payments. Thus, in equilibrium, firms offer a single, but maximal bonus.

Importantly, even though paying a bonus creates an inefficiency, bonus contracts are more prevalent in competitive rather than monopolistic markets. This follows from the fact that firms standing in competition only care about beating the best offer of their competitors and not necessarily about maximizing the consumers' willingness to pay. More precisely, by increasing the regular payments in order to cover a bonus payment, a firm in a competitive market makes not only her own offer less attractive, but it also makes her competitors' offers look worse. Hence, since only the incremental change over and above the competitors matters, paying a bonus is indeed a good idea. In contrast, even if regular payments are relatively low, such an increase in regular payments may not pay off for a monopolist, since it can lower the consumers' willingness to pay due to the fact that inframarginal payments are also weighted more.

Corollary 1. *If the product's value to consumers is sufficiently high, the contractual inefficiencies are strictly lower in a monopolistic than in a competitive market.*[13]

3.3 Robustness

In this subsection, we argue in how far our findings take over to more general setups.

Outside Option. So far, we have assumed that the outside option—a vector of zeros—affects attribute ranges and therefore decision weights. This seems particularly plausible in the monopoly case, where the consumer only decides whether or not to sign a certain contract. This way our model captures the intuitive feature of the focusing model that large payments attract more attention than smaller ones. For that reason, we regard it as a plausible assumption that the outside option determines attribute ranges.

Importantly, our main insight—that bonus contracts are more widely used in competitive than in monopolistic markets—does not depend on whether the outside option affects decision

[13] Notice that, if Eq. (1) is violated, then consumers will sign a contract without bonus payment(s) in any competitive equilibrium. In this sense Corollary 1 relies on the assumption that (1) is satisfied. But, as we discuss in Section 2, Eq. (1) seems to be an extremely weak restriction on the inefficiency arising from a bonus payment.

weights or not. Suppose the outside option does not impact on the focus-weighted utility derived from offered contracts. Independent of the consumers' valuation for the product, a monopolist will never set a bonus contract as it could not manipulate the relative weight on regular payments by setting a bonus: all attribute ranges are zero. Under competition, any equilibrium contract (that generates a positive demand) involves at least one bonus payment. For the sake of contradiction, suppose that firms do not offer any bonus in equilibrium. Then, as firms earn zero profits in any equilibrium, at least two firms offer the contract $(v, 0, \ldots, 0)$ and serve the market. But this gives rise to the same incentives to deviate to a bonus contract as if the outside option affects focus weights. Hence, by the same arguments as in the proof of Proposition 2, we cannot have an equilibrium without a bonus payment. In addition, it is easily verified that an equilibrium with the same properties as described in Proposition 2 exists.[14]

Maximal Bonus Payment. We have also assumed that the maximal bonus a firm can pay is bounded and that this upper bound does not depend on other primitives of the model. In the following, we extend Example 1 to show that our qualitative results do not change if we impose the plausible assumption that the maximal bonus increases mildly in the product's value v.

Example 2. *Suppose that $g(x) = x$ and that the maximal bonus is given by $\bar{b}(v) := \gamma v + \delta$ with $\gamma, \delta > 0$. This implies a threshold value $\hat{\kappa} = (\sqrt{N} - 1)\delta$. In addition, there exists some $\hat{\gamma} > 0$ such that, for any $\gamma < \hat{\gamma}$, the monopolist offers a bonus contract if and only if $\kappa < \hat{\kappa}$ as well as*

$$v < \hat{v}(\gamma, \kappa) = \frac{\gamma\big[(N-1)\delta - \kappa\big] + \sqrt{N}\left[\sqrt{(\kappa+\delta)^2 + \gamma\kappa\big(\gamma\kappa - 2\sqrt{N}\delta\big)} - (\kappa+\delta)\right]}{\gamma\big[2\sqrt{N} - \gamma(N-1)\big]},$$

[14]Suppose all firms offer the contract $\big(v, \bar{b}, 0, \ldots, 0, \frac{\kappa+\bar{b}}{N}, \ldots, \frac{\kappa+\bar{b}}{N}\big)$ in equilibrium. Decreasing the sum of regular payments by some amount $\epsilon > 0$ and at the same time decreasing the bonus—still using the same bonus attribute—by an amount $\epsilon' > \epsilon$ is not a profitable deviation, as the decrease in the bonus attracts more attention than the decrease in regular payments (irrespective of how ϵ is spread across regular payments). In addition, shifting a share $\alpha \in (0, 1]$ of the bonus to another bonus attribute does not increase focus-weighted utility relative to the competitors (and therefore does not allow a firm to attract consumers), since the range in both bonus dimensions would be exactly the same, namely: $\alpha\bar{b}$. Combining the two arguments implies that there does not exist a profitable deviation involving a positive bonus. Finally, setting all bonus payments equal to zero (in order to save κ and reduce regular payments) is not a profitable deviation either, since consumers always choose $\big(v, \bar{b}, 0, \ldots, 0, \frac{\kappa+\bar{b}}{N}, \ldots, \frac{\kappa+\bar{b}}{N}\big)$ over $(v, 0, \ldots, 0)$ by Eq. (1). Hence, even if the outside option does not affect focus weights, there exists a pure-strategy equilibrium with the same properties as described in Proposition 2.

whereby $\hat{v}(\gamma, \kappa) > 0$ *holds for any combination of* γ *and* κ *with* $\gamma < \hat{\gamma}$ *and* $\kappa < \hat{\kappa}$.

As examplified above, it depends on the product's value v, and on the curvature of the focusing function $g(\cdot)$ whether a monopolist offers a bonus contract if the maximal bonus increases in the value v. In contrast, our qualitative results on the structure of contracts offered by firms that stand in competition generalizes to the case where the maximal bonus is an increasing function of v. More specifically, also if the maximal bonus increases in v, in any (symmetric) competitive equilibrium firms offer a bonus contract (as in Proposition 2). Altogether, our finding that a bonus payment is always made under competition, but not necessarily in a monopolistic market, is robust to allowing for a maximal bonus that increases in v.

Attribute Space. In addition, we have assumed that each contract is characterized by M bonus payments, N regular payments, and the consumption value. We can relax this assumption in the following two ways, without changing our qualitative results.

Reduced Attribute Space. Consumers might not consider each regular payment as a separate attribute, but instead think about the monthly payments as one dimension. In this case, the fact that the monthly payments all have the same size would not be an endogenous equilibrium property anymore, but would represent an exogenous assumption. Besides this, our results carry over to this alternative specification of the attribute space. Indeed, all proofs remain the same, except for skipping the part in which we verify that all regular payments are of the same size.

Endogenous Attribute Space. So far, we have considered the case with a fixed number of bonus and regular payments, respectively. Obviously, as firms want to pay at most a single bonus, such a restriction on the number of bonus payments is without loss of generality. Just assuming a fixed number of regular payments without imposing further restrictions (e.g., transaction costs for regular payments) entails a loss of generality, however, as then firms would always want to increase the number of regular payments. Instead, we could assume that firms can freely choose the number of bonus and regular payments, but that consumers incur transaction costs that are increasing and convex in the number of non-zero regular payments.

As we prove in Appendix B, given these assumptions, the qualitative insights from Propositions 1 and 2 remain valid. If in addition consumers' transaction costs are sufficiently convex, then also our result on the comparison of the monopolistic and the competitive outcome remains valid; that is, if transaction costs are sufficiently convex and the consumers' valuation for the product is sufficiently high, the contractual inefficiencies are strictly lower in a monopolistic than in a competitive market. In order to illustrate this result, assume a cost function that is relatively flat first and steep afterwards. In this case, it is easy to see that a monopolist would choose the same number of regular payments as competitive firms would do, so that the only welfare-relevant difference between the monopolistic and the competitive outcome refers to the question of whether the monopolist pays a bonus or not.

Heterogeneous Consumers. So far, we have assumed that consumers are homogeneous, both with respect to their valuation for the product, v, and their focusing function, $g(\cdot)$. In the following, we subsequently relax each of these assumptions.

First, suppose that consumers have the same valuation for the product, but are heterogeneous with respect to the curvature of their focusing function, and that firms can only offer a single contract. Then, a monopolist offers a contract that sets the consumer type that has, among those that should be attracted, the flattest focusing function indifferent between buying and not buying. Depending on v and the focusing function of the indifferent type, either no bonus or the maximal bonus will be set. Consumers with a stronger focusing bias (i.e., a steeper focusing function) will also be attracted by that contract as they appreciate the bonus even more. If there are at least two firms competing for consumers, the maximal bonus will be set (at least in the symmetric equilibrium) and the equal-sized regular payments will be chosen in such a way that firms earn zero profits. As an illustration, suppose that consumers were not susceptible to focusing and were therefore indifferent between a contract with a maximal bonus and no bonus, respectively, as long as the net payment (and hence firms' profits) was held constant. But this implies that, if a small fraction of consumers have focused-weighted utility, competing firms want to exploit this by offering a bonus contract.

Second, suppose that consumers differ only in their valuation for the product, and that each firm can offer a single contract. Also in this case, the main insights of Propositions 1 and 2 still hold. A monopolist makes the consumer type that has, among those that should be attracted, the lowest valuation indifferent between buying and not buying, and maximizes profits along the lines of Proposition 1. Competitive firms set the maximal bonus in any case, charging equal-sized regular payments that allow them to break even.

Third, suppose that consumers are heterogeneous (in some or both dimensions) and that firms can perfectly discriminate between them, that is, each firm can offer a different contract to each consumer type. In addition, assume that each consumer type can only see the contracts(s) tailored to it. Then, all of our preceding results apply separately to each consumer group.

4 Applications

In order to apply our results to three exemplary markets, we first extend our model by including two types of consumers. Subsequently, we discuss anecdotal evidence in line with our predictions.

4.1 Extending the Model: Loyal and Switching Consumers

Suppose there are $L \geq 2$ firms, where each firm $k \in \{1, \ldots, L\}$ has a share of *loyal consumers* $\alpha_k > 0$ with $\sum_{k=1}^{L} \alpha_k < 1$. A consumer who is loyal to firm k only considers her tailor-made contract offered by firm k, and buys as long as this contract gives her a non-negative focus-weighted utility. The remaining consumers, a share $1 - \sum_{k=1}^{L} \alpha_k$, we call *switching consumers*. They observe all contracts except for those tailored to the loyal consumers, and they choose among these contracts so as to maximize their focus-weighted utility. Finally, suppose that the consumers' valuation for the good is so high that a monopolist would not offer a bonus contract.

We then predict that firms offer different contracts for loyal and for switching consumers. Since each firm k acts as a monopolist for its loyal consumers and since the product's value is assumed to be high, it offers a contract without a bonus payment as defined in Lemma 1. In contrast, firms fiercely compete for switching consumers and offer them the bonus contract defined in Proposition 2. The following corollary summarizes this result.

Corollary 2. *Each firm serves its loyal consumers using a contract without a bonus payment, thereby making positive profits. In addition, at least two firms serve the switching consumers, offering them the maximal bonus via a single bonus payment, thereby making zero profits. Both contracts involve regular payments that are equally spread across all periods.*

4.2 Anecdotal Evidence

Application I: Electricity Supply Contracts. In practice, power consumption is not binary, but continuous. Consumers not only decide whether or not to consume, but also how much to consume. However, in many countries, such as Germany, electricity suppliers charge fixed monthly pre-payments that are based on a consumer's estimated power consumption. Arguably, even though the actual monthly fees are not fixed, this contract design (involving pre-payments) might make consumers ex-ante reason as if the regular payments were fixed, in particular because demand for electricity has been found to be highly inelastic with respect to the actual marginal price (e.g., Ito, 2014). Furthermore, in many countries (e.g., in Germany, the UK, or the US), the electricity market consists of local default providers and several smaller entrants. Empirical studies suggest that a substantial share of consumers do not consider switching from their default provider to a cheaper alternative as a feasible option (Hortaçsu *et al.*, 2017). As a consequence, our formal setup that distinguishes between loyal and switching consumers matches the market for electricity supply quite well.

As predicted by our model, loyal consumers are typically charged high regular payments and do not receive a bonus. In contrast, but in line with our model, electricity suppliers fiercely compete for the remaining consumers who are willing to switch (i.e., who compare offers across providers) by offering large bonus payments that are paid after the subscription (often around 60 days later).[15] As mentioned above, it is a common feature of electricity supply contracts that the (often monthly) payments to be made by consumers are constant over the contractual period, even though actual usage is measured and therefore also billed only once a year.[16] Such a contract design involving regular (pre-)payments that are equally dispersed over the contractual period is also optimal according to our model as—unlike contracts conditioning payments in each period on the actual per-period usage—it minimizes the consumers' focus on costs.

Application II: Telephony and Internet Contracts. Our model also fits the market for mobile phone contracts. Although in most OECD countries there are several providers of telecommunication services, a substantial share of consumers have never switched their provider.[17] While contracts designed for new customers typically include valuable features (such as smartphones, tablets, special discounts, or bonus payments), customers who extend an already exist-

[15] See, for instance, `http://www.handelsblatt.com/politik/konjunktur/oekonomie/nachrichten/anbieterwechsel-die-teure-traegheit-der-verbraucher/3560414-all.html`, accessed on October 1, 2018.

[16] See, for instance, `https://www.gov.uk/guidance/gas-meter-readings-and-bill-calculation`, or `https://www.verbraucherzentrale.de/energieversorger-rechnungen`, both accessed on October 1, 2018.

[17] See, for instance, `https://www.oecd.org/sti/consumer/40679279.pdf`, pp. 32, accessed on October 1, 2018.

ing contract usually obtain worse offers that do not involve such a bonus.[18] This observation is suggestive for our prediction that firms offer contracts without bonus payments to their loyal consumers, and compete for switching consumers with bonus contracts. In line with our model, firms often advertise flat-rate contracts, that is, contracts involving payments to be made by consumers that are equally spread over the contractual period and that do not depend on actual usage frequency. Analogous tariff structures are common in the market for Internet contracts.[19]

Application III: Bank Accounts. The retail banking industry serves as another example that our setup applies to. In most EU countries, a considerable share of consumers do not even consider the option to switch their bank as a viable alternative, although there are several competitors in the market.[20] While account management fees are usually dispersed over the contractual period, banks try to attract new customers by offering a large switching bonus that is typically paid after the contract is signed and certain conditions (e.g., minimal monthly deposits) are satisfied.[21] As predicted by our model, banks offer bonus payments only to those consumers who are willing to switch and open a new account, but not to their existing customers.

Summary of Stylized Facts. In order to compare the explanatory power of our model and alternative approaches (that we discuss in the next section), we find it useful to sum up the stylized features of bonus contracts observed in the exemplary markets, as delineated above:

(a) If a firm pays a bonus to consumers, it offers a single, but relatively high bonus payment.

(b) Consumers receive the bonus not immediately when signing the contract, but with a delay.

(c) Regular payments made by consumers are equally dispersed over the contractual period.

(d) Only those consumers who search for the best deal are offered bonus contracts, others not.

[18] See, for instance, `http://money.cnn.com/2015/03/18/smallbusiness/tmobile-uncarrier/index.html`, accessed on October 1, 2018.

[19] Our assumption of unit demand is particularly plausible for flat-rate contracts that are very common for telephony and Internet services. To be precise, however, not only flat-rate contracts exist, but also contracts that depend on actual usage, in which case the periodical payments differ to some degree.

[20] See, for instance, `http://www.ec.europa.eu/competition/sectors/financial_services/inquiries/sec_2007_106.pdf`, p. 66, accessed on October 1, 2018.

[21] See, for instance, `https://www.welt.de/finanzen/geldanlage/article126159643/Hohe-Praemien-fuer-Girokonten-bringen-Nachteile.html`, accessed on October 1, 2018.

5 Alternative Approaches and Related Literature

In this section, we review the related literature and discuss alternative explanations for bonus contracts, with an emphasis on predictions that allow us to distinguish between our model and these different approaches. For that, we first derive the predictions of models on time preferences (Section 5.1) and switching costs (Section 5.2) in our context, and second relate these alternative approaches to the anecdotal evidence that we have presented in the previous section. We argue that features (a), (b), and (d) remain largely unexplained by the existing literature. Finally, we relate our paper also to the literature on partitioned pricing and shrouding.

5.1 Exponential and (Quasi-)Hyperbolic Discounting

Exponential Discounting. According to the classical model, as proposed by Samuelson (1937), an agent maximizes her expected intertemporal utility, which (i) is additively separable across payoffs received at different points in time, and (ii) satisfies exponential discounting (i.e., payoffs t periods ahead are discounted by δ^t for some discount factor $\delta < 1$). A classical agent should be indifferent between any allocation of payments across time that has the same net present value. Therefore, firms will avoid inefficient bonus payments. If in addition we impose the common assumption that the marginal utility from money decreases (i.e., preferences can be represented by a concave utility function over monetary wealth), the use of large bonus payments becomes even less attractive. Thus, the classical model can only explain (c).

(Quasi-)Hyperbolic Discounting. In order to match evidence on present-biased behavior, more recent approaches to intertemporal decision making have assumed that discounting is hyperbolic (for seminal contributions, see, Chung and Herrnstein, 1967, and Loewenstein and Prelec, 1992) or quasi-hyperbolic (Laibson, 1997). In principle, a model of quasi-hyperbolic discounting could explain the use of bonus contracts. In the following, we discuss its predictions on monopolistic and competitive outcomes in more detail. In order to give quasi-hyperbolic discounting the best chance to explain the anecdotal evidence, we assume that firms can indeed offer bonus payments that lie in the *present*, while they can shift all regular payments into the future. We discuss these assumptions subsequently. As usual, denote the short-term discount factor by $\beta < 1$ and, for the sake of comparability, let the long-term discount factor equal 1.

Monopolistic Market. If the monopolist offers a bonus payment, it will always choose to pay the bonus immediately, as otherwise the consumer would not overweight the bonus relative to the regular payments. More precisely, if consumers derive linear utility from money, a monopolist would move regular payments into the future and offer immediate bonus payments such that

$$\sum_{i=1}^{N} p_i = \frac{1}{\beta}\left(v + \sum_{j=1}^{M} b_j\right) \tag{2}$$

is satisfied. Since each bonus implies an additional cost of $\kappa > 0$, we conclude from Eq. (2) that the monopolist chooses at most one bonus payment. If the monopolist sets a bonus, then its maximization problem is given by

$$\max_{b\in(0,\bar{b}]} \frac{1}{\beta}(v+b) - b.$$

Since $\beta < 1$, the monopolist offers either the maximal bonus or no bonus at all, whereby a bonus contract is indeed optimal if and only if $\frac{\kappa}{\bar{b}} < \frac{1-\beta}{\beta}$. Given linear utility of money, a model of quasi-hyperbolic discounting makes no prediction on the structure of regular payments.

If we instead assume that consumers have a decreasing marginal utility from consumption, which implies that the marginal disutility from making a payment is increasing, the monopolist equally disperses the regular payments across the N periods. But given a decreasing marginal utility from consumption, a model of quasi-hyperbolic discounting does no longer make a clear-cut prediction on bonus payments. If consumers integrate the different bonus payments, the monopolist still prefers to offer at most one bonus. If consumers perceive bonus payments as separate (which seems particularly plausible for a mix of monetary and non-monetary bonuses), however, setting multiple bonuses might be more profitable than offering a single bonus payment.

Interestingly, given that consumers integrate bonus payments, the monopolist is more likely to offer a bonus the higher the valuation v is. As an illustration, denote utility from the bonus by $u(\cdot)$ with $u(0) = 0$, $u' > 0$, and $u'' < 0$, and utility from a regular payment by $-u(\cdot)$. Then, the monopolist sets the uniform regular payment, p, and the bonus payment, b, such that

$$Nu(p) = \frac{1}{\beta}(v + u(b)).$$

We denote as $p^*(v, b)$ the unique solution to this equation. Consequently, conditional on offering a positive bonus, the monopolist's problem is given by

$$\max_{b\in(0,\bar{b}]} Np^*(v,b) - b,$$

so that the marginal profit from increasing the bonus payment equals

$$\frac{1}{\beta}\frac{u'(b)}{u'\big(p^*(v,b)\big)} - 1.$$

Since $u(\cdot)$ is concave and $p^*(v,b)$ increases with v, the marginal profit from a bonus payment also increases with v, so that the monopolist is more likely to pay a bonus the higher v is.

Competitive Market. A standard Bertrand-type argument implies that firms earn zero profits in equilibrium. In addition, any equilibrium contract has to maximize consumers' perceived utility, as otherwise a firm could design a contract that consumers strictly prefer, thereby earning positive profits. If consumers derive linear utility from money, this immediately implies that firms offer at most one bonus payment. Then, conditional on firm k offering a bonus contract (and serving a positive share of consumers), the zero-profit condition implies $\sum_{i=1}^{N} p_i^k = \kappa + b^k$. If firm k does not offer a bonus but serves at least some consumers, the zero-profit condition yields $\sum_{i=1}^{N} p_i^k = 0$. Using these zero-profit conditions, it is easy to see that consumers prefer a bonus contract with bonus payment b if and only if $\frac{\kappa}{b} < \frac{1-\beta}{\beta}$. This further implies that firms offer either the maximal bonus or no bonus at all. Altogether, we conclude that competitive firms offer a bonus contract if and only if a monopolist would also do so. If we instead assume a decreasing marginal utility from consumption, a monopolist not only sets a bonus whenever competitive firms do so, but might even set a bonus when competitive firms do not.[22]

(Quasi-)Hyperbolic Discounting vs. Focusing. While a model of focusing predicts that a monopolist offers a bonus contract only for low-value goods, a model of quasi-hyperbolic discounting either makes the opposite prediction (under decreasing marginal consumption utility) or predicts that the choice to offer a bonus does not depend on the product's value (under linear consumption utility). In addition, according to quasi-hyperbolic discounting, the frequency of bonus contracts does not depend on the degree of competition (under linear consumption utility)

[22] For the sake of argument, suppose that both a monopolist and competitive firms set either a single, maximal bonus or no bonus at all, which is indeed the case as long as consumption utility is not too concave. Let $u^{-1}(\cdot)$ denote the inverse of $u(\cdot)$. Then, the monopolist will offer a bonus if and only if $N\big[u^{-1}\left(\frac{v+\bar{b}}{\beta N}\right) - u^{-1}\left(\frac{v}{\beta N}\right)\big] - \bar{b} > \kappa$ holds, while competitive firms offer a bonus contract in equilibrium if and only if $Nu^{-1}\left(\frac{\bar{b}}{\beta N}\right) - \bar{b} > \kappa$ is satisfied. Since $u(0) = 0$ and since $u(\cdot)$ is monotonically increasing and concave, $u^{-1}(\cdot)$ is super-additive. This gives

$$u^{-1}\left(\frac{v+\bar{b}}{\beta N}\right) > u^{-1}\left(\frac{v}{\beta N}\right) + u^{-1}\left(\frac{\bar{b}}{\beta N}\right),$$

which in turn implies that a monopolist offers a bonus contract whenever competitive firms do so.

or bonus contracts are even more frequent in monopolistic rather than in competitive markets (under decreasing marginal consumption utility). Thus, stylized fact (d) remains unexplained by quasi-hyperbolic discounting. In contrast and in line with anecdotal evidence, focusing predicts that bonus contracts are more prevalent in competitive markets.

Most importantly, quasi-hyperbolic discounting can only explain bonus payments that are perceived by the consumer as payments in the *present*. Recent studies have estimated $\beta \approx 1$ for payments that are obtained not immediately, but later on the same day (see, for instance, Andreoni and Sprenger, 2012). And even for immediate payments the extent of present bias is small or negligible (Andersen *et al.*, 2014; Augenblick *et al.*, 2015; Balakrishnan *et al.*, 2017).[23] Also a model of hyperbolic discounting suggests that any bonus payment is made immediately, while in practice bonus payments are often delivered with a substantial delay: in the case of power supply contracts, for instance, most bonuses are paid only after 60 days. Due to this inevitable delay in payments, the recent experimental literature on time-discounting suggests that (quasi-)hyperbolic discounting could not explain the observed design of bonus contracts.[24] Consequently, also stylized fact (b) is inconsistent with models of (quasi-)hyperbolic discounting.

5.2 Switching Costs and Automatic-Renewal Contracts

Switching Costs. Models on switching costs (e.g., Klemperer, 1995) can explain why consumers might be reluctant to switch providers. Accordingly, consumers need to be compensated for their switching costs, which may in principle be achieved by paying a bonus to consumers (see, Farrell and Klemperer, 2007, for a discussion). For the sake of comparability, we again abstract from discounting. In addition, we assume a decreasing marginal utility from consumption.

A Simple Model. Consider the following two-period game. In each period firms simultaneously offer a contract and consumers choose from the set of contracts. In the second period, consumers incur some cost $s > 0$ when switching to another firm. We solve for a subgame-perfect Nash equilibrium, whereby we assume that firms cannot commit to second-period prices. In the

[23]Notably, a comprehensive review of the literature on time-discounting not only questions whether subjects indeed discount future payoffs (quasi-)hyperbolically, but makes the point that the discount function framework in general might not be adequate for understanding intertemporal decisions (Cohen *et al.*, 2016).

[24]Also the theoretical behavioral IO literature has argued that payments specified in common contracts necessarily affect future, not present consumption (see, Heidhues and Kőszegi, 2010, for a discussion of credit contracts).

markets discussed in the previous section (i.e., electricity, telephony, and bank accounts), prices are typically fixed for the contractual period of one or two years. Thus, to meaningfully apply a model on switching costs without commitment to our setup, we need to assume that each period lasts for the duration of a contract.

Suppose that a positive share of consumers bought at firm i in the first period. Because of switching costs, firm i serves as a de-facto monopolist for these consumers, and since any bonus payment implies an inefficiency, it has no incentive to offer a bonus contract. In addition, due to decreasing marginal consumption utility, it equally disperses regular payments across the N payment dates in the second period. More specifically, firm i chooses its regular payments such that $Nu(p_i) = s$, and therefore earns $Nu^{-1}\left(\frac{s}{N}\right)$ per consumer.

In the first period, firms fiercely compete for consumers and—depending on the magnitude of switching costs—might indeed offer bonus payment(s). In fact, firms offer at least one bonus payment if and only if $\kappa < \frac{L-1}{L} Nu^{-1}\left(\frac{s}{N}\right)$, where the optimal number of bonus payments depends on the curvature of consumption utility relative to the bonus-related inefficiency. In any bonus contract, regular payments are equally spread across payment dates and are chosen in a way that, as long as $\bar{b}$ is large enough, second-period profits are handed back to consumers. If no bonus is paid, firms set regular payments equal to zero, thereby earning positive profits, because of the floor on the regular payments.

Switching Costs vs. Focusing. In order to explain inefficient bonus payments, models on switching costs need to assume that firms cannot ex-ante commit to second-period prices (a point that has been made already in DellaVigna and Malmendier, 2004). Given this assumption, firms fiercely compete for a large customer base in the first period, which may result in consumers signing bonus contracts in equilibrium. In contrast, our approach explains bonus contracts also if firms can commit to second-period prices. In addition, when marginal production costs were positive, firms would rather charge lower regular payments in the first period than give a bonus payment to their consumers, due to the inherent inefficiency of bonus payments. Only if lowering regular payments does not suffice for attracting consumers, switching costs could explain why a firm might set a bonus payment. Then, the equilibrium bonus payment must exceed the sum of all regular payments, however, which we regard as implausible, as we are not aware of any markets where this is the case.[25] Thus, features (a) and (d) remain unexplained by models on switching costs.

Automatic-Renewal Contracts. Relatedly, Johnen (2018) studies a market in which firms offer automatic-renewal contracts to consumers who are inert in the sense that they forgo benefits from switching to another firm. If a consumer underestimates the probability of failing to cancel a contract (e.g., due to limited attention or a naive present-bias), firms can exploit this consumer by offering an attractive teaser rate that increases after the automatic renewal of the contract. Although his approach provides a plausible explanation for offering attractive teaser rates, it does not make specific predictions on whether firms should use a bonus payment to attract consumers or whether they should simply lower the regular payments, as in his model only the predicted net present value matters. Indeed, if the bonus payment is inefficient, firms will never offer it (i.e., feature (a) is not explained). In order to explain bonus contracts, therefore, the model by Johnen (2018) needs to be augmented, for instance, along the lines that we suggest in this paper (whereby consumers are assumed to be focused thinkers).[26] As a consequence, we regard the explanation for bonus contracts offered by our model and by Johnen (2018) as complementary. Interestingly, also in Johnen (2018), competitive firms focus more on exploiting consumer mistakes than a monopolist does, so that similar to our findings also in his model the

[25] Only under the specific assumption that consumers face high switching costs, but have very limited access to credit, inefficient bonus payments may be made if firms can commit to future prices. This scenario, however, does not fit to the examples that we have in mind, as switching power suppliers, banks, or providers of mobile phone services seems to be typically very cheap or even for free.

[26] In order to explain the use of bonus contracts one could augment Johnen (2018) also in other ways, for instance, by assuming that a consumer is more likely to remember to switch contracts if the per-period payments increase over time relative to the benchmark with constant payments. We thank a referee for suggesting this possible extension of Johnen (2018). If the change in regular payments occurs after the auto-renewal or the cancellation period, however, we would expect this effect to be rather small. More importantly, we are unaware of any direct empirical support for this mechanism. One could, of course, extend Johnen (2018) also in other ways, and we think of the focusing model as one natural candidate. In particular, the focusing mechanism that we explore is supported by lab evidence (Dertwinkel-Kalt *et al.*, 2017). Thus, we regard our focusing-based explanation as the more parsimonious extension of Johnen (2018).

monopolistic outcome can be more efficient than the competitive one.[27]

5.3 Salience Theory of Consumer Choice

Salience theory, as proposed by Bordalo *et al.* (2013b), shares Kőszegi and Szeidl's central assumption that dimensions along which alternatives differ much attract much attention. Salience theory, however, has more degrees of freedom than the model by Kőszegi and Szeidl, as the salience of an attribute is determined by a bivariate *salience function* that compares an option's attribute value against the average value that this attribute takes in the choice set. Besides the contrast effect, a salience function exhibits a second fundamental property, that is the *level effect*, whereby a fixed contrast attracts less attention for larger overall attribute values. Also, unlike the focusing model, the salience model predicts that it depends on the specific option at hand which of its attributes grabs a consumer's attention; that is, the attribute that is most salient and therefore attracts most attention can vary across options.

Due to its greater flexibility, salience theory can, depending on the exact context, predict both concentration bias or its inverse, dispersion bias. Put differently, the results derived in this paper are consistent with salience theory, but also various other equilibrium structures could be consistent with the salience model. As a consequence, we build our analysis on the focusing model by Kőszegi and Szeidl (2013), which allows for the clear-cut prediction of concentration bias.

5.4 Partitioned Pricing, Shrouding, and Socially Wasteful Products

We have shown that attentional focusing can explain why firms frequently partition a product's total price into several price components in order to increase a consumer's willingness to pay (for empirical evidence, see Morwitz *et al.*, 1998). Indeed, a number of older studies observe that consumers systematically underestimate a product's overall price if it is partitioned into several price components that the consumer is simultaneously (*partitioned pricing*) or sequentially (*drip pricing*) informed about (e.g., Carlson and Weathers, 2008; Ahmetoglu *et al.*, 2014). More recent experimental evidence from the lab (Dertwinkel-Kalt *et al.*, 2017) and from the field (Dertwinkel-

[27] There is a small, but recently growing literature on the distorting effects of competition (e.g., Carlin, 2009; Gabaix *et al.*, 2016; Friedrichsen, 2018). See Johnen (2018) for a discussion of the mechanisms in these papers.

Kalt *et al.*, 2018) suggests, however, that simply splitting up the total price without dispersing the price components over time does not affect a consumer's willingness to pay. Hence, in line with our interpretation, it seems to be the correct timing of regular payments (e.g., monthly) that gives partioned pricing the potential to increase a consumer's willingness to pay.

While also the literature on shrouded price components (e.g., Gabaix and Laibson, 2006; Heidhues *et al.*, 2016) can explain the success of partitioned pricing, it necessarily assumes that a share of consumers are not aware of some price components when making the purchase decision or the importance thereof when mispredicting their future behavior. In contrast, a model of focusing can also explain these effects if all information is readily available. Because several smaller prices attract less attention than a single, but large one, they are underweighted. As a consequence, the focusing model can account for the fact that a uniform dispersion of the total price over time increases a consumer's willingness to pay even if the consumer is fully informed about all price components.

Finally, our study connects to the literature demonstrating that even socially wasteful products can survive competition and may be sold in a competitive equilibrium. Heidhues *et al.* (2016) argue that, if a part of the product's price is shrouded, some consumers may not anticipate the product's total price at the moment of making the purchase decision, so that these consumers may purchase a good at a price that strictly exceeds their valuation. Thus, even a socially wasteful product—that is, a product for which the production costs lie above the consumers' valuation—may generate positive demand. Since firms typically have no incentive to unshroud the additional price, selling a socially wasteful product can be the equilibrium outcome in a perfectly competitive market. Also in our model socially wasteful products might be sold in a competitive equilibrium even if consumers are aware of the entire price.[28] By focusing on the contract's outstanding feature (i.e., the large bonus payment), consumers may overestimate the value of a deal and sign contracts for socially wasteful products.

[28] Since our model assumes zero production costs, a product that consumers value at $v < 0$ is socially wasteful. While we assume $v \geq 0$, in fact our analysis also holds if v is negative but sufficiently close to zero.

6 Conclusion

Bonus contracts create two distinct inefficiencies. On the one hand, bonus payments create administrative costs, both for the issuing firm and for the consumer, and non-monetary bonuses such as included premiums may give an imperfect match between the bonus and the consumer's preferences. On the other hand, bonus contracts yield an imbalanced decision situation—benefits are concentrated in the form of a single, large bonus payment while costs are dispersed over many small payments—in which focused thinkers tend to make suboptimal decisions. In fact, a consumer chooses an inefficient bonus contract in the competitive equilibrium even if some firm offers an efficient contract with no bonus payment. If we follow Kőszegi and Szeidl (2013) in assuming that focus weights only affect the consumer's *decision utility* but not her *experienced utility* that defines her surplus, then this decision can be considered a mistake by the consumer: firms earn zero profits in any competitive equilibrium, so that consumers bear the additional costs that adjoin bonus payments (in the form of higher regular payments). Hence, under the assumption that focusing does not affect experienced utility, the use of bonus payments strictly lowers consumer welfare, both in monopolistic and competitive markets.

We have shown that these inefficiencies are not eliminated by competition, but can only be overcome by regulation. Indeed, firms *have to* exploit attentional focusing under competitive pressure, so that bonus contracts are even more frequent in competitive than in monopolistic markets. By enhancing the use of bonus payments, competition therefore exacerbates the inefficiencies arising from contracting with focused agents.[29]

From a policy perspective, our study suggests that a legal ban on bonus payments could have favorable consequences. On the one hand, a legal ban on bonus payments eliminates the inherent inefficiency of paying bonuses. On the other hand, it creates choice environments that are balanced, that is, where in equilibrium all payments receive the same amount of attention. Notably, making bonus payments is not necessary to encourage consumers to switch providers, as firms could instead lower the regular payments to attract consumers (see, Farrell and Klemperer, 2007, for a discussion of different modeling approaches). Hence, even if consumers incur costs for switching to another provider, a ban on bonus payments does not impair competitive forces.

[29] As in other behavioral IO models (see, for instance, DellaVigna and Malmendier, 2004; Gabaix and Laibson, 2006), competition drives down firms' profits to zero, *even though* consumers' decision biases are fully exploited.

Altogether, we argue that prohibiting the use of bonus contracts not only reduces the direct inefficiencies arising from bonus payments, but could also induce better decisions by consumers.

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Appendix A: Proofs

For brevity, we denote $\tilde{v} := g(v)v$ the focus-weighted consumption value of the product. In addition, we suppress the consumption value dimension of a contract throughout the Appendix.

Proof of Lemma 1. The proof proceeds in two steps. First, we rewrite the monopolist's maximization problem and characterize the optimal payments to be made by consumers. Second, we argue that the monopolist offers either the maximal bonus (i.e., $\sum_{j=1}^{M} b_j = \bar{b}$) or no bonus. Third, we show that the monopolist pays at most one bonus.

1. STEP: In order to solve the monopolist's maximization problem, we set up the Lagrangian

$$\mathcal{L}(\boldsymbol{c}, \boldsymbol{\mu}, \eta, \boldsymbol{\gamma}, \lambda) := \sum_{i=1}^{N} p_i - \sum_{j=1}^{M} b_j - \lambda \left(\sum_{i=1}^{N} g(p_i)p_i - \tilde{v} - \sum_{j=1}^{M} g(b_j)b_j \right) - \eta \left(\sum_{j=1}^{M} b_j - \bar{b} \right) - \sum_{j=1}^{M} \gamma_j(-b_j) - \sum_{i=1}^{N} \mu_i(-p_i),$$

where $\lambda, \eta, \gamma_j, \mu_i \geq 0$, which yields the following Karush-Kuhn-Tucker Conditions:

$$\frac{\partial \mathcal{L}}{\partial p_i} = 1 - \lambda \left[g(p_i) + g'(p_i)p_i \right] + \mu_i \leq 0, \tag{KKT^p_i-1}$$

holding with equality if $p_i > 0$, and

$$\frac{\partial \mathcal{L}}{\partial b_j} = -1 + \lambda \left[g(b_j) + g'(b_j)b_j \right] - \eta + \gamma_j \begin{cases} \leq 0 & \text{if } b_j = 0, \\ = 0 & \text{if } 0 < b_j < \bar{b}, \\ \geq 0 & \text{if } b_j = \bar{b}, \end{cases} \tag{KKT^b_j-1}$$

as well as the condition on the participation constraint

$$\lambda \cdot \left(\sum_{i=0}^{N} g(p_i)p_i - \tilde{v} - \sum_{j=1}^{M} g(b_j)b_j \right) = 0, \tag{KKT-PC}$$

and conditions on price constraints, that is,

$$\mu_i \cdot (-p_i) = 0 \tag{KKT^p_i-2}$$

for any $i \in \{1, \ldots, N\}$, and conditions on bonus constraints, that is,

$$\gamma_j \cdot (-b_j) = 0 \tag{KKT^b_j-2}$$

for any $j \in \{1, \ldots, M\}$ as well as

$$\eta \left(\sum_{j=1}^{M} b_j - \bar{b} \right) = 0. \tag{KKT-BC}$$

First, we characterize the optimal payments to be made by consumers (i.e., part (i) of our lemma). We observe that at least one p_i has to be larger than zero, as otherwise $\lambda = 0$ by (KKT-PC) and therefore $\frac{\partial \mathcal{L}}{\partial p_i} > 0$ by (KKT^p_i-1); a contradiction. Hence, from now on suppose $p_i > 0$ for some $i \in \{1, \ldots, N\}$. Then, since $1 + \mu_i > 0$, Condition (KKT^p_i-1) gives $\lambda > 0$. Together with (KKT-PC), this yields

$$\sum_{i=1}^{N} g(p_i)p_i = \tilde{v} + \sum_{j=1}^{M} g(b_j)b_j. \tag{3}$$

Next, we show that this implies $p_i > 0$ for any $i \in \{1, \ldots, N\}$. For the sake of contradiction, suppose $p_j = 0$ for some $j \in \{1, \ldots, N\}$. Then, Condition (KKT^p_j-1) yields $1 + \mu_j \leq \lambda g(0)$. As there is at least one $p_i > 0$, Conditions (KKT^p_i-1) and (KKT^p_i-2) yield $1 = \lambda\big[g(p_i) + g'(p_i)p_i\big]$. Together, these considerations give

$$1 \leq 1 + \mu_j \leq \lambda \underbrace{\left[g(0) + g'(0)0\right]}_{=g(0)} \overset{\text{A.3}}{<} \lambda \left[g(p_i) + g'(p_i)p_i\right] = 1, \tag{4}$$

a contradiction. Thus, we have $p_i > 0$ for any $i \in \{1, \ldots, N\}$. In addition, Conditions (KKT^p_i-1) and (KKT^p_j-1) require $p_i = p_j$ for $i, j \in \{1, \ldots, N\}$, which completes the proof of part (i).

2. STEP: Given the results derived above, we show that *the monopolist either offers the maximal bonus (i.e., $\sum_{j=1}^{M} b_j = \bar{b}$) or no bonus at all.* For the sake of contradiction, suppose that the monopolist offers a contract with $0 < \sum_{j=1}^{M} b_j < \bar{b}$. Thus, we have $\eta = 0$ by (KKT-BC), and, for any $b_j > 0$, also $\gamma_j = 0$ by (KKT^b_j-2). Using the same arguments as in the first step, we conclude from Conditions (KKT^p_i-1) and (KKT^b_j-1) that either $b_j = p_i = p'$ or $b_j = 0$. Let $m \in \{1, \ldots, M\}$ bonus payments be non-zero and notice that $m < N$, as otherwise profits would be zero. Then, (KKT-PC) implies that $g(p')p' = \tilde{v}/(N - m)$, so that the monopolist earns

$$\pi' = \frac{\tilde{v}}{g(p')} - m \cdot \kappa. \tag{5}$$

Suppose that the monopolist instead does not offer any bonus payments; that is, $b_j = 0$ for any $j \in \{1, \ldots, M\}$. By the first step, we have $p_i = p''$ for any $i \in \{1, \ldots, N\}$, and Condition (KKT-PC) yields $g(p'')p'' = \tilde{v}/N$, so that the monopolist earns

$$\pi'' = \frac{\tilde{v}}{g(p'')}. \tag{6}$$

Since $g(x)x$ is a strictly increasing function, by Assumption 2, we conclude $p'' < p'$ from

$$g(p'')p'' = \frac{\tilde{v}}{N} < \frac{\tilde{v}}{N-m} = g(p')p'.$$

Then, for any $\kappa \geq 0$, we obtain $\pi'' > \pi'$ by Assumption 2; a contradiction. As a consequence, the monopolist will never offer a contract with $0 < \sum_{j=1}^{M} b_j < \bar{b}$.

3. STEP: Given the results from the preceding steps, we next show that *the monopolist offers at most one bonus payment.* Suppose that $\sum_{j=1}^{M} b_j = \bar{b}$ and that $m \geq 1$ bonus payments are non-zero. By the first step, we have $p_i = p'''(m)$, $i \in \{1, \ldots, N\}$, and (KKT-PC) yields

$$g(p'''(m))p'''(m) = \frac{1}{N} \cdot \left[\tilde{v} + \sum_{j=1}^{m} g(b_j)b_j\right]. \tag{7}$$

For any $m > 1$, Assumption 2 immediately implies that

$$g(\bar{b})\bar{b} = \sum_{j=1}^{m} g(\bar{b})b_j \overset{\text{A.2}}{>} \sum_{j=1}^{m} g(b_j)b_j$$

holds. Thus, by Assumption 2 and Eq. (7), we have $p'''(1) > p'''(m)$ for any $m > 1$. As the bonus payment is fixed and as less bonus payments imply lower costs, the monopolist will choose at most one bonus payment, which was to be proven. □

Proof of Proposition 1. By Lemma 1, the monopolist offers either a bonus contract with a single bonus payment, $\boldsymbol{c}^{bon} := (\bar{b}, 0, \ldots, 0, p^{bon}, \ldots, p^{bon})$, or a contract without any bonus payments, $\boldsymbol{c}^{no} := (0, \ldots, 0, p^{no}, \ldots, p^{no})$. We have also seen in the proof of Lemma 1 that $p^{bon} = p^{bon}(v, \bar{b})$ is implicitly defined by $g(p^{bon})p^{bon} = \frac{1}{N}\left[\tilde{v} + g(\bar{b})\bar{b}\right]$, and that $p^{no} = p^{no}(v)$ is implicitly given by $g(p^{no})p^{no} = \frac{\tilde{v}}{N}$. We proceed in two steps. First, we neglect the cost of paying a bonus, that is, we set $\kappa = 0$. Second, we allow for positive costs of paying a bonus, that is, $\kappa > 0$.

1. STEP: Let $\kappa = 0$. Then, the monopolist offers a bonus contract $\boldsymbol{c}^{bon}$ if and only if

$$\frac{\tilde{v} + \left(g(\bar{b}) - g(p^{bon})\right)\bar{b}}{g(p^{bon})} > \frac{\tilde{v}}{g(p^{no})}$$

or, equivalently,

$$\bar{b}\left(g(\bar{b}) - g(p^{bon})\right) > \tilde{v}\left(\frac{g(p^{bon}) - g(p^{no})}{g(p^{no})}\right). \tag{8}$$

We proceed as follows: first, we verify that $\pi(\boldsymbol{c}^{bon}) - \pi(\boldsymbol{c}^{no})$ monotonically decreases in v, which implies that (8) is more likely to hold for small values of v. Second, we argue that (8) is violated as v approaches infinity while it is fulfilled as v approaches zero.

Recall that $p^{bon} > p^{no}$. Then, by the Implicit Function Theorem, we obtain

$$\frac{\partial}{\partial v}\Big(\pi(\boldsymbol{c}^{bon}) - \pi(\boldsymbol{c}^{no})\Big) = N \cdot \left(\frac{\partial}{\partial v}p^{bon}(v,\bar{b}) - \frac{\partial}{\partial v}p^{no}(v)\right)$$
$$= \frac{\partial \tilde{v}}{\partial v} \cdot \left(\frac{1}{g(p^{bon}) + g'(p^{bon})p^{bon}} - \frac{1}{g(p^{no}) + g'(p^{no})p^{no}}\right),$$

which is strictly negative by Assumption 3 and $p^{bon} > p^{no}$. Thus, $\pi(\boldsymbol{c}^{bon}) - \pi(\boldsymbol{c}^{no})$ monotonically decreases in v, which was to be proven.

Next, suppose that v approaches infinity, and notice that the left-hand side of (8) is negative for sufficiently large values of v, while the right-hand side of (8) is non-negative for any $v \geq 0$. Hence, (8) is violated in the limit of v approaching infinity. Finally, we consider the limit for v approaching zero. By Assumption 2, this implies that also $\tilde{v} := g(v)v$ approaches zero. First, it is easy to see that in this limit the left-hand side of Inequality (8) is strictly larger than zero, as

$$\lim_{v \to 0} g\big(p^{bon}(v,\bar{b})\big)p^{bon}(v,\bar{b}) = \frac{g(\bar{b})\bar{b}}{N} < g(\bar{b})\bar{b},$$

and thus $\lim_{v\to 0} p^{bon}(v,\bar{b}) < \bar{b}$. Second, as $\lim_{v\to 0} \frac{\tilde{v}}{g(p^{no})} = N \lim_{v \to 0} p^{no}(v) = 0$ by definition of $p^{no}(v)$ and Assumption 2, the right-hand side of (8) is zero in the limit of v approaching zero.

Combining the above results and using the fact that $\pi(\boldsymbol{c}^{bon}) - \pi(\boldsymbol{c}^{no})$ is continuous in v, we conclude by the Intermediate Value Theorem that there exists some threshold value $v' > 0$ such that the monopolist offers a bonus contract if and only if $v < v'$.

2. STEP: Let $\kappa > 0$. Then, the monopolist offers a bonus contract $\boldsymbol{c}^{bon}$ if and only if

$$\frac{\tilde{v} + \big(g(\bar{b}) - g(p^{bon})\big)\,\bar{b}}{g(p^{bon})} - \kappa > \frac{\tilde{v}}{g(p^{no})}$$

or, equivalently,

$$\kappa < \underbrace{\frac{\bar{b}\left(g(\bar{b}) - g(p^{bon})\right)}{g(p^{bon})} - \tilde{v}\left(\frac{g(p^{bon}) - g(p^{no})}{g(p^{no})g(p^{bon})}\right)}_{=\pi(\boldsymbol{c}^{bon})\big|_{\kappa=0} - \pi(\boldsymbol{c}^{no})}. \tag{9}$$

We have already seen in the first step that the right-hand side of Inequality (9) monotonically decreases in v. Hence, the monopolist offers a bonus contract for some $v > 0$ only if

$$\kappa < \lim_{v \to 0}\left[\frac{\bar{b}\left[g(\bar{b}) - g\big(p^{bon}(v,\bar{b})\big)\right]}{g\big(p^{bon}(v,\bar{b})\big)} - \tilde{v}\left(\frac{g\big(p^{bon}(v,\bar{b})\big) - g\big(p^{no}(v)\big)}{g\big(p^{no}(v)\big)g\big(p^{bon}(v,\bar{b})\big)}\right)\right]$$
$$= \lim_{v \to 0} \frac{\bar{b}\left[g(\bar{b}) - g\big(p^{bon}(v,\bar{b})\big)\right]}{g\big(p^{bon}(v,\bar{b})\big)}$$
$$=: \hat{\kappa}.$$

By the same arguments as in the first step, for any $\kappa < \hat{\kappa}$, there exists some $\hat{v}(\kappa)$ such that the monopolist offers a bonus contract if and only if $v < \hat{v}(\kappa)$, which was to be proven.

Specifically, the function $\hat{v} : [0, \hat{\kappa}) \to \mathbb{R}_+$ is implicitly given by

$$\underbrace{\frac{\tilde{v}(\hat{v}) + \left[g(\bar{b}) - g\big(p^{bon}(\hat{v}, \bar{b})\big)\right]\bar{b}}{g\big(p^{bon}(\hat{v}, \bar{b})\big)} - \frac{\tilde{v}(\hat{v})}{g\big(p^{no}(\hat{v})\big)} - \kappa}_{=:F(\hat{v},\kappa)} = 0.$$

By construction, we have $\hat{v}(\hat{\kappa}) = 0$. In addition, the Implicit Function Theorem yields

$$\frac{\partial \hat{v}}{\partial \kappa} = -\frac{\frac{\partial}{\partial \kappa} F(\hat{v}, \kappa)}{\frac{\partial}{\partial \hat{v}} F(\hat{v}, \kappa)} = -\frac{(-1)}{\frac{\partial}{\partial \hat{v}}\left[\pi(\boldsymbol{c}^{bon})\big|_{\kappa=0} - \pi(\boldsymbol{c}^{no})\right]} < 0,$$

since we have seen above that the denominator is strictly negative. This completes the proof. □

Proof of Proposition 2. For illustrative purposes, we only consider the case of $L = 2$, but we solve for the essentially unique equilibrium without restricting ourselves to symmetric equilibria. The generalization of the arguments to the case of $L > 2$ is straightforward when restricting the analysis to symmetric equilibria.

The proof proceeds in seven steps. First, we show that the standard Bertrand logic applies, so that in equilibrium firms earn zero profits, and consumers are indifferent between both offers. Second, we show that equilibrium payments made by consumers are equally spread across periods. Third, we argue that both firms charge the same regular payments, which also implies that both offer the same overall bonus. Fourth, we show that firms offer at most one bonus payment, which in turn implies that both firms offer essentially the same contract. Fifth, we argue that firms either offer the maximal bonus or no bonus. Sixth, we show that firms offer a bonus in equilibrium. Seventh, we prove that a unique equilibrium exists.

1. STEP: We show that *firms earn zero profits in any equilibrium.* For the sake of contradiction, suppose firm $k \in \{1, 2\}$ earns strictly positive profits in equilibrium, which implies

$$\tilde{v} + \sum_{i=1}^{M} g(\Delta_j^b) b_j^k \geq \sum_{i=1}^{N} g(\Delta_i^p) p_i^k, \text{ and } \sum_{i=1}^{M} g(\Delta_j^b)(b_j^k - b_j^{-k}) \geq \sum_{i=1}^{N} g(\Delta_i^p)(p_i^k - p_i^{-k}), \text{ and } \sum_{i=1}^{N} p_i^k > \sum_{j=1}^{M} b_j^k.$$

Hence, we have $p_i^k > 0$ for at least one $i \in \{1, \ldots, N\}$. Without loss of generality, let $p_1^k > 0$ and $\sum_{i=1}^N p_i^k - \sum_{j=1}^M \left[b_j^k + \mathbb{1}[b_j^k > 0] \cdot \kappa\right] \geq \sum_{i=1}^N p_i^{-k} - \sum_{j=1}^M \left[b_j^{-k} + \mathbb{1}_{\mathbb{R}_{>0}}(b_j^k) \cdot \kappa\right]$. This immediately implies that firm $-k$ earns at most

$$(1 - D_k) \cdot \left(\sum_{i=1}^{N} p_i^k - \sum_{j=1}^{M} \left[b_j^k + \mathbb{1}_{\mathbb{R}_{>0}}(b_j^k) \cdot \kappa\right] \right) \tag{10}$$

for some $D_k \leq 1$. By deviating to another contract $\boldsymbol{c}^{-k} = (b_1^k, \dots, b_M^k, p_1^k - \epsilon, \dots, p_N^k)$ for some $\epsilon > 0$, firm $-k$ can earn $\sum_{i=1}^N p_i^k - \sum_{j=1}^M \left[b_j^k + \mathbb{1}_{\mathbb{R}_{>0}}(b_j^k) \cdot \kappa\right] - \epsilon$, which exceeds (10) for ϵ sufficiently small. Hence, firm $-k$ has an incentive to deviate; a contradiction. As a consequence, firms earn zero profits in equilibrium. Finally, it is straightforward to see that, in equilibrium, consumers are indifferent between both firms' offers. Otherwise, the firm that serves the market could slightly adjust its contract and earn strictly positive profits.

2. STEP: We show that *all payments to be made by consumers are of the same size, that is,* $p_i^k = p^k$ *for any* $i \in \{1, \dots, N\}$. For the sake of contradiction, suppose that there exist $i, j \in \{1, \dots, N\}$ such that firm k offers a contract with $p_i^k \neq p_j^k$ in equilibrium. In this case, maximal payment $p_{\max} := \max\{p_1^k, \dots, p_N^k\}$ strictly exceeds minimal payment $p_{\min} := \min\{p_1^k, \dots, p_N^k\}$. Without loss of generality, let $p_{\max} = p_1^k$ and $p_{\min} = p_2^k$. As firms earn zero profits in equilibrium, firm $-k$ could profitably deviate to a contract $\tilde{\boldsymbol{c}}^{-k} = (b_1^k, \dots, b_M^k, p_1^k - \epsilon, p_2^k + \epsilon + \epsilon', p_3^k, \dots, p_N^k)$ for some $\epsilon, \epsilon' > 0$ such that $p_1^k > p_2^k + \epsilon + \epsilon'$. Obviously, all consumers choose contract $\tilde{\boldsymbol{c}}^{-k}$ if

$$g(p_1^k)[p_1^k - \epsilon] + g(p_2^k + \epsilon + \epsilon')[p_2^k + \epsilon + \epsilon'] + \sum_{l=3}^N g(\Delta_l^p) p_l^k < \sum_{l=1}^N g(\Delta_l^p) p_l^k,$$

or, equivalently,

$$g(p_1^k) p_1^k - g(p_1^k)[p_1^k - \epsilon] > g(p_2^k + \epsilon + \epsilon')[p_2^k + \epsilon + \epsilon'] - g(p_2^k + \epsilon + \epsilon') p_2^k.$$

Rearranging this inequality yields

$$g(p_1^k)\epsilon > g(p_2^k + \epsilon + \epsilon')[\epsilon + \epsilon'],$$

which is satisfied for ϵ' sufficiently small. Hence, firm $-k$ indeed has a profitable deviation; a contradiction. As a consequence, in equilibrium, we must have $p_i^k = p_j^k$ for any two payments $i, j \in \{1, \dots, N\}$, and any firm $k \in \{1, 2\}$.

3. STEP: We show that *both firms offer the same regular payments, that is,* $p_i^k = p$ *for any* $k \in \{1, 2\}$ *and any* $i \in \{1, \dots, N\}$. For the sake of contradiction, let $p_i^k = p^k > p^{-k} = p_i^{-k}$. Since consumers are indifferent between both contracts, we conclude that $\sum_{j=1}^M b_j^k > \sum_{j=1}^M b_j^{-k}$. Hence, at least one bonus payment of firm k exceeds the corresponding bonus payment of firm $-k$. Without loss of generality, let $b_1^k > b_1^{-k}$.

In a first step, we argue that firm k offers a single bonus payment. For the sake of contradiction, suppose further that firm k offers at least two bonus payments in equilibrium. Then, firm k could profitably deviate to a contract $\tilde{\boldsymbol{c}}^k = (\sum_{j=1}^M b_j^k, 0, \ldots, 0, p^k + \epsilon, p^k, \ldots, p^k)$ for some $\epsilon > 0$ since all consumers choose the contract $\tilde{\boldsymbol{c}}^k$ if

$$\begin{aligned}
&-g(p^k)(N-1)p^k - g(p^k+\epsilon)[p^k+\epsilon] + g\left(\sum_{j=1}^M b_j^k\right)\left(\sum_{j=1}^M b_j^k\right) \\
&\quad = -g(p^k)Np^k + \sum_{j=1}^M g(\max\{b_j^k, b_j^{-k}\})b_j^k \\
&\qquad - p^k[g(p^k+\epsilon) - g(p^k)] - g(p^k+\epsilon)\epsilon + \sum_{j=1}^M \left[g\left(\sum_{j=1}^M b_j^k\right) - g(\max\{b_j^k, b_j^{-k}\})\right] b_j^k \\
&\quad > -g(p^k)Np^{-k} + \sum_{j=1}^M g(\max\{b_j^k, b_j^{-k}\})b_j^{-k} \\
&\qquad - p^{-k}[g(p^k+\epsilon) - g(p^k)] + \left[g\left(\sum_{j=1}^M b_j^k\right) - g(\max\{b_1^k, b_1^{-k}\})\right] b_1^{-k} + \sum_{j=2}^M \left[g(b_j^{-k}) - g(\max\{b_j^k, b_j^{-k}\})\right] b_j^{-k} \\
&\quad = -g(p^k)(N-1)p^{-k} - g(p^k+\epsilon)p^{-k} + g\left(\sum_{j=1}^M b_j^k\right) b_1^{-k} + \sum_{j=2}^M g(b_j^{-k})b_j^{-k}.
\end{aligned}$$

As consumers must be indifferent between both contracts in equilibrium, we have

$$-g(p^k)Np^k + \sum_{j=1}^M g(\max\{b_j^k, b_j^{-k}\})b_j^k = -g(p^k)Np^{-k} + \sum_{j=1}^M g(\max\{b_j^k, b_j^{-k}\})b_j^{-k},$$

so that the above inequality holds if and only if

$$\begin{aligned}
&\overbrace{g(p^k+\epsilon)\epsilon}^{\to 0 \text{ as } \epsilon\to 0} + [p^k - p^{-k}]\overbrace{[g(p^k+\epsilon) - g(p^k)]}^{\to 0 \text{ as } \epsilon\to 0} < \overbrace{\left[g\left(\sum_{j=1}^M b_j^k\right) - g(\max\{b_1^k, b_1^{-k}\})\right]}^{>0 \text{ by A.2}} \overbrace{[b_1^k - b_1^{-k}]}^{>0} \\
&\qquad + \underbrace{\sum_{j=2}^M \left[g\left(\sum_{j=1}^M b_j^k\right) - g(\max\{b_j^k, b_j^{-k}\})\right] b_j^k}_{>0 \text{ by our assumption towards a contradiction}} - \sum_{j=2}^M \underbrace{\left[g(b_j^{-k}) - g(\max\{b_j^k, b_j^{-k}\})\right]}_{\leq 0 \text{ by A.2}} b_j^{-k};
\end{aligned}$$

that is, if and only if ϵ is sufficiently small; a contradiction. Thus, given our initial assumption that k charges higher regular payments than $-k$, firm k must offer a single bonus payment. Thus, from now on, let $b_j^k = 0$ for any $j \neq 1$, and notice that $b_1^k > p^k$, as otherwise firm k would earn positive profits

In a second step, we argue that—given that firm k offers a single bonus payment and higher regular payments—firm $-k$ could profitably deviate to a contract

$$\tilde{\boldsymbol{c}}^{-k} = (b_1^{-k} + \epsilon, b_2^{-k}, \ldots, b_M^{-k}, p^{-k} + \epsilon + \epsilon', p^{-k}, \ldots, p^{-k})$$

for some $\epsilon, \epsilon' > 0$ such that $b_1^k > b_1^{-k} + \epsilon$ and $p_1^{-k} + \epsilon + \epsilon' < p_1^k$ since all consumers choose $\tilde{\boldsymbol{c}}^{-k}$ if

$$-g(p^k)[Np^{-k} + \epsilon + \epsilon'] + g(b_1^k)[b_1^{-k} + \epsilon] + \sum_{j=2}^{M} g(b_j^{-k})b_j^{-k} > -g(p^k)Np^k + g(b_1^k)b_1^k$$

or, equivalently,

$$-g(p^k)[\epsilon + \epsilon'] + g(b_1^k)\epsilon > \underbrace{g(p^k)N[p^{-k} - p^k] + g(b_1^k)[b_1^k - b_1^{-k}] - \sum_{j=2}^{M} g(b_j^{-k})b_j^{-k}}_{=0 \text{ as consumers must be indifferent between contracts}}.$$

This inequality is satisfied for ϵ' sufficiently small since $g(b_1^k) > g(p^k)$; a contradiction. As a consequence, in equilibrium, both firms offer the same regular payments. This further implies that $\sum_{j=1}^{M} b_j^k = \sum_{j=1}^{M} b_j^{-k}$, as otherwise at least one firm would earn positive profits; that is, either both firms offer a bonus contract or none does so.

4. STEP: We show that *firms offer at most one bonus payment in equilibrium.* For the sake of contradiction, suppose that firm k offers at least two bonus payments. By STEP 3, we have $\sum_{j=1}^{M} b_j^k = \sum_{j=1}^{M} b_j^{-k}$, and therefore $\sum_{j=1}^{M} b_j^{-k} > b_j^k$ for any $j \in \{1, \ldots, N\}$. Denote the payment to be made by consumers in each period by p, which is the same across periods by STEP 2 and the same across firms by STEP 3. Then, firm $-k$ could profitably deviate to $\tilde{\boldsymbol{c}}^{-k} = (\sum_{j=1}^{M} b_j^{-k}, 0, \ldots, 0, p + \epsilon, p, \ldots, p)$ for some $\epsilon > 0$ since all consumers choose $\tilde{\boldsymbol{c}}^{-k}$ if

$$\begin{aligned} -g(p)(N-1)p - g(p+\epsilon)[p+\epsilon] + g\left(\sum_{j=1}^{M} b_j^{-k}\right)\left(\sum_{j=1}^{M} b_j^{-k}\right) \\ > -g(p)(N-1)p - g(p+\epsilon)p + g\left(\sum_{j=1}^{M} b_j^{-k}\right) b_1^k + \sum_{j=2}^{M} g(b_j^k)b_j^k, \end{aligned}$$

or, equivalently,

$$g(p+\epsilon)\epsilon < \overbrace{g\left(\sum_{j=1}^{M} b_j^{-k}\right)\left(\sum_{j=1}^{M} b_j^{-k}\right) - \underbrace{\left[g\left(\sum_{j=1}^{M} b_j^{-k}\right) b_1^k + \sum_{j=2}^{M} g(b_j^k)b_j^k\right]}_{<\sum_{j=1}^{M} g\left(\sum_{j=1}^{M} b_j^{-k}\right)b_j^k \text{ by A.2}}}^{>0},$$

which holds if and only if ϵ is sufficiently small; a contradiction. Thus, firms offer at most one bonus payment in equilibrium.

5. STEP: We show that *firms either offer the maximal bonus or no bonus at all.* We already know that each firm offers at most one bonus and that bonus firms offer the same overall

bonus payment. Without loss of generality, we can assume that both firms use the same bonus attribute; that is, we can solve the game as if there is only one bonus attribute, say, $b_1^k = b_1$. Again, denote the payment to be made by consumers in each period by p.

For the sake of contradiction, suppose that $0 < b_1 < \bar{b}$ in equilibrium. Then, firm k could profitably deviate to a contract $\tilde{\boldsymbol{c}}^k = (b_1 + \epsilon, 0, \ldots, 0, p + \frac{\epsilon+\epsilon'}{N}, \ldots, p + \frac{\epsilon+\epsilon'}{N})$ for some $\epsilon, \epsilon' > 0$ such that $\bar{b} \geq b_1 + \epsilon > p + \frac{\epsilon+\epsilon'}{N}$ since all consumers choose the contract $\tilde{\boldsymbol{c}}^k$ if

$$-g\left(p + \frac{\epsilon+\epsilon'}{N}\right) N \left[p + \frac{\epsilon+\epsilon'}{N}\right] + g(b_1+\epsilon)[b_1+\epsilon] > -g\left(p + \frac{\epsilon+\epsilon'}{N}\right) Np + g(b_1+\epsilon)b_1,$$

or, equivalently,

$$g(b_1+\epsilon)\epsilon > g\left(p + \frac{\epsilon+\epsilon'}{N}\right)[\epsilon+\epsilon'].$$

This inequality is satisfied for ϵ' sufficiently small since $g(b_1+\epsilon) > g\left(p + \frac{\epsilon+\epsilon'}{N}\right)$. As a consequence, firms either pay the maximal bonus or no bonus at all.

6. STEP: Notice that there are only two equilibrium candidates left (again we assume that both firms use the same bonus attribute, which is in fact without loss): either both firms offer $\boldsymbol{c}^{no} = (0, \ldots, 0)$ or both firms offer $\boldsymbol{c}^{bon} = \left(\bar{b}, 0, \ldots, 0, \frac{\kappa+\bar{b}}{N}, \ldots, \frac{\kappa+\bar{b}}{N}\right)$. We show that *both firms offering $\boldsymbol{c}^{no}$ cannot be an equilibrium.*

For the sake of contradiction, suppose that both firms offer the contract $\boldsymbol{c}^{no}$ in equilibrium. Now, firm k could profitably deviate to a contract $\tilde{\boldsymbol{c}}^k = \left(\bar{b}, 0, \ldots, 0, \frac{\kappa+\bar{b}}{N} + \epsilon, \frac{\kappa+\bar{b}}{N}, \ldots, \frac{\kappa+\bar{b}}{N}\right)$ for some $\epsilon > 0$ since all consumers choose $\tilde{\boldsymbol{c}}^k$ if

$$\underbrace{-g\left(\frac{\kappa+\bar{b}}{N}\right)(N-1)\left(\frac{\kappa+\bar{b}}{N}\right) - g\left(\frac{\kappa+\bar{b}}{N}+\epsilon\right)\left(\frac{\kappa+\bar{b}}{N}+\epsilon\right)}_{\to -g\left(\frac{\kappa+\bar{b}}{N}\right)[\kappa+\bar{b}] \text{ as } \epsilon\to 0} + g(\bar{b})\bar{b} > 0,$$

which holds for sufficiently small ϵ by Eq. (1). Hence, both firms offering $\boldsymbol{c}^{no}$ is not an equilibrium, which was to be proven.

7. STEP: It remains to be proven that *both firms offering contract $\boldsymbol{c}^{bon}$ is indeed an equilibrium.* We show that firm k has no incentive to deviate. In order to attract consumers, firm k has to reduce some payment p_i^k for $i \in \{1, \ldots, N\}$ by an amount $\epsilon > 0$, as increasing the bonus payment is not feasible. In order to benefit from this deviation, it has to increase some other payments p_j^k, $j \neq i$, to be made by consumers, or decrease the bonus payment b_1^k by an overall amount $\epsilon' > \epsilon$. As $g(\cdot)$ is increasing by Assumption 2, the most effective way of increasing

payments is to equally spread ϵ' over all payments to be made by consumers, namely p_j^k for $j \neq i$. Then, the price cut ϵ is weighted by $g\left(\frac{\kappa+\bar{b}}{N}\right)$, while each price increase $\frac{\epsilon'}{N-1}$ is weighted by $g\left(\frac{\kappa+\bar{b}}{N} + \frac{\epsilon'}{N-1}\right)$. Thus, this deviation attracts consumers if and only if

$$g\left(\frac{\kappa+\bar{b}}{N}\right)\epsilon > g\left(\frac{\kappa+\bar{b}}{N} + \frac{\epsilon'}{N-1}\right)\epsilon',$$

which can only be satisfied for $\epsilon > \epsilon'$; a contradiction. Hence, firm k has no incentive to deviate, so that both firms offering the contract $\boldsymbol{c}^{bon}$ is an equilibrium. Since this was the last remaining equilibrium candidate, the equilibrium is unique.

Finally, consider the case of $L > 2$ firms. It is straightforward to show that in the essentially unique symmetric equilibrium all firms offer the contract $\boldsymbol{c}^{bon}$. This completes the proof. □

Appendix B: Endogenous Attribute Space

B.1: Model

We extend our baseline model from Section 2 in the following two ways: suppose first that the number of bonus payments and the number of regular payments are unbounded and second that consumers buying at firm k incur transaction costs, $\tau(N^k)$, depending on the number of non-zero regular payments specified in firm k's contract, which we denote as N^k.

We assume that consumers' transaction costs are strictly increasing and convex in the number of non-zero regular payments. For technical reasons and without loss of generality, we treat τ as a twice continuously differentiable function from $\mathbb{R}_+$ to $\mathbb{R}_+$ and we further assume that $\tau', \tau'' > 0$. In addition, we impose assumptions on the cost function: (i) $\tau(2) + 2g\left(\frac{\kappa+\bar{b}}{2}\right)\left(\frac{\kappa+\bar{b}}{2}\right) < g(\bar{b})\bar{b}$, and (ii) $\tau'(2) < g'\left(\frac{\kappa+\bar{b}}{2}\right)\left(\frac{\kappa+\bar{b}}{2}\right)^2$, and (iii) $\lim_{N\to\infty} \tau'(N) = \infty$. Notice that Assumptions (i) and (ii) are the natural extensions of Equation (1), which we imposed on the inefficiency arising from paying a bonus in order to allow firms to increase a consumer's focus-weighted utility using a bonus payment and to break even at the same time. Assumption (iii) is a typical Inada-Condition to ensure that a profit-maximizing number of regular payments exists.

The remainder of Appendix B is organized as follows. In Section B.2 we derive a monopolist's optimal contract offer. In Section B.3 we characterize (symmetric) competitive equilibria and compare the competitive outcome to the monopolistic one. Importantly, as long as transaction costs are sufficiently convex, our main result (i.e., Corollary 1) is robust to this extension.

B.2: Monopolistic Market

The monopolist's optimal contract offer is characterized in the following lemma.

Lemma 2. *A contract maximizes the monopolist's profit only if*

(i) the regular payments made by consumers are equally spread across N^{mon} periods, that is, $p_i(N^{mon}) = p(N^{mon})$ for any $i \in \{1, \dots, N^{mon}\}$, whereby $N^{mon} \in \{\lfloor N^* \rfloor, \lceil N^* \rceil\}$ and N^* is the unique solution to

$$g'(p(N))p(N)^2 = \tau'(N),$$

(ii) and, if bonus payment(s) are made, the maximal bonus is paid using a single payment.

Proof. In order to prove the statement, we can make use of the insights derived in Lemma 1, where we have characterized the optimal contract offer for a fixed number of regular and bonus payments, respectively. Indeed, the second part immediately follows from Lemma 1 since, even if the number of potential bonus payments is fixed, the monopolist will not want to pay more than one bonus. Thus, it remains to be shown that also (i) holds.

Remember that we have seen in the proof of Lemma 1 that regular payments have to be of equal size and that there is either a single bonus payment that is maximal or none bonus at all. In addition, we know that for a given number of regular payments, $N \in \mathbb{N}$, it has to hold that

$$Ng(p(N))p(N) = \tilde{V} - \tau(N), \tag{11}$$

where $\tilde{V} = \tilde{v} + g(\bar{b})\bar{b}$ if a bonus is paid and $\tilde{V} = \tilde{v}$ otherwise. Notice that consumers are willing to buy at a price of zero only if $N \leq \tau^{-1}(\tilde{V}) =: \overline{N}^{mon}$, where τ^{-1} is the inverse of the transaction cost function, which indeed exists as τ is strictly increasing. This in turn implies that the optimal regular payments are characterized by (11) as long as N lies weakly below $\overline{N}^{mon}$.

Now ignore the integer constraint for a moment and suppose that $N \in (0, \overline{N}^{mon})$ holds. Then, when applying the Implicit Function Theorem to (11), we obtain

$$p'(N) = -\frac{1}{N}\frac{g(p(N))p(N) + \tau'(N)}{g(p(N)) + g'(p(N))p(N)} < 0. \tag{12}$$

Since the size of the bonus payment is independent of N and as both $N = 0$ and $N = \overline{N}^{mon}$ imply zero profit, the monopolist chooses N as to maximize $Np(N)$ subject to $N \in (0, \overline{N}^{mon})$. In addition, as the function $Np(N)$ is continuous in N on the interval $(0, \overline{N}^{mon})$ and also strictly positive by (11), it has at least one local maximum in this interval, so that—ignoring integer constraints—the optimal number of regular payments solves

$$
\begin{aligned}
0 &= p(N) + Np'(N) \\
&= p(N) - \frac{g(p(N))p(N) + \tau'(N)}{g(p(N)) + g'(p(N))p(N)} \\
&= \frac{1}{g(p(N)) + g'(p(N))p(N)} \cdot \Big[g'(p(N))p(N)^2 - \tau'(N) \Big].
\end{aligned} \tag{13}
$$

Here, the second equality follows from (12) and the last equality is a simple re-arrangement. Hence, we conclude that the optimal number of payments has to solve

$$
g'(p(N))p(N)^2 = \tau'(N). \tag{14}
$$

Since Assumption 3 and Eq. (12) imply that the left-hand side of (14) strictly decreases in N and since $\tau'' > 0$ implies that the right-hand side of (14) strictly increases in N, there exists a unique solution to (14), which further implies that $Np(N)$ has a unique local maximum, N^*, on the interval $(0, \overline{N}^{mon})$. Finally, as $Np(N)$ strictly increases (decreases) for any $N < N^*$ ($N > N^*$), the statement follows immediately when taking the integer constraint into account. □

Before we can prove the analogue to Proposition 1, the next lemma derives further properties of the monopolist's optimal contract that will be useful in the proof later on.

Lemma 3. *The monopolist's contract offer delineated in Lemma 2 satisfies:*

(i) $\frac{\partial N^*}{\partial v} > 0$ *and* $\lim_{v \to \infty} \frac{\partial N^*}{\partial v} = 0$.

(ii) There exists some $v' \in \mathbb{R}_+$ *such that for any* $v > v'$ *we have* $\frac{\partial N^{mon}}{\partial v} = 0$.

(iii) There exists some $v'' \in \mathbb{R}_+$ *such that for any* $v > v''$ *we have* $p(N^{mon}) > \bar{b}$.

(iv) There is some $\overline{\tau} \in \mathbb{R}_+$ *so that for any cost function with* $\tau''(\cdot) > \overline{\tau}$ *the monopolist chooses the same number of regular payments irrespective of whether she pays a bonus or not.*

Proof. First, we derive some preliminary results. Subsequently, we directly prove the statements.

PRELIMINARIES: First, when applying the Implicit Function Theorem to (11), we obtain

$$
\frac{\partial}{\partial \tilde{V}} p(N, \tilde{V}) = \frac{1}{N} \frac{1}{g'(p(N, \tilde{V}))p(N, \tilde{V}) + g(p(N, \tilde{V}))}. \tag{15}
$$

Second, when applying the Implicit Function Theorem to (14), we obtain

$$\begin{aligned}
\frac{dN^*}{d\tilde{V}} &= -\frac{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2\frac{\partial p}{\partial \tilde{V}} + 2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})\frac{\partial p}{\partial \tilde{V}}}{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2\frac{\partial p}{\partial N} + 2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})\frac{\partial p}{\partial N} - \tau''(N^*)} \\
&= -\left(\frac{\partial p/\partial \tilde{V}}{\partial p/\partial N}\right)\cdot\left(\frac{1}{1-\frac{\tau''(N^*)}{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2\frac{\partial p}{\partial N}+2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})\frac{\partial p}{\partial N}}}\right) \\
&= \left(\frac{1}{g(p(N^*,\tilde{V})p(N^*,\tilde{V})+\tau'(N^*)}\right)\cdot\left(\frac{1}{1-\frac{1}{\frac{\partial p}{\partial N}}\frac{\tau''(N^*)}{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2+2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})}}\right) \\
&= \frac{1}{g(p(N^*,\tilde{V})p(N^*,\tilde{V})+\tau'(N^*)+N^*\tau''(N^*)\cdot\left(\frac{g(p(N^*,\tilde{V}))+g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})}{g''(p(N^*,\tilde{V}))p(N^*,\tilde{V})^2+2g'(p(N^*,\tilde{V}))p(N^*,\tilde{V})}\right)} \\
&> 0,
\end{aligned}$$

where the second equality is a simple re-arrangement, the third equality follows from inserting (12) and (15), and the last equality follows from inserting (12) once more.

PART (i): Since $\frac{dN^*}{d\tilde{V}} > 0$ and since $\frac{dN^*}{d\tilde{V}} < \frac{1}{\tau'(N^*)}$, we obtain (i) simply from the fact that $\tilde{V}$ increases with v and goes to infinity as v approaches infinity and that $\lim_{N\to\infty}\tau'(N) = \infty$.

PART (ii): Follows immediately from (i).

PART (iii): Follows immediately from (11), when taking the limit of v to infinity and keeping in mind that N^{mon} is constant for sufficiently large values of v by (ii).

PART (iv): Follows immediately from the fact that $N^* \geq 1$ and that $\lim_{\overline{\tau}\to\infty}\frac{dN^*}{d\tilde{V}} = 0$. □

Using the above lemmata, we can fully characterize the monopolist's contract offer. In particular, the following proposition shows that our previous result on the monopolistic outcome still holds if transaction costs are sufficiently convex.

Proposition 3. *The following statements hold true:*

(i) There exist a threshold value $\check{\kappa} > 0$ and, for any $\kappa < \check{\kappa}$, a threshold value $\check{v}_1(\kappa) > 0$ such that the monopolist offers a bonus contract if $\kappa < \check{\kappa}$ and $v < \check{v}_1(\kappa)$.

(ii) For any $\kappa > 0$, there exists a threshold value $\check{v}_2(\kappa) \geq 0$ such that the monopolist does not offer a bonus contract if $v > \check{v}_2(\kappa)$.

(iii) If transaction costs are sufficiently convex (i.e., if $\tau''(N)$ is sufficiently large for any N), then $\check{v}_1(\kappa) = \check{v}_2(\kappa) = \check{v}(\kappa)$ for any $\kappa < \check{\kappa}$ and $\check{v}$ monotonically decreases in κ on $[0, \check{\kappa})$.

Proof. PART (i): Obviously, if $v = 0$, the monopolist can earn positive profits only when offering a bonus contract. As $\tau(2) + g(\kappa/2)(\kappa/2) < g(\bar{b})\bar{b}$ by assumption, the monopolist can indeed earn strictly positive profits using a bonus contract even if $v = 0$. The statement then follows from the fact that the monopolist's profit is continuous in v conditional on offering a certain type of contract (i.e., a bonus contract or a contract without a bonus payment).

PART (ii): Follows immediately from Lemma 3 Part (iii) using basically the same arguments as in the proof of Proposition 1.

PART (iii): By Lemma 3 Part (iv), the monopolist chooses the same number of regular payments irrespective of whether she pays a bonus or not. Given this fact, the proof is analogous to that of Proposition 1. □

B.3: Competitive Market

Next, we analyze the competitive outcome in our extended model with transaction costs.

Proposition 4. *If $L = 2$, an equilibrium exists and any equilibrium has the following properties:*

(i) the market is covered and firms earn zero profits,

(ii) the regular payments made by consumers are equally spread across N_k^{com} periods, that is, $p_i^k(N_k^{com}) = p^k(N_k^{com})$ for $i \in \{1, \ldots, N_k^{com}\}$ and $k \in \{1, 2\}$, whereby $\underline{N} \leq N_k^{com} \leq \lceil N^{**} \rceil$ and N^{**} is the unique solution to

$$g'\left(\frac{\kappa + \bar{b}}{N}\right)\left(\frac{\kappa + \bar{b}}{N}\right)^2 = \tau'(N)$$

while $\underline{N}$ is the maximum of two and the smallest natural number N that satisfies

$$\frac{\kappa + \bar{b}}{N + 1} \leq \frac{\tau(N + 1) - \tau(N)}{g\left(\frac{\kappa + \bar{b}}{N}\right) - g\left(\frac{\kappa + \bar{b}}{N+1}\right)}, \tag{16}$$

(iii) both firms offer the maximum bonus using a single bonus payment, and

(iv) both firms provide the exact same focus-weighted utility to consumers.

If $L \geq 3$, a symmetric equilibrium exists and any such equilibrium satisfies properties (i) – (iv). In addition, for any $L \geq 2$, there exists a symmetric equilibrium with $N_k^{com} \in \{\lfloor N^{**} \rfloor, \lceil N^{**} \rceil\}$.

Proof. We prove the statement for $L = 2$, while the proof for $L \geq 3$ is a straightforward extension. Again, we can make use from the insights derived in the main text, namely, Proposition 2. For instance, we already know that firms earn zero profits in equilibrium and that consumers are indifferent between both offers, that is, Part (iv) immediately follows from Proposition 2. In addition, it follows directly from Proposition 2 that firms offer at most one bonus payment. Hence, without loss of generality, let $M = 1$ in the following.

The remainder of the proof proceeds in four steps. In a first step, we show that in any equilibrium $N_k^{com} \geq 2$, which in turn implies that firms offer bonus contracts. In a second step, we prove that in any equilibrium $N_k^{com} \leq \lceil N^{**} \rceil$. In a third step, we show that a symmetric equilibrium with $N^{com} \in \{\lfloor N^{**} \rfloor, \lceil N^{**} \rceil\}$ exists. In a fourth step, we show that an equilibrium with $N^k \in \{2, \ldots, \lfloor N^{**} \rfloor\}$ exists if and only if (16) holds at $N = N^k$ and that (16) is more likely to be fulfilled for larger values of N.

1. STEP: By Proposition 2, we know that for $M = 1$ and a fixed number of regular payments $N \geq 2$, there exists a unique equilibrium in which both firms offer the contract

$$\boldsymbol{c}^{bon}(M = 1, N) = \left(\bar{b}, \frac{\kappa + \bar{b}}{N}, \ldots, \frac{\kappa + \bar{b}}{N}\right).$$

Moreover, if firms choose at most one non-zero regular payment, they cannot profitably offer a bonus. But then the only other equilibrium candidate is setting all regular payments to zero.

For the sake of contradiction, suppose that firms do not offer a bonus payment in equilibrium, but set all regular payments to zero. Then, firm k could profitably deviate to a contract

$$\tilde{\boldsymbol{c}}^k(M = 1, N^k = 2) = \left(\bar{b}, \frac{\kappa + \bar{b} + \epsilon}{2}, \frac{\kappa + \bar{b} + \epsilon}{2}\right)$$

for some $\epsilon > 0$ since all consumers choose $\tilde{\boldsymbol{c}}^k(M = 1, N^k = 2)$ if

$$g(\bar{b})\bar{b} - 2g\left(\frac{\kappa + \bar{b} + \epsilon}{2}\right)\left(\frac{\kappa + \bar{b} + \epsilon}{2}\right) - \tau(2) > 0,$$

which holds for sufficiently small values ϵ by the assumption that $\tau(2) + 2g\left(\frac{\kappa+\bar{b}}{2}\right)\left(\frac{\kappa+\bar{b}}{2}\right) < g(\bar{b})\bar{b}$; a contradiction. Hence, we have $N_k^{com} \geq 2$ in any equilibrium.

2. STEP: For the sake of contradiction, suppose that $N_k^{com} > \lceil N^{**} \rceil$ holds in equilibrium. Then, firm k could profitably deviate to a contract

$$\tilde{\boldsymbol{c}}^k(M = 1, N^k) = \left(\bar{b}, \frac{\kappa + \bar{b} + \epsilon}{N^k}, \ldots, \frac{\kappa + \bar{b} + \epsilon}{N^k}\right)$$

for $N^k \in \{\lfloor N^{**} \rfloor, \lceil N^{**} \rceil\}$ and some $\epsilon > 0$ since all consumers choose $\tilde{\boldsymbol{c}}^k(M = 1, N^k)$ if

$$-N^k g\left(\frac{\kappa + \overline{b} + \epsilon}{N^k}\right)\left(\frac{\kappa + \overline{b} + \epsilon}{N^k}\right) - \tau(N^k)$$
$$> -N^k g\left(\frac{\kappa + \overline{b} + \epsilon}{N^k}\right)\left(\frac{\kappa + \overline{b}}{N^{com}_{-k}}\right) - [N^{com}_{-k} - N^k] g\left(\frac{\kappa + \overline{b}}{N^{com}_{-k}}\right)\left(\frac{\kappa + \overline{b}}{N^{com}_{-k}}\right) - \tau(N^{com}_{-k}). \tag{17}$$

Notice that the right-hand side of the above inequality is smaller than

$$-N^{com}_{-k} g\left(\frac{\kappa + \overline{b}}{N^{com}_{-k}}\right)\left(\frac{\kappa + \overline{b}}{N^{com}_{-k}}\right) - \tau(N^{com}_{-k})$$

by Assumption 2 and that

$$-N^k g\left(\frac{\kappa + \overline{b} + \epsilon}{N^k}\right)\left(\frac{\kappa + \overline{b} + \epsilon}{N^k}\right) - \tau(N^k) > -N^{com}_{-k} g\left(\frac{\kappa + \overline{b}}{N^{com}_{-k}}\right)\left(\frac{\kappa + \overline{b}}{N^{com}_{-k}}\right) - \tau(N^{com}_{-k})$$

by our assumption toward a contradiction and the definition of N^{**} as the unique minimizer of $Ng\left(\frac{\kappa+\overline{b}}{N}\right)\left(\frac{\kappa+\overline{b}}{N}\right) + \tau(N)$. Consequently, Inequality (17) holds for sufficiently small values of ϵ; a contradiction. Hence, we conclude that $N^{com}_k \leq \lceil N^{**} \rceil$ in any equilibrium.

3. STEP: Suppose that both firms offer the contract

$$\boldsymbol{c}^{bon}(M = 1, N^{com}) = \left(\overline{b}, \frac{\kappa + \overline{b}}{N^{com}}, \ldots, \frac{\kappa + \overline{b}}{N^{com}}\right),$$

where N^{com} is chosen as to minimize $Ng\left(\frac{\kappa+\overline{b}}{N}\right)\left(\frac{\kappa+\overline{b}}{N}\right) + \tau(N)$; that is, $N^{com} \in \{\lfloor N^{**} \rfloor, \lceil N^{**} \rceil\}$. By STEP 3, no firm has an incentive to increase the number of regular payments, which by the way implies that $\underline{N} \leq \lceil N^{**} \rceil$. In addition, notice that the regular payments of firm k would determine the focus-weights if it decides to decrease the number of regular payments in a way that allows for non-negative profits. But then, by the definition of N^{com}, decreasing the number of regular payments cannot increase focus-weighted utility and yield non-negative profits at the same time. Hence, no firm has an incentive to decrease the number of regular payments and therefore no incentive to deviate, which was to be proven.

4. STEP: Suppose that both firms offer the contract

$$\boldsymbol{c}^{bon}_k(M = 1, N^{com}_k) = \left(\overline{b}, \frac{\kappa + \overline{b}}{N^{com}_k}, \ldots, \frac{\kappa + \overline{b}}{N^{com}_k}\right),$$

where $N^{com}_k \in \{2, \ldots, \lfloor N^{**} \rfloor\}$. First, suppose that both firms choose the same number of regular payments, that is, $N^{com}_1 = N^{com}_2 = N^{com}$. Since $N^{com} \leq \lfloor N^{**} \rfloor$, by same argument as in STEP

3, firms do not have an incentive to decrease the number of regular payments. In addition, firms do not have an incentive to increase the number of payments if and only if

$$N^{com} g\left(\frac{\kappa+\bar{b}}{N^{com}}\right)\left(\frac{\kappa+\bar{b}}{N^{com}}\right) - \tau(N^{com})$$
$$> N^{com} g\left(\frac{\kappa+\bar{b}}{N^{com}}\right)\left(\frac{\kappa+\bar{b}}{N^{com}+1}\right) + g\left(\frac{\kappa+\bar{b}}{N^{com}+1}\right)\left(\frac{\kappa+\bar{b}}{N^{com}+1}\right) - \tau(N^{com}+1),$$

which holds if and only if (16) holds at $N = N^{com}$.

Second, notice that

$$\underbrace{\frac{\partial}{\partial N}\frac{\kappa+\bar{b}}{N+1}\left[g\left(\frac{\kappa+\bar{b}}{N}\right) - g\left(\frac{\kappa+\bar{b}}{N+1}\right)\right]}_{<0 \text{ by A.3}} - \underbrace{\frac{\partial}{\partial N}\left[\tau(N+1)-\tau(N)\right]}_{<0 \text{ as } \tau''>0} < 0,$$

which in turn implies that (16) is more likely to hold for larger values of N.

Third, let $N_1^{com} \neq N_2^{com}$. If $N_k^{com} \leq N_{-k}^{com} - 2$, firm k could profitbaly deviate to the contract

$$\tilde{\boldsymbol{c}}_k^{bon}(M=1, N_k^{com}+1) = \left(\bar{b}, \frac{\kappa+\bar{b}+\epsilon}{N_k^{com}+1}, \ldots, \frac{\kappa+\bar{b}+\epsilon}{N_k^{com}+1}\right)$$

for some sufficiently small $\epsilon > 0$, as $N_k^{com} \leq \lfloor N^{**} \rfloor - 1$ and as firm k's regular payments would fully determine the focus-weights. If $N_k^{com} = N_{-k}^{com} - 1$, then

$$N_k^{com} g\left(\frac{\kappa+\bar{b}}{N_k^{com}}\right)\left(\frac{\kappa+\bar{b}}{N_k^{com}}\right) - \tau(N_k^{com})$$
$$= N_k^{com} g\left(\frac{\kappa+\bar{b}}{N_k^{com}}\right)\left(\frac{\kappa+\bar{b}}{N_k^{com}+1}\right) + g\left(\frac{\kappa+\bar{b}}{N_k^{com}+1}\right)\left(\frac{\kappa+\bar{b}}{N_k^{com}+1}\right) - \tau(N_k^{com}+1),$$

has to hold, as consumers have to be indifferent between both contracts in equilibrium. But then (16) holds at $N = N_k^{com}$ and as it is more likely to hold for larger values of N it also holds at $N = N_{-k}^{com}$. This completes the proof. □

The preceding proposition shows that the competitive equilibrium has the same qualitative properties as before, namely, firms offer a single, maximum bonus payment and the regular payments are of equal size. The only difference compared to our baseline model is that there can exist multiple equilibria that differ in the number of non-zero regular payments. It is easy to see, however, that this multiplicity vanishes for sufficiently convex transaction costs. Consequently, as long as the transaction cost function is sufficiently convex, also our result on the comparison of monopolistic and competitive outcomes remains qualitatively the same.

Corollary 3. *If transaction costs are sufficiently convex (i.e., if $\tau''(N)$ is sufficiently large for any N), there is a unique (symmetric) competitive equilibrium. In this equilibrium all firms choose the same number of regular payments as a monopolist would do. If in addition the consumers' valuation for the product is sufficiently high, the contractual inefficiencies are strictly lower in a monopolistic than in a competitive market.*

Proof. As τ becomes more convex, the right-hand side of (16) becomes smaller for small values of N and larger for large values of N. Hence, $\underline{N}$ becomes larger as τ becomes more convex and eventually only one equilibrium candidate survives. In addition, as τ becomes more convex, both N^* and N^{**} become less sensitive to the level of the regular payments, so that for sufficiently convex transaction costs $N^{mon} = N^{com}$. □