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Steering the Energy Transition in a World of Intermittent Electricity Supply: Optimal Subsidies and Taxes for Renewables Storage

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JEL Code: H23, Q42, Q58, O33
Keywords: Intermittent renewable energies, electricity storage, carbon externality, subsidies, peak-load pricing, optimal control

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* This is a substantially revised and extended version of a working paper that was circulated under the title "Subsidising Renewables but Taxing Storage? Second-Best Policies with Imperfect Carbon Pricing". We gratefully acknowledge helpful comments from participants at the 2019 Annual Conference of the German Economic Association, the 2019 Annual Meeting of the European Economic Association, the 2019 Conference of the European Association of Environmental and Resource Economists, the 2019 Toulouse Conference on the Economics of Energy and Climate, 2018 Conference on Sustainable Resource Use and Economic Dynamics (SURED), the 2018 International Energy Workshop, at the 2018 meeting of the German Association of Environmental and Resource Economists, and at research seminars and workshops at the ETH Zurich and Hamburg University.

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Carsten Helm† *and Mathias Mier*‡

June 22, 2020

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1 Introduction

Dramatic cost reductions and substantial subsidies have created a worldwide boom of renewable energies. In most parts of the world, they now have lower LCOE (levelised cost of electricity) than conventional fossil energies (IRENA, 2019). Therefore, intermittency of supply and low reliability of wind and solar energies are increasingly becoming the main obstacles to transition to an energy system based primarily on renewable energy sources.

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This problem can be alleviated by technological improvements of wind turbines and solar panels, as well as by enhanced power transmission grids that are able to exploit spatial differences in the availability of intermittent renewables. Moreover, it is widely perceived that electricity storage is an essential part of the solution. For example, according to IEA's sustainable development scenarios (SDS) generation capacity of energy storage must increase from 176.5 GW in 2017 to 266 GW in 2030 (see also IRENA, 2017). Such storage will probably be a mix of traditional pumped hydro storage, small- (as in electric vehicles) and large-scale batteries, power-to-gas (mainly hydrogen), and compressed air storage. Since the deployment of pumped hydro storage is limited (Gimeno-Gutiérrez and Lacal-Arántegui, 2015; Sinn, 2017), much of the build-up must come from technologies that are not competitive yet.[1]

We build on the peak-load pricing model to analyse policy interventions in an economy with three types of firms: those that produce with a polluting fossil energy, those that use carbon-neutral but intermittent renewable energies, and those that engage in electricity storage. Starting in a greenfield setting, firms make long-term investments in their respective capacities. Thereafter, firms produce electricity and interact with consumers in a perfectly competitive market. Storage firms have a dual role. They buy electricity—that is, act like consumers—at the low prices that prevail during times of high availability of renewables, but supply electricity at the high prices that obtain during times of low availability. This exploitation of price differences and the increasing role of flexible pricing schemes motivates our assumption of dynamic pricing.[2]

The first policy that we consider is a Pigouvian tax per unit of carbon emissions from fossils. Given our assumptions of competitive markets, dynamic pricing, and lump-sum taxation, this instrument would implement the first-best solution; at least as long as there is no other externality such as R&D spillovers that requires a separate intervention. Since we neglect dynamic aspects of resource extraction and of the climate system, the tax would even be constant for constant marginal damage costs (see Lemoine and Rudik (2017) for dynamic taxing schemes).

However, a Pigouvian tax may not be implementable, for example, due to political economy reasons.[3] Abrell, Rausch, and Streitberger (2019) as well as Ambec and Crampes (2019) have shown that a policy which combines subsidies for the renewable (and storage) technology with a consumption tax can also decentralise the first-best solution. We also find this, but only until renewable capacities have reached a level at which their supply at times of high availability exceeds electricity demand at a price of zero. Such situations of excess capacities, that countries with a high market penetration of renewables are already experiencing occasionally, cannot obtain in the different model set-ups of the above contributions (see below). Moreover, the usual framing of this policy is to emphasize the need to subsidise renewables and to consider the consumption tax as a complementary policy. We show that the underlying economics suggest a reverse perspective. In particular, it is well known that the tax incidence does not depend on who pays the tax. Therefore, a consumption tax at the same level as a Pigouvian tax imposes the same tax burden on fossils. However, the consumption tax must also be paid for electricity from renewables and from the storage. This distorts investment decisions, which requires subsidies for renewables and, somewhat surprisingly, taxes for electricity storage as complementary policy interventions.

In the real world, such transfer payments are nearly always costly so that we find it difficult to

[1] Costs of batteries fell by 22% from 2016 to 2017 (https://www.iea.org/tcep/energy-integration/energystorage/). Schmidt, Hawkes, Gambhir, and Staffell (2017) predict (using experience curves) that battery storage will be competitive in the next 10 (electric vehicle transportation) to 20 years (residential energy storage) (see Kittner, Lill, and Kammen (2017) for similar predictions), although other studies are less optimistic (e.g., Brouwer, van den Broek, Zappa, Turkenburg, and Faaij, 2016).

[2] Dynamic pricing of electricity is still often restricted to larger commercial customers (e.g., Borenstein and Holland, 2005; Joskow and Wolfram, 2012), but according to Helm and Mier (2019) this may be sufficient to create appropriate price signals. Moreover, recent technological advances have dramatically lowered the costs of smart metering technologies, and many regions have set ambitious targets for their deployment (e.g., in the EU Third Energy Package). In addition, several studies have found evidence that households actually do respond to higher electricity prices by reducing usage (e.g., Faruqui and Sergici, 2010; Jessoe and Rapson, 2014).

[3] The literature discusses equity issues (e.g., Polinsky, 1979), lobbying and rent seeking (e.g., Fredriksson, 1997), and distributional implications (see Goulder and Parry (2008) for a discussion and Reguant (2019) for empirical evidence).

argue for such a disguised Pigouvian tax for fossils. Therefore, the focus of our analysis lies on optimal support policies for renewables and storage technologies that are financed by lump-sum taxes. For parsimony, we consider a per unit subsidy for capacity investments rather than the widely used feed-in tariffs, market premiums, and, more recently, tenders.[4] These instruments also imply an implicit subsidy for investments in renewables and storage so that their effects are quite similar.[5] Indeed, we later argue that they are often identical within our specific modelling framework where support policies are financed by lump-sum taxation.

Subsidies for renewables reduce pollution only indirectly. First, more renewables capacities lower the expected electricity price and, thus, incentives to invest in fossils. Second, fossil capacities may remain unused when the availability of intermittent renewables, which have lower variable costs, is high. Storage capacities even out the intermittent supply of renewables and, thereby, raise their competitiveness compared to fossils. This makes subsidising storage seem reasonable, but it turns out that this intuition is wrong because a direct subsidy for renewables is better suited for this.

Storage reduces the electricity price when stored energy is supplied to the market, but it raises the price when the storage is filled. This has countervailing effects on average electricity prices and, therefore, on the incentives to invest in fossil capacities. Due to round-trip efficiency losses during a storage cycle, more electricity has to be taken from the market than can be supplied to it during times of destorage.[6] Therefore, as long as fossils contribute to electricity production during times of storage, the price increasing effect dominates and storage capacities should be taxed to make investments in fossils less attractive. Once the level of renewable capacities is large enough to fill the storage, fossils no longer benefit from the price increasing effect and it becomes optimal to subsidise storage. This subsidy is constant until fossils are no longer used; under the same conditions that lead to a decreasing renewable subsidy. Roughly speaking, as the market share of fossil energies falls, it is optimal to gradually shift from the subsidisation of renewables to subsidising storage.

The analytical model is restricted to the most interesting case where all three technologies are used. However, in a numerical simulation we also consider cost parameters for which fossils are no longer employed in the efficient solution. We find that implementing this as a decentralized solution still requires substantial subsidies in order to keep fossils out of the market. Finally, since renewable energies are still a less mature technology than electricity generation from fossils, the model accounts (in a very stylised way) for economy-wide economies of scale or learning spillovers that reduce unit costs. Internalising this externality requires an additional subsidy, as one would expect. To summarize, the main contribution of the paper is twofold. First, it extends the peak-load pricing model by developing an analytically tractable model that integrates the optimal control problem of storage firms and accounts for rather general intermittency patterns of renewables. Second, we use this model to examine subsidies for storage and renewable technologies as an alternative to Pigouvian taxation to address the carbon externality of fossils.

Accordingly, our paper is related to several literatures. First there is the literature on the economics of intermittent sources of electricity production, of which Ambec and Crampes (2012, p. 321) wrote some years ago that they are "still in their infancy". Since then, the literature has grown substantially, but most contributions rely heavily on numerical simulations (e.g., Després, Mima, Kitous, Criqui, Hadjsaid, and Noirot, 2017) or are empirical (e.g., Abrell, Kosch, and Rausch, 2019; Liski and Vehviläinen, 2020). Ambec and Crampes (2019) share our focus on optimal support policies, and they also consider a storage technology. However, the storage pattern is trivial because the availability of renewables is restricted to be binary, i.e., either 0 or 1. In the scenarios without storage, this and their assumption of non-reactive consumer demand imply that renewables must be fully backed up by fossils. Therefore, if fossils turn out to be more harmful, the optimal policy response may be to

[4] See, e.g., Eichner and Runkel (2014) for a similar approach. In 2016, 83 countries used feed-in tariffs or premiums to promote renewable energy, 58 countries used investment subsidies (capital subsidies, grants, or rebates), and 73 countries used auctions that do not exclude the use of an investment subsidy (IRENA and CPI, 2018). Moreover, most of storage subsidization is constructed as an investment subsidy (ESC, 2015).

[5] Using data from a Belgian program, De Groote and Verboven (2019) find that investment subsidies are more effective than production subsidies like feed-in tariffs because households significantly discount their future benefits.

[6] Round-trip efficiency is usually in the range of 65 to 90 per cent, depending on storage technology (IRENA, 2017).

reduce renewable capacities so that less fossils are needed to back them up.

Abrell et al. (2019), using a simulation and a simpler analytical model, analyse a larger set of renewables support policies but abstract from storage. More importantly, they deviate from the standard peak-load pricing paradigm by not distinguishing between production and capacity choices of fossil energies. This neglects that fossils need prices above their marginal costs to recoup capacity costs, and that this becomes increasingly difficult as capacities are underutilised more often when supply from renewable energies rises. Like Fell and Linn (2013), the authors include two renewable technologies (wind and solar) with different times of binary availability. Our model could be extended relatively straightforwardly to several renewable technologies too, but this would raise the notational complexity. Moreover, the main effects are very intuitive; hence we only provide an informal discussion in the concluding section. Andor and Voss (2016) also consider subsidies for renewables, but their model includes neither fossils nor a storage technology. Finally, Helm and Mier (2019) use a peak-load pricing model with a very general intermittency pattern similar to this paper. However, they do not account for storage and do not examine policy instruments.

Another strand of literature to which this paper relates is the economics of storage. Traditional applications include balancing stochastic production disturbances in agriculture (e.g., Newbery and Stiglitz, 1979; Wright and Williams, 1984) and the combination of thermal capacity with mainly pumped hydro storage (e.g., Crampes and Moreaux, 2001). In a seminal contribution, Gravelle (1976) studies the implications of storage for peak-load pricing with variable demand. He finds that peak consumption increases less than off-peak production increases, due to round-trip losses of storage. This is similar to the effect of storage during times with high and low availability of intermittent renewables in our model. More recently, the focus has shifted toward the role of pumped storage as a natural complement to the intermittency of renewables (e.g., Crampes and Moreaux, 2010; Heal, 2016; Schmalensee, 2019). Similar to us, Steffen and Weber (2013) determine optimal capacity investments, but only for the fossil and storage technologies. They then use a load duration curve to determine the effect of intermittent renewable energies and demonstrate their results numerically by using a case study for Germany. In a related contribution, Steffen and Weber (2016) use optimal control theory to provide a more precise representation of storage dynamics. However, like Horsley and Wrobel (2002), they only consider the problem of an individual storage firm, and they focus on differences between large (unconstrained) and small (constrained) reservoirs. Durmaz (2014) uses discrete time and dynamic programming to determine the optimal storage pattern. However, he does not consider policy instruments and his problem is analytically not fully tractable. Finally, Pommeret and Schubert (2019) also integrate storage into a model with electricity production from renewable and fossil technologies. Their focus is on the optimal allocation of a fixed carbon budget over time, whereas the availability of sufficient storage capacities is taken as exogenously given.

Our paper also contributes to the more general literature on second-best policies and the ranking of policy instruments to incentivize pollution abatement. For a given abatement cost function, pollution taxes and abatement subsidies are usually seen as equivalent in the short run, whereas in the long run subsidies lead to excessive firm entry (e.g., Kohn, 1992). In an extension of this literature that is more similar to our approach, firms can decide whether to incur the fixed cost of a new technology that reduces costs of emission abatement. In this framework, taxes on emissions and subsidies for emission abatement are usually equivalent (e.g., Milliman and Prince, 1989; Requate and Unold, 2003). Although this literature is often motivated by the problem of mitigating CO_2 emissions, specific aspects of energy markets such as the intermittency of renewables and their interaction with storage are usually neglected (see also Fischer, Preonas, and Newell, 2017). We show that accounting for them fundamentally affects the comparison of instruments, as it compromises the efficiency of subsidies, but not that of a Pigouvian tax.

In accordance with our results, there is a broad consensus that no additional subsidies are necessary to tackle an environmental externality if perfect carbon taxation is possible (Golosov, Hassler, Krusell, and Tsyvinski, 2014; Van Der Ploeg and Withagen, 2014). Positive externalities from R&D may require renewables subsidisation (Acemoglu, Aghion, Bursztyn, and Hemous, 2012), but Parry, Pizer, and Fischer (2003) argue that the welfare effect from tackling climate change externalities is greater than the positive effect of R&D subsidisation (see also Goulder and Parry, 2008). Other reasons that have been put forward to motivate renewables subsidies are international tax competition with mobile capital (Eichner and Runkel, 2014), learning externalities and imperfect competition (Reichenbach and Requate, 2012), lumpy entry cost (Antoniou and Strausz, 2017), and imperfections in demand for energy efficiency (Fischer et al., 2017). We account for learning externalities, but focus on the role of intermittency of renewable energies and of storage when addressing the carbon externality.

The remainder of the paper is structured as follows. In Section 2, we introduce the model and the timing of decisions. The game is then solved by backward induction, examining electricity production and storage decisions in Section 3 and capacity choices in Section 4. We then turn to the analysis of policy instruments in Section 5. A numerical simulation in Section 6 illustrates the results and extends the analysis to the situations where storage is not yet efficient, and after fossils have left the market. Section 7 concludes, and an appendix contains the proofs.

2 The Model

Consider an electricity market with three technologies, indexed $j = f, r, s$. Technology f represents a dispatchable fossil technology—like conventional power plants that burn coal or gas. Dispatchability means that electricity production can be freely varied at every point in time up to the limit of its installed capacity (see Joskow, 2011). Since we abstract from uncertainty, we can ignore ramp-up times because conventional power plants are well able to adapt production some time ahead. Technology r is a renewable technology with intermittent supply—like wind turbines, solar PV, or solar thermal plants. The third technology s does not generate electricity, but is able to store it for later usage.

For each of the three technologies there are a large number, n_j, of identical firms that interact on competitive markets. We use lower-case letters to denote choices of firms and upper-case letters for aggregate values. Accordingly, the overall capacity level of firms that produces with technology j is $Q_j = n_j q_j$. To avoid tedious case distinctions, the formal analysis is restricted to the most interesting situation where strictly positive capacities are installed for all three technologies. The numerical simulation in Section 6 extends this to situations where only a subset of technologies is used. Obviously, which of the cases occurs depends on the relative costs of the technologies.

A firm operating with technology j has capacity costs $c_j(Q_j)q_j$, where $c_j(Q_j) > 0$ are the costs of providing one unit of capacity, which are constant from the perspective of an individual firm. If one thinks of $c_j(Q_j)$ as the unit costs of, e.g., solar panels or batteries for electricity storage, this coincides with the standard assumption that firms on competitive markets are too small to affect input prices. However, unit costs depend on the overall capacity level, which allows us to account for different assumptions in the literature regarding to renewables. In particular, $c_j'(Q_j) < 0$ would capture the idea that economy-wide economies of scale or learning reduces unit costs (as in Green and Léautier, 2017).[7] By contrast, if one wants to emphasize that the most efficient sites for wind and solar energies are used first, then $c_j'(Q_j) > 0$ seems more appropriate (as in Abrell et al. (2019) and Ambec and Crampes (2019)). Similarly, for storage, increasing unit costs could result from less suitable pump storage locations and the scarcity of the rare earths that are needed for batteries.[8] Note that due to our assumption that individual firms take unit costs as given, these effects would constitute an externality. For the renewable and storage technology we impose no restriction on the sign of $c_j'(Q_j)$. For the established fossil technology we assume $c_f'(Q_f) = 0$ and denote the unit costs by c_f.

Electricity produced by the fossil and renewable technology is $y_j \geq 0$, $j = f, r$. We assume constant costs, $k_f > 0$, of producing one unit of output with the fossil technology, which are mainly variable costs for coal, oil, or natural gas. Moreover, fossil production leads to an environmental unit cost, $\delta > 0$ that may be (fully or partly) internalised by a carbon tax, τ. Hence a fossil firm's total unit

[7] In a seminal paper, Ghemawat and Spence (1985) argue that unit costs are decreasing in accumulated output of the industry. The simple specification $c_j'(Q_j) < 0$ reflects this idea, but omits the time dimension of accumulating capacity.

[8] For a simple 2-period model that accounts for learning and site scarcity see Lancker and Quaas (2019).

costs are $b_f = k_f + \tau$, which equals social costs if $\tau = \delta$. Variable costs of renewables are negligible and, therefore, ignored.

Turning to the storage technology, $y_s(t) > 0$ is supply of stored electricity, and $y_s(t) < 0$ is electricity taken from the market to fill the storage. Storage leads to conversion losses so that the change in the level of stored electricity, $s(t)$, differs from the quantity of electricity that is fed into or taken out of the storage. Specifically, $\dot{s} := \frac{ds}{dt} = -\eta(y_s)\, y_s(t)$, where the parameter $\eta(y_s) > 0$ represents conversion losses per unit of y_s that differ for storage and destorage. We assume that $\eta(y_s) = \eta_s \in (0, 1]$ during times of storage ($y_s(t) < 0$), and $\eta(y_s) = \eta_d \geq 1$ during destorage ($y_s(t) > 0$). Hence more than one unit of electricity is needed to fill the storage by one unit, and less than one unit taken from the storage arrives at the market. For intermediate periods during which the storage capacity is not used ($y_s(t) = 0$), we assume that no electricity is lost ($\dot{s} = 0$) and $\eta(y_s) = 1$. Finally, we do not model limits or costs of the charging speed, but assume that firms prefer a smoother storage pattern if this does not lead to additional costs.

Intermittency of renewables is represented by an availability factor $\alpha(t) \in [0, 1]$ that is a continuous function of time. Therefore, renewable capacities available at time t are $\alpha(t)\, Q_r$. To keep the analysis tractable, we assume that $\alpha(t)$ can be forecasted perfectly and follows an identical repetitive pattern, described in more detail below.[9] For example, this pattern could represent daily fluctuations of solar power or seasonal fluctuations of wind. Storage serves to balance these fluctuations so that we choose one cycle during which the storage is filled and emptied as a "representative" period. The lifetime of installed capacities is the same for all technologies and consists of m such representative cycles.

The timing is as follows. In Stage 1, the government chooses one or several of the following policy instruments: a tax on fossil production τ, a tax on electricity consumption χ, and subsidies for renewable σ_r and storage capacities σ_s. In Stage 2, competitive firms build their respective fossil, renewable, or storage capacities. In line with the literature on peak-load pricing, we assume a greenfield setting that disregards any capacity that is currently in place. Finally, in Stage 3, firms choose production levels and interact with consumers on a competitive electricity market.

3 Production and Consumption Decisions

3.1 Derivation of Optimality Conditions

The game is solved by backward induction, and we first analyse production and consumption decisions during the lifetime of installed capacities for given subsidies and taxes on fossil production, τ, and electricity consumption, χ. The competitive market equilibrium follows from firms' profit maximisation and consumers' utility maximisation, subject to electricity prices, $p(t)$, that balance supply and demand. The after-tax price is $p(t) - \tau$ for fossil producers and $p(t) + \chi$ for consumers, whereas storage firms pay no taxes as this would lead to double taxation. Due to our assumption that the availability of the renewable technology follows a repetitive pattern, the market outcome will be the same for each representative storage cycle. We denote the initial and terminal time of a storage cycle by t_0 and T, respectively, and ignore discounting within a cycle for parsimony.

First, consider production decisions of fossil and renewable firms. Capacity costs are sunk so that firms' objective is to maximise revenues, $p(t)\, y_j(t)$, over the length of a representative period, minus variable production costs, $b_f y_f(t)$, for fossil firms. Production is restricted by the (available) capacity, $y_f(t) \leq q_f$, $y_r(t) \leq \alpha(t)\, q_r$, and must be non-negative, $y_f(t), y_r(t) \geq 0$. The latter constraint can be ignored because profit maximising renewable and fossil firms will never choose negative quantities in the unconstrained equilibrium. Thus, a fossil firm's profit maximisation problem for a representative cycle in Stage 3 is

[9] Short-term forecasts over a day-night cycle are actually quite accurate (e.g., Iversen, Morales, Møller, and Madsen, 2016), and seasonal wind availability is, at least in the historic average, well known. Moreover, in their empirical study for southeastern Arizona, Gowrisankaran, Reynolds, and Samano (2016) find that social costs of unforecastable intermittency are small in comparison to those of intermittency overall.

$$\pi_f\left(y_f^*\left(q_f\right)\right) := \max_{y_f(t)} \int_{t_0}^{T} \left(p\left(t\right)-\tau-k_f\right) y_f\left(t\right) dt \ \text{ such that} \tag{1}$$

$$y_f\left(t\right) \leq q_f. \tag{2}$$

Using asterisks to characterise values in the competitive market solution, $\pi_f(y_f^*(q_f))$ denotes the value function of this problem, that is, the maximum profits a firm can achieve by optimising production y_f for all $t \in [t_0, T]$, given the fixed capacity parameter q_f. Differentiation of the corresponding Lagrangian yields the first-order and complementary slackness conditions ($\mu_f\left(t\right)$ is the Lagrangian multiplier) for each $t \in [t_0, T]$:

$$p\left(t\right)-\tau-k_f-\mu_f\left(t\right) \leq 0 \qquad \left[=0, \text{ if } y_f^*\left(t\right)>0\right], \tag{3}$$

$$q_f - y_f\left(t\right) \geq 0, \qquad \mu_f\left(t\right) \geq 0, \ \mu_f\left(t\right)\left[q_f - y_f\left(t\right)\right] = 0. \tag{4}$$

Due to the linearity of the objective function, the first-order condition is sufficient and leads to corner solutions. Specifically, if the price exceeds variable production costs, the firm produces at full capacity; i.e., $y_f\left(t\right) = q_f$ if $p\left(t\right) > b_f = k_f + \tau$. By contrast, fossil firms do not produce during times t for which $p\left(t\right) < b_f$, while any $y_f\left(t\right) \in [0, q_f]$ is optimal if $p\left(t\right) = b_f$.

Renewable firms face no variable costs, but their capacity constraint depends on the availability, $\alpha\left(t\right)$, of renewable capacities. Thus, the profit maximisation problem is

$$\pi_r\left(y_r^*\left(q_r\right)\right) := \max_{y_r(t)} \int_{t_0}^{T} p\left(t\right) y_r\left(t\right) dt \ \text{ such that} \tag{5}$$

$$y_r\left(t\right) \leq \alpha\left(t\right) q_r, \tag{6}$$

and for each $t \in [t_0, T]$ the first-order and complementary slackness conditions are

$$p\left(t\right)-\mu_r\left(t\right) = 0, \tag{7}$$

$$\alpha\left(t\right) q_r - y_r \geq 0, \qquad \mu_r\left(t\right) \geq 0, \ \mu_r\left(t\right)\left[\alpha\left(t\right) q_r - y_r\left(t\right)\right] = 0. \tag{8}$$

Here, the binding condition (7) reflects that $y_r^*\left(t\right) > 0$ for any $\alpha\left(t\right), p\left(t\right) > 0$ because, in contrast to fossils, renewables have no variable costs. Moreover, the complementary slackness condition in (8) then implies $y_r^*\left(t\right) = \alpha\left(t\right) q_r$ for all $p\left(t\right) > 0$, i.e., renewables are used at full capacity. However, if the level of available renewable capacities is very large, supply at full capacity may exceed demand from consumers and storage firms, leading to an equilibrium price of zero.

Storage firms control the level of stored electricity $s(t)$ (the state variable) so as to exploit price differences. They buy and store electricity ($y_s\left(t\right) < 0$) during times of low prices, and they destore ($y_s\left(t\right) > 0$) when prices are high. Their optimal control problem is:

$$\pi_s\left(y_s^*\left(q_s\right)\right) := \max_{y_s(t)} \int_{t_0}^{T} p\left(t\right) y_s\left(t\right) dt \text{ such that} \tag{9}$$

$$\dot{s}\left(t\right) = -\eta\left(y_s\right) y_s\left(t\right), \tag{10}$$

$$s\left(t_0\right) = s\left(T\right), \tag{11}$$

$$s\left(t\right) \leq q_s, \tag{12}$$

$$s\left(t\right) \geq 0. \tag{13}$$

The first constraint (10) is the equation of motion for the level of stored energy, $s(t)$. Condition (11) requires that the initial and terminal storage level must be the same, which follows from our assumption of a representative storage cycle. Finally, (12) is the capacity constraint of storage firms, and (13) is the constraint that the level of stored energy must be non-negative. The Hamiltonian is

$$\mathcal{H}_s\left(y_s\left(t\right)\right) = p\left(t\right) y_s\left(t\right) - \lambda\left(t\right) \eta\left(y_s\right) y_s\left(t\right), \tag{14}$$

where $\lambda(t)$ is the adjoint variable of $s(t)$. Conditions (12) and (13) are pure state space constraints that can be accounted for by forming the Lagrangian

$$\mathcal{L}_s\left(t\right) = \mathcal{H}_s\left(y_s\left(t\right)\right) + \varphi_s\left(t\right)\left(q_s - s\left(t\right)\right) + \varphi_d\left(t\right) s\left(t\right), \tag{15}$$

where $\varphi_s(t)$ and $\varphi_d(t)$ are the Lagrangian multipliers for the respective constraints. The Hamiltonian $\mathcal{H}_s$ is linear in $y_s(t)$ and the constraints (12) and (13) are linear in $s(t)$. Therefore, the following conditions are sufficient for optimality, if $\lambda(t)$ is continuous (see Seierstad and Sydsaeter (1987, p. 317-318) and Section 3.2 for the continuity of $\lambda(t)$):

$$\begin{aligned}
\max_{y_s(t)} \mathcal{H}_s\left(y_s\left(t\right)\right) &= \left[p\left(t\right) - \lambda\left(t\right)\eta\left(y_s\right)\right] y_s\left(t\right), & (16)\\
\dot{s}\left(t\right) &= \frac{\partial \mathcal{L}_s(t)}{\partial \lambda\left(t\right)} = -\eta\left(y_s\right) y_s\left(t\right), & (17)\\
\dot{\lambda}\left(t\right) &= -\frac{\partial \mathcal{L}_s(t)}{\partial s\left(t\right)} = \varphi_s\left(t\right) - \varphi_d\left(t\right), & (18)\\
\frac{\partial \mathcal{L}_s(t)}{\partial \varphi_s\left(t\right)} &= q_s - s\left(t\right) \geq 0, \;\; \varphi_s\left(t\right) \geq 0, \; \varphi_s\left(t\right)\left[q_s - s\left(t\right)\right] = 0, & (19)\\
\frac{\partial \mathcal{L}_s(t)}{\partial \varphi_d\left(t\right)} &= s\left(t\right) \geq 0, \;\; \varphi_d\left(t\right) \geq 0, \; \varphi_d\left(t\right) s\left(t\right) = 0, & (20)\\
s\left(t_0\right) &= s\left(T\right). & (21)
\end{aligned}$$

Here, (16) is the optimality condition for the control variable $y_s(t)$, conditions (17) and (18) are the differential equations for the state and adjoint variable, and conditions (19) and (20) account for the pure state space constraints.

Turning to consumers, utility maximisation leads to a demand function $x(t) = x(p(t) + \chi)$, for which we impose no restrictions other than $\frac{\partial x}{\partial p} < 0$. Consumption choices on the competitive electricity market maximise consumer surplus and are restricted by aggregate production. It is straightforward to show that in equilibrium $x(t) = \sum_j Y_j(t)$, that is, demand equals supply. In conclusion, this market clearing condition, the inverse demand function, $p(x(t))$, and the optimality conditions of fossil firms, (3) and (4), renewable firms, (7) and (8), and storage firms, (16) to (21) determine electricity production, demand and the electricity price as functions of the environmental and consumption tax τ, χ, and of installed capacities Q_j, which in turn depend on related subsidies and taxes $\sigma_r, \sigma_s, \tau, \chi$ (see Section 4).

3.2 Determination of Competitive Equilibrium

We can distinguish three outcomes: Storage periods ($y_s(t) < 0$), destorage periods ($y_s(t) > 0$), and intermediate periods with neither storage nor destorage ($y_s(t) = 0$). First, consider the two outcomes with $y_s(t) \neq 0$. If we had $p(t) \neq \lambda(t)\eta(y_s)$, it would be impossible for any $y_s^*(t)$ to maximize (16); hence $p(t) = \lambda(t)\eta(y_s)$ during storage and destorage. Intuitively, the adjoint variable $\lambda(t)$ is usually interpreted as the change in the value function due to a unit increase in the state variable, $s(t)$. Thus, $\lambda(t)$ is the value of stored electricity which, after being weighted by conversion losses, must equal the price of electricity. Moreover, during storage and destorage periods the storage can neither be full nor empty (except at the boundaries), i.e., $s(t) < q_s$ and $s(t) > 0$. Thus, $\varphi_s(t) = \varphi_d(t) = 0$ from the complementary slackness conditions in (19) and (20) so that $\dot{\lambda}(t) = \varphi_s(t) - \varphi_d(t) = 0$ from (18). Finally, by assumption the round-trip efficiency loss parameter is constant at $\eta(y_s) = \eta_s$ during storage and at $\eta(y_s) = \eta_d$ during destorage. Using $p(t) = \lambda(t)\eta(y_s)$ it follows that not only $\lambda(t)$, but also prices are constant during each storage and destorage period. Otherwise, firms would have arbitrage opportunities and use their storage to buy electricity at low prices and sell it at high prices.

To this point, we have not restricted the evolution of $\alpha(t)$ over time. To keep the model tractable, we need to impose more structure as otherwise nearly any sequence of storage, destorage, and intermediate periods is conceivable. Therefore, the remainder of the paper is based on the following additional assumption.

Assumption 1. *For each representative cycle $t \in [t_0, T]$, the availability of renewable energies, $\alpha(t)$, is the same single-peaked function with $\alpha(t_0) = \alpha(T) = \min\{\alpha(t)\}$ and maximum availability $\alpha_{\max}$.*

The assumption $\alpha(t_0) = \alpha(T)$ captures that, by continuity of $\alpha(t)$, the availability at the end of the current and at the beginning of the next representative cycle must be the same. The black solid curve in Figure 1 depicts the availability from a mix of solar PV, wind onshore, and wind offshore in Germany, for a (representative) cycle of 24 hours.[10] This distribution satifies Assumption 1, except that there is a mild wind peak during the destorage period at night. Nevertheless, the subsequent analysis would still hold as long as this peak is not too large (smaller than α_d, see below) so that destorage would remain optimal. The transparent segments to the left and to the right illustrate our simplifying assumption that the availability of renewables is the same in periods prior and subsequent to the depicted one.

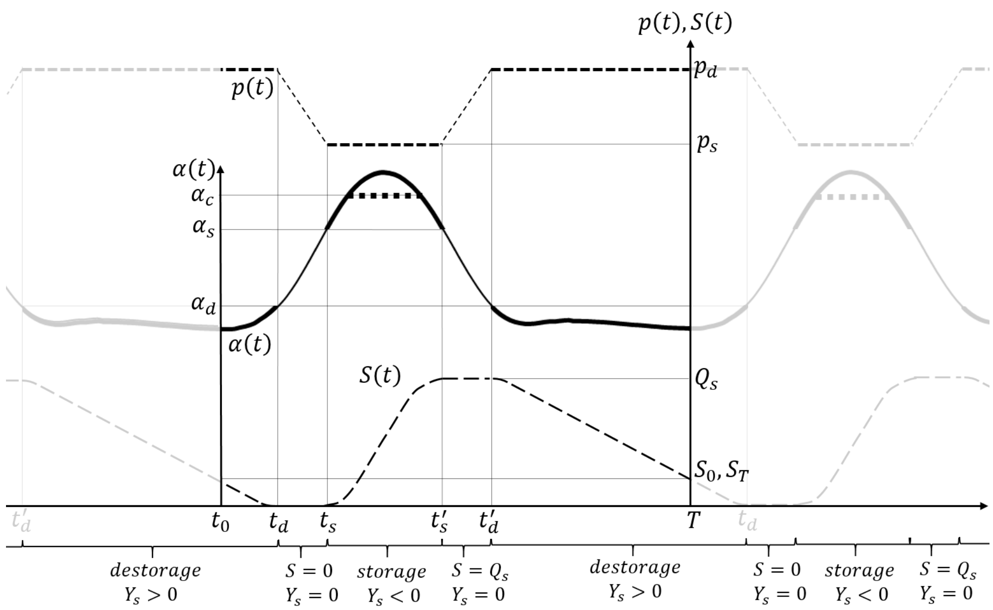

Fig. 1: Availability of renewables and competitive equilibrium

Obviously, firms should destore electricity when the availability of renewables is low (lower bold parts of the curve), and store electricity when the availability is high (upper bold part of the curve), leading to the following sequence of periods: destorage, intermediate, storage, intermediate, destorage, ... Together with our assumption that we consider a representative cycle, a repeated pattern of identical destorage and storage periods obtains. Therefore, the storage should be completely emptied at the end of each destorage period, and completely filled at the end of each storage period. Otherwise, some stored electricity and/or some storage capacity would never be used, which cannot be optimal. This property keeps the analysis tractable and is the reason for Assumption 1. In the remainder of

[10] We calculate $\alpha(t)$ by aggregating production of solar PV, wind onshore, and wind offshore for every quarterly hour of a day from 2016 to the end of 2018, and divide by the installed capacity. The data is downloaded from https://data.open-power-system-data.org/time_series/ on 13 January 2020.

this subsection, we derive an intuitive solution for the competitive equilibrium and show in Appendix A that it satisfies all optimality conditions from Subsection 3.1. For later reference, we state the solution in terms of aggregate values, $Y_j = n_j y_j$, $Q_j = n_j q_j$, and $S = n_s s$.

Dispatchable electricity from storage and fossils is most valuable when the availability of renewables is minimal, i.e., at t_0. Hence there is destorage and fossils are fully used as, otherwise, some capacities would always lie idle. Moreover, we have already shown that the electricity price is constant at $p_d = \lambda(t_0)\eta_d$ during destorage (upper dashed curve in Figure 1). Therefore, fossils—and obviously also renewables due to their lower variable costs—are fully used during the whole destorage interval. In Figure 1, these are the two bold segments $[t_0, t_d]$ and $[t'_d, T]$ that comprise all times with availabilities below $\alpha_d := \alpha(t_d) = \alpha(t'_d)$. The level of destorage balances the fluctuation of renewables so as to keep electricity supply and, thus, the price constant, i.e.,[11]

$$Y_s(t) \quad = \quad (\alpha_d - \alpha(t))\, Q_r \quad \text{for all } t \in [0, t_d]\,. \tag{22}$$

Noting that we consider a representative cycle, the destorage interval $[t'_d, T]$ is identical to the one that precedes t_0. Accordingly, the two destorage periods can be viewed as being connected. This implies that the storage must be full at $t = t'_d$ and run empty at $t = t_d$ (lower dashed curve in Figure 1). Hence, integration of the equation of motion (17) over the destorage periods must satisfy $Q_s = \eta_d \int_{t_0}^{t_d} Y_s(t)\, dt + \eta_d \int_{t'_d}^{T} Y_s(t)\, dt$. Substitution from (22) gives

$$Q_s \quad = \quad \eta_d \int_d (\alpha_d - \alpha(t))\, Q_r dt, \tag{23}$$

where $\int_d dt := \int_{t_0}^{t_d} dt + \int_{t'_d}^{T} dt$ denotes the combined duration of the two destorage periods. Condition (23) implicitly determines the critical availability α_d where destorage ends. Intuitively, the destorage period is shorter when the storage capacity Q_s is low, and when conversion losses, η_d, and the level of renewable capacities that has to be substituted by destorage, $(\alpha_d - \alpha(t))\, Q_r$, are large. The first line in Table 1 summarises production in the destorage period.

Tab. 1: Solution of production stage for fossils, renewables, and storage

<table>
<tr><th>period</th><th colspan="2">availability of renewables</th><th>$Y_r(t)$</th><th>$Y_f(t)$</th><th>$Y_s(t)$</th></tr>
<tr><td>d</td><td colspan="2">$0 \le \alpha(t) \le \alpha_d$</td><td>$\alpha(t)\, Q_r$</td><td>$Q_f$</td><td>$(\alpha_d - \alpha(t))\, Q_r$</td></tr>
<tr><td>case 1</td><td colspan="2">$\alpha_d < \alpha(t) \le \min\{\alpha_1, \alpha_s\}$</td><td>$\alpha(t)\, Q_r$</td><td>$Q_f$</td><td>0</td></tr>
<tr><td>case 2</td><td colspan="2">$\alpha_1 < \alpha(t) \le \min\{\alpha_2, \alpha_s\}$</td><td>$\alpha(t)\, Q_r$</td><td>$x\left(b_f + \chi\right) - \alpha(t)\, Q_r$</td><td>0</td></tr>
<tr><td>case 3</td><td colspan="2">$\alpha_2 < \alpha(t) \le \alpha_s$</td><td>$\alpha(t)\, Q_r$</td><td>0</td><td>0</td></tr>
<tr><td rowspan="3">s</td><td>$\alpha_s < \frac{x(0+\chi)}{Q_r}$</td><td>$\alpha_s < \alpha(t) \le \alpha_{\max}$,</td><td>$\alpha(t)\, Q_r$</td><td>$Y_f(\alpha_s)$</td><td>$(\alpha_s - \alpha(t))\, Q_r$</td></tr>
<tr><td rowspan="2">$\alpha_s = \frac{x(0+\chi)}{Q_r}$</td><td>$\alpha_s < \alpha(t) \le \alpha_c$</td><td>$\alpha(t)\, Q_r$</td><td rowspan="2">0</td><td>$x(0+\chi) - \alpha(t)\, Q_r$</td></tr>
<tr><td>$\alpha_c < \alpha(t) \le \alpha_{\max}$</td><td>$a_c Q_r$</td><td>$x(0+\chi) - a_c Q_r$</td></tr>
</table>

α_d implicitly solves $Q_s = \eta_d \int_d (\alpha_d - \alpha(t))\, Q_r dt$, $\alpha_1 = \frac{x\left(b_f+\chi\right) - Q_f}{Q_r}$, $\alpha_2 = \frac{x\left(b_f+\chi\right)}{Q_r}$,
$\alpha_s = \min\left\{\alpha_s \text{ that solves (25)}, \frac{x(0+\chi)}{Q_r}\right\}$, and α_c implicitly solves (26)

When $\alpha(t)$ starts to exceed α_d, we enter the first intermediate period where neither storage nor destorage occurs so that $y_s(t) = 0$ (similar to Helm and Mier (2019)). By continuity of $\alpha(t)$, fossils and renewables continue to be fully used initially. This is case 1 in Table 1. As renewable supply rises together with $\alpha(t)$, the equilibrium price $p(t)$ falls until it equals the total unit costs of fossils,

[11] More formally, t_d is characterised by $Y_f(t_d) = Q_f$, $Y_r(t_d) = \alpha(t_d)\, Q_r$, and $Y_s(t_d) = 0$. By continuity of $\alpha(t)$ and, thus, of available production capacities, we must have $p(t_d) = p_d$. This implies $x(t_d) = x_d$, where demand x_d is constant during destorage. Solving the market clearing condition for $Y_s(t) = x_d - Y_f(t) - Y_r(t)$ and using $x_d = x(t_d) = Q_f + \alpha(t_d)\, Q_r$ yields (22).

$b_f = k_f + \tau$. Thereafter, case 2 obtains, for which the maximum feasible electricity output from renewables and fossils exceeds demand at the constant after tax price $p(t) + \chi = b_f + \chi$. Hence only renewable capacities are fully used (due to lower variable costs), and fossils serve the remaining demand, $x(b_f + \chi) - \alpha(t) Q_r$. For even larger values of $\alpha(t)$, the equilibrium price $p(t)$ falls below b_f so that only renewables are used, but still at full capacity (case 3).

Let $\alpha_1 < \alpha_2$ denote the availabilities where the respective cases end, and $t_1 < t_2$ the associated times. Hence $\alpha_i = \alpha(t_i)$ for $i = 1, 2$. Depending on the size of storage and renewable capacities, not all cases need obtain. Ceteris paribus, a larger storage capacity takes longer to fill so that storage starts earlier, that is, already during case 1 or 2. Conversely, larger renewable capacities imply that a given storage can be filled faster, hence storage starts later and more cases obtain. In Table 1, this is represented by the minimum operator in the column for availability, where $\alpha_s = \alpha(t_s) = \alpha(t'_s)$ is the availability when the intermediate period ends and storage starts (t'_s denotes the end of the storage period; see Figure 1).

Accordingly, any one of cases 1 to 3 can prevail at the start of the storage period, during which the price remains constant (see above). By continuity of the available production capacities, this price, p_s, must be the same as that at the end of the intermediate period, i.e., $p_s = p(t_s)$. This results in constant demand, $x_s = x(t_s)$, and supply from fossils, $Y_f(t) = Y_f(t_s)$, during storage. Moreover, supply of renewables, $Y_r(t)$, above the level required to satisfy demand, $Y_r(t_s) = \alpha_s Q_r$, is used to fill the storage, i.e.,[12]

$$Y_s(t) \quad = \quad \alpha_s Q_r - Y_r(t) \quad \text{for all } t \in [t_s, t'_s]. \tag{24}$$

To determine $Y_r(t)$, we need to account for the possibility that the level of available renewable capacities exceeds demand at an equilibrium price $p(t) = 0$ plus the quantity required to fill the storage. First, consider the case where no such excess capacities exist. Accordingly, $Y_r(t) = \alpha(t) Q_r$ for all t during storage and substitution into (24) yields $Y_s(t) = (\alpha_s - \alpha(t)) Q_r$.[13] The empty storage is completely filled during the storage period so that integration of the equation of motion (17) yields $Q_s = -\eta_s \int_{t_s}^{t'_s} Y_s(t)\, dt$. After substitution for Y_s, we obtain

$$Q_s \quad = \quad -\eta_s \int_s (\alpha_s - \alpha(t)) Q_r dt, \tag{25}$$

where $\int_s dt := \int_{t_s}^{t'_s} dt$ denotes the duration of the storage period.

Second, consider the case of excess capacities. Obviously, this leads to an equilibrium price $p_s = 0$ and demand $x_s = x(0 + \chi)$ throughout the whole storage period, and also at its boundaries so that $\alpha_s Q_r = x(0 + \chi)$.[14] Given our assumption that firms prefer smoother storage patterns, only the production peaks of renewables where storage would be maximal will be capped. This is illustrated by the dotted bold line in Figure 1. In particular, let α_c denote the critical availability that separates the uncapped from the capped part of the storage period. Then renewable production is $Y_r(t) = \alpha(t) Q_r$ in the uncapped region, i.e., for all $\alpha(t) \in [\alpha_s, \alpha_c]$, and $Y_r(t) = \alpha_c Q_r$ for all $\alpha(t) > \alpha_c$. Substitution of this into (24) yields $Y_s(t)$ for the uncapped and the capped region, which are stated in the last two lines in Table 1. Moreover, substituting these expressions into $Q_s = -\eta_s \int_{t_s}^{t'_s} Y_s(t)\, dt$ gives

$$Q_s \quad = \quad -\eta_s \left[\int_{\alpha_s}^{\alpha_c} (x(0 + \chi) - \alpha Q_r)\, d\alpha + \int_{\alpha_c}^{\alpha_{\max}} (x(0 + \chi) - \alpha_c Q_r)\, d\alpha \right], \tag{26}$$

[12] More formally, solving the market clearing condition for $Y_s(t) = x_s - Y_f(t) - Y_r(t)$ and noting that there is no storage at t_s so that $x_s = Y_r(t_s) + Y_f(t_s)$, we obtain $Y_s(t) = Y_r(t_s) + Y_f(t_s) - Y_f(t) - Y_r(t)$ during storage. Substitution of $Y_f(t) = Y_f(t_s)$ and $Y_r(t_s) = \alpha_s Q_r$ yields (24).

[13] Note that if storage starts in case 2, the storage period is characterised by excess capacities of fossils and a price that equals variable production costs. These idle fossil capacities could be used to reschedule some storage without affecting profits. However, if fossil generators prefer a smooth pattern of production (due to ramping cost and constraints), the suggested pattern is the only optimal one.

[14] Remember that $Y_f(t) = 0$ if $p(t) = 0$ and $Y_s(t_s) = 0$ so that at the boundary of the storage cycle available renewable capacities are equal to demand.

where we integrate over α to simplify notation.

It remains to determine α_s, α_c and which of the two cases obtains. Without excess capacities, the critical availability, α_s, when the storage period starts is implicitly determined by (25). With excess capacities, it follows immediately from $\alpha_s Q_r = x(0+\chi)$. Moreover, $\alpha_s Q_r \leq x(0+\chi)$ as otherwise there would be excess capacities during case 3. Therefore, $\alpha_s = \min\left\{\alpha_s \text{ that solves (25)}, \frac{x(0+\chi)}{Q_r}\right\}$ so that for the case of no excess capacities $\alpha_s < \frac{x(0+\chi)}{Q_r}$. Finally, α_c only exists for the situation with excess capacities and follows implicitly from (26) after substitution of $\alpha_s = \frac{x(0+\chi)}{Q_r}$. This completes the derivation of the results summarised in Table 1.

For the second intermediate period from t'_s to t'_d (see Figure 1) the solution follows from the same equilibrium conditions as for the first one. Thus, for each $\alpha(t)$, the solution is the same as already summarised by cases 1 to 3 in Table 1, but the cases obtain in reverse order because $\alpha(t)$ is now (weakly) decreasing in t. Lemma 1 summarises these results.

Lemma 1. *Equilibrium levels for production and storage are as given in Table 1. Demand and prices follow straightforwardly from the market clearing condition, $x(t) = \sum_j Y_j(t)$, the inverse demand function $p(x(t))$.*

The later analysis of optimal subsidies depends on how the triggered changes in capacities affect production and demand. For the intermediate period, this follows from the expressions in Figure 1, but for the storage and destorage periods it depends in a non-trivial way on effects via the boundaries α_d, α_s that determine the lengths of these periods. Lemma 2 summarises the comparative statics.

Lemma 2. *Marginal changes in capacities Q_f, Q_r, Q_s have the following comparative static effects for the storage and destorage periods.*

(a) Fossil capacities: $\frac{\partial x_d}{\partial Q_f} = 1$ and $\frac{\partial \alpha_s}{\partial Q_f} = \frac{\partial \alpha_d}{\partial Q_f} = 0$. Moreover, $\frac{\partial x_s}{\partial Q_f} = 1$ if storage starts during case 1, whereas for all other cases $\frac{\partial x_s}{\partial Q_f} = 0$.

(b) Renewable and storage capacities for $\alpha_s Q_r < x(0+\chi)$ (no excess capacities of renewables): If case 2 obtains at the beginning of the storage period, then $\frac{\partial x_s}{\partial Q_r} = \frac{\partial x_s}{\partial Q_s} = 0$. Otherwise,

	$\partial\alpha_d/\partial$	$\partial x_d/\partial$	$\partial\alpha_s/\partial$	$\partial x_s/\partial$
Q_r	$-\frac{\int_d(\alpha_d-\alpha(t))dt}{Q_r\int_d dt} < 0$	$\frac{\int_d \alpha(t)dt}{\int_d dt} > 0$	$-\frac{\int_s(\alpha_s-\alpha(t))dt}{Q_r\int_s dt} > 0$	$\frac{\int_s \alpha(t)dt}{\int_s dt} > 0$
Q_s	$\frac{1}{\eta_d Q_r\int_d dt} > 0$	$\frac{1}{\eta_d\int_d dt} > 0$	$-\frac{1}{\eta_s Q_r\int_s dt} < 0$	$-\frac{1}{\eta_s\int_s dt} < 0$

(c) Renewable and storage capacities for $\alpha_s Q_r = x(0+\chi)$ (excess capacities of renewables): For the destorage period, derivatives are as in (b). For the storage period, demand is constant at $x(0+\chi)$ so that $\frac{\partial x_s}{\partial Q_r} = \frac{\partial x_s}{\partial Q_s} = 0$, $\frac{\partial \alpha_s}{\partial Q_s} = 0$, and $\frac{\partial \alpha_s}{\partial Q_r} = -\frac{x(0+\chi)}{Q_r^2} < 0$.

The non-trivial effects that require some intuition are those in the table and concern the most relevant case of no excess capacities. With higher *capacities of renewables*, storage starts later ($\frac{\partial \alpha_s}{\partial Q_r} > 0$) because the storage can be filled faster. The magnitude of this effect is given by the additional production of a marginal renewable capacity unit over the storage cycle, $\int_s (\alpha_s - \alpha(t))\,dt$, weighted by the overall capacity, Q_r, and the length of the storage period, $\int_s dt$. Similarly, the destorage period lasts shorter ($\frac{\partial \alpha_d}{\partial Q_r} < 0$) because a given level of stored electricity, Q_s, has to substitute for a larger amount of renewables over the destorage period. The corresponding marginal changes in demand during destorage and storage, $\frac{\partial x_d}{\partial Q_r}, \frac{\partial x_s}{\partial Q_r}$, are simply average additional renewable production over the destorage and storage period.

An increase in *storage capacities* leads to longer storage and destorage periods ($\frac{\partial \alpha_s}{\partial Q_s} < 0$ and $\frac{\partial \alpha_d}{\partial Q_s} > 0$). The size of this effect is smaller if more intermittent renewables, Q_r, have to be balanced by storage and destorage, and if the respective periods last longer. In addition, with larger conversion losses of storage (small η_s) it takes longer to fill the storage, and with larger conversion losses of destorage (high η_d) the storage is depleted more quickly. Turning to demand, a larger storage requires more electricity to be filled. This raises the price and reduces demand ($\frac{\partial x_s}{\partial Q_s} < 0$), and conversion losses (small $\eta_s \int_s dt$) accentuate this effect. Conversely, during the destorage period the additional electricity feed-in of a larger storage reduces the price and raises demand ($\frac{\partial x_d}{\partial Q_s} > 0$). The effect is smaller when conversion losses are high (large $\eta_d \int_d dt$).

4 Capacity Choices of Competitive Firms

We now turn to Stage 2, in which fossil, renewable, and storage firms choose their respective capacities, thereby anticipating the outcome of production decisions in Stage 3. Remember that the value functions, $\pi_j\left(y_j^*\left(q_j\right)\right)$, $j = f, r, s$ as given by (1), (5), and (9), represent the maximum profits that the respective firms firms can achieve for given capacities, q_j, during one representative cycle, $t \in [t_0, T]$. By construction, production choices in one cycle have no effect on other cycles. Therefore, the net present value of profits over the lifetime of capacities—which is m representative cycles—is simply $\sum_{z=1}^{m} \frac{1}{(1+r)^z} \pi_j\left(y_j^*\left(q_j\right)\right) = \rho \pi_j\left(y_j^*\left(q_j\right)\right)$, where $\rho := \frac{1}{r} - \frac{1}{r(1+r)^m}$ and r is the discount factor. After substitution for the value functions and accounting for capacity costs, $c_j\left(Q_j\right) q_j$, as well as subsidies and taxes ($\boldsymbol{\theta} = (\sigma_r, \sigma_s, \tau, \chi)$ denotes the vector of policy instruments), the profits that the respective firms maximise in Stage 2 are

$$\pi_f\left(q_f^*(\boldsymbol{\theta}), \boldsymbol{\theta}\right) := \max_{q_f} \rho \int_{t_0}^{T} \left(p\left(t\right) - \tau - k_f\right) y_f^*(t, q_f) dt - c_f q_f, \tag{27}$$

$$\pi_r\left(q_r^*(\boldsymbol{\theta}), \boldsymbol{\theta}\right) := \max_{q_r} \rho \int_{t_0}^{T} p\left(t\right) y_r^*(t, q_r) dt - \left(c_r(Q_r) - \sigma_r\right) q_r, \tag{28}$$

$$\pi_s\left(q_s^*(\boldsymbol{\theta}), \boldsymbol{\theta}\right) := \max_{q_s} \rho \int_{t_0}^{T} p\left(t\right) y_s^*(t, q_s) dt - \left(c_s(Q_s) - \sigma_s\right) q_s. \tag{29}$$

When choosing capacity levels, competitive firms take as given the capacity choices of other firms, unit capacity costs, $c_j(Q_j)$, the equilibrium electricity demand and price, $x\left(t\right), p\left(t\right)$, as well as the occurrence of cases and the t where they start (columns 1 and 2 of Table 1). Using this, differentiation of the objective functions in (27) to (29) with respect to the respective capacities yields the following first-order conditions for fossil, renewable, and storage firms ($\pi_{jj} := d\pi_j\left(q_j^*(\boldsymbol{\theta}), \boldsymbol{\theta}\right)/dq_j$ for $j = f, r, s$):

$$\pi_{ff} = \rho \int_{t_0}^{T} \left(p\left(t\right) - \tau - k_f\right) \frac{\partial y_f^*\left(t, q_f\right)}{\partial q_f} dt - c_f = 0, \tag{30}$$

$$\pi_{rr} = \rho \int_{t_0}^{T} p\left(t\right) \frac{\partial y_r^*\left(t, q_r\right)}{\partial q_r} dt - c_r\left(Q_r\right) + \sigma_r = 0, \tag{31}$$

$$\pi_{ss} = \rho \int_{t_0}^{T} p\left(t\right) \frac{\partial y_s^*\left(t, q_s\right)}{\partial q_s} dt - c_s\left(Q_s\right) + \sigma_s = 0, \tag{32}$$

where the derivatives $\partial y_j^*\left(t, q_j\right)/\partial q_j$ follow straightforwardly from Table 1 for the respective cases. Intuitively, firms equalise the net present value of additional production from a marginal capacity unit—the integral terms—and its costs, $c_j\left(Q_j\right)$, thereby accounting for subsidies, and a tax on fossils if implemented. A priori, corner solutions might obtain. However, our focus on situations where optimal capacity levels are positive for all three technologies excludes the case that $\pi_{jj} < 0$. Conversely, $\pi_{jj} > 0$ and, thus, positive marginal (and total) profits would lead to entry until the conditions bind. Indeed, by substituting $c_j\left(Q_j\right), j = f, r, s$ and, thus, the equilibrium capacity levels from (30) to (32) into the profit functions (27) to (29), it is straightforward to see that all firms make zero profits in equilibrium.

5 First- and Second-best Policies

Now consider the regulator's choice of the optimal tax-subsidy system for fossil production, electricity consumption as well as renewable and storage capacities in Stage 1. Denote by $\mathbf{Q} = (Q_f, Q_r, Q_s)$ the vector of overall capacities and remember that $\boldsymbol{\theta} = (\sigma_r, \sigma_s, \tau, \chi)$ is the vector of policy instruments. We use (non-bold) $\theta = \sigma_r, \sigma_s, \tau, \chi$ to refer to an element of this vector. Assuming lump-sum taxation, welfare is given by (we omit the asterisks for optimised values from stages 2 and 3)

$$W := \rho \int_{t_0}^{T} \left(\int_0^{x(t,\chi,\tau,\mathbf{Q})} p(\tilde{x})\, d\tilde{x} \right) dt - \rho \int_{t_0}^{T} (k_f + \delta)\, Y_f(t, \chi, \tau, \mathbf{Q})\, dt - \sum_{j=f,r,s} c_j(Q_j)\, Q_j, \tag{33}$$

where the notation clarifies that equilibrium demand, $x(t, \chi, \tau, \mathbf{Q})$, and production of fossils, $Y_f(t, \chi, \tau, \mathbf{Q})$, at time t depend directly on taxes χ, τ, but only indirectly via capacity levels $\mathbf{Q}$ on subsidies σ_r, σ_s (see Table 1). The first term is the net present value of gross consumer surplus, i.e., the area under the inverse demand function $p(x)$. The second term is the net present value of variable production costs and environmental damage costs. The third term are capacity costs. This takes into account that revenues and costs of taxes and subsidies cancel in the aggregate.

The optimal policy vector maximises welfare W. Differentiation of (33) yields for $\theta = \sigma_r, \sigma_s, \tau, \chi$ (skipping arguments for parsimony):[15]

$$\frac{dW}{d\theta} = \rho \int_{t_0}^{T} \left(\frac{dx(t)}{d\theta} (p(t) + \chi) - (k_f + \delta) \frac{dY_f(t)}{d\theta} \right) dt - \sum_{j=f,r,s} c_j(Q_j) \frac{dQ_j}{d\theta} - \sum_{i=r,s} Q_i \frac{\partial c_i}{\partial Q_i} \frac{dQ_i}{d\theta}. \tag{34}$$

The first term under the integral reflects that the value of the inverse demand function at the equilibrium consumption level equals the equilibrium price plus consumption tax, i.e., $p(x(t, \chi, \tau, \mathbf{Q})) = p(t) + \chi$. Accordingly, the integral terms stipulate that if capacity levels were exogenously given, then policy instruments should be chosen such that the marginal value of electricity consumption should be equal to marginal production cost after accounting for the environmental externality. The remaining two terms extend this to effects via capacities, where the last term reflects our assumption that $\frac{\partial c_f}{\partial Q_f} = 0$ for the established fossil technology. Using $\sum_j Y_j(t) = x(t)$, rearranging terms and substituting from the conditions (30) to (32) for firms' capacity choices, the four first-order conditions $\frac{dW}{d\theta} = 0$ for the optimal policy instruments $\theta = \sigma_r, \sigma_s, \tau, \chi$ are (see Appendix C for the calculations):

$$\sum_{i=r,s} \left(\sigma_i + Q_i \frac{\partial c_i}{\partial Q_i} - \rho\chi \int_{t_0}^{T} \frac{\partial Y_i(t)}{\partial Q_i} dt \right) \frac{dQ_i}{d\theta} - \rho(\tau - \delta + \chi) \int_{t_0}^{T} \frac{dY_f(t)}{d\theta} dt - z = 0, \tag{35}$$

where

$$z = \begin{cases} 0 & \text{if } \alpha_s Q_r < x(0+\chi) \\ \rho\chi \frac{1}{\eta_s} \frac{dQ_s}{d\theta} & \text{if } \alpha_s Q_r = x(0+\chi) \text{ for } \theta = \sigma_r, \sigma_s, \tau \\ \rho\chi \left(\int_s \frac{\partial x(0+\chi)}{\partial \chi} dt + \frac{1}{\eta_s} \frac{dQ_s}{d\chi} \right) & \text{if } \alpha_s Q_r = x(0+\chi) \text{ for } \theta = \chi \end{cases} \tag{36}$$

Accordingly, z is only non-zero when there are excess capacities of renewables—i.e., if $\alpha_s Q_r = x(0+\chi)$—and we explain the expression when we discuss this situation in Section 5.2. We now examine the different options for policy interventions.

5.1 Pigouvian Taxation

First, suppose that the consumption tax is set at $\chi = 0$ so that the related terms in (35) cancel and $z = 0$. Hence the first-order conditions for the optimal policy instruments $\theta = \sigma_r, \sigma_s, \tau$ simplify to

[15] All terms under the integral as well as their derivatives are continuous so that one can apply the Leibniz rule and differentiate under the integral sign (see Sydsaeter, Hammond, Seierstad, and Strom 2005, p. 156).

$$\sum_{i=r,s}\left(\sigma_i + Q_i\frac{\partial c_i}{\partial Q_i}\right)\frac{dQ_i}{d\theta} = \rho\left(\tau-\delta\right)\int_{t_0}^{T}\frac{dY_f\left(t\right)}{d\theta}dt. \tag{37}$$

Intuitively, the effects of subsidies and taxes on capacity, production and consumption choices should correct the environmental and the capacity cost externality. Abstracting from the latter by first considering the case $\frac{\partial c_i}{\partial Q_i}=0$, $i=r,s$, a Pigouvian tax $\tau=\delta$ and $\sigma_r=\sigma_s=0$ would solve (37). Next, suppose that a cost externality obtains ($\frac{\partial c_i}{\partial Q_i}\neq 0$), whereas the individual firms take unit costs $c_i\left(Q_i\right)$ as given. This provides a separate reason to subsidise capacities if it reduces unit costs—i.e., if $\frac{\partial c_i}{\partial Q_i}<0$ due to economy-wide economies of scale or learning—and to tax capacities if $\frac{\partial c_i}{\partial Q_i}>0$—e.g., due to the scarcity of suitable sites for wind and solar energies. It is straightforward to see that the the tax/subsidy scheme, $\tau^*=\delta$, $\chi^*=0$, and $\sigma_i^*=-Q_i^*\frac{\partial c_i}{\partial Q_i}$, $i=r,s$, satisfies the first-order conditions as given in (37).

To see that this instrument mix even implements the first-best solution, note that production and consumption choices on competitive markets as analysed in Section 3.2 were only distorted by the pollution externality. Hence the resulting equilibrium levels in Lemma 1 are obviously first-best if no consumption tax is levied and the environmental externality is internalised by a Pigouvian tax $\tau=\delta$, provided that the underlying capacity levels are first-best. Welfare maximizing capacity levels follow from maximizing welfare W in (33) with respect to capacities for $\chi=0$. Using $\frac{\partial c_f}{\partial Q_f}=0$, we have for $j=f,r,s$:

$$\frac{\partial W}{\partial Q_j}=\rho\int_{t_0}^{T}\left(\frac{\partial x\left(t\right)}{\partial Q_j}p\left(t\right)-\left(k_f+\delta\right)\frac{\partial Y_f\left(t\right)}{\partial Q_j}\right)dt-c_j\left(Q_j\right)-\frac{\partial c_j}{\partial Q_j}Q_j=0. \tag{38}$$

In the proof of Proposition 1, we show that imposing a Pigouvian tax and a capacity subsidy on profit maximizing firms leads to exactly the same first-order conditions and, hence, the same capacity levels. This leads to the following result.

Proposition 1. *The social optimum can be implemented by a Pigouvian tax on fossils, $\tau^*=\delta$, subsidies for renewable and storage capacities that are equal to the cost externality, $\sigma_i^*=-Q_i^*\frac{\partial c_i}{\partial Q_i}$, $i=r,s$, and no consumption tax, $\chi^*=0$.*

Proposition 1 confirms the expectation that Pigouvian taxation also works in a model that accounts for intermittency of renewables and storage. However, this provides no guidance for second-best policies when Pigouvian taxes are not feasible—for example, due to political economy reasons. In the next subsection, we consider the alternative policy option of subsidizing renewable and storage capacities in combination with a consumption tax on electricity.

5.2 Consumption Taxation

Now suppose that—in contrast to the previous Section—an incomplete fossil tax, $\tau<\delta$, is taken as given, but the regulator can implement a consumption tax χ. Remember that the first-order conditions for policy instruments are as stated in (35). First, consider the situation without excess capacities of renewables ($\alpha_s Q_r < x(0+\chi)$). For this case, we have $z=0$, $\frac{\partial Y_r(t)}{\partial Q_r}=\alpha\left(t\right)$, and $\int_{t_0}^{T}\frac{\partial Y_s(t)}{\partial Q_s}dt=\int_d\frac{\partial\alpha_d}{\partial Q_s}Q_r dt+\int_s\frac{\partial\alpha_s}{\partial Q_s}Q_r dt=\frac{1}{\eta_d}-\frac{1}{\eta_s}\leq 0$ (from Table 1 and Lemma 2). Hence the optimal policy instruments $\theta=\sigma_r,\sigma_s,\chi$ must satisfy

$$\left(\sigma_r+Q_r\frac{\partial c_r}{\partial Q_r}-\rho\chi\int_{t_0}^{T}\alpha\left(t\right)dt\right)\frac{dQ_r}{d\theta}+\left(\sigma_s+Q_s\frac{\partial c_s}{\partial Q_s}-\rho\chi\left(\frac{1}{\eta_d}-\frac{1}{\eta_s}\right)\right)\frac{dQ_s}{d\theta}=\rho\left(\tau-\delta+\chi\right)\int_{t_0}^{T}\frac{dY_f\left(t\right)}{d\theta}dt. \tag{39}$$

Denoting the optimal tax/subsidy scheme with a consumption tax by superscript $^{\#}$, it is straightforward to see that $\chi^{\#}=\delta-\tau$, $\sigma_r^{\#}=-Q_r\frac{\partial c_r}{\partial Q_r}+\rho\chi^{\#}\int_{t_0}^{T}\alpha\left(t\right)dt$, and $\sigma_s^{\#}=-Q_s\frac{\partial c_s}{\partial Q_s}+\rho\chi^{\#}\left(\frac{1}{\eta_d}-\frac{1}{\eta_s}\right)$

satisfies all three first-order conditions. Moreover, as shown in the proof of Proposition 2 it implements the first-best solution if capacity levels are first-best.

For the fossil technology this is very intuitive because the combined tax level on electricity from fossils is $\tau + \chi^{\#} = \delta$—i.e., just at the Pigouvian level—and the tax incidence does not depend on who pays the tax. Moreover, from Table 1 supply $Y_j(t)$ and, thus, also demand $x(t) = \sum_{j=f,r,s} Y_j(t)$ are the same under the Pigouvian and under the consumption tax. Therefore, $x\left(p^{\#}(t) + \chi^{\#}\right) = x(p^*(t))$, where $p^*(t)$ and $p^{\#}(t)$ denote the (before tax) equilibrium prices with the Pigouvian or the consumption tax, respectively. It follows that $p^{\#}(t) = p^*(t) - \chi^{\#}$, i.e., equilibrium prices under the consumption tax are lower. This reflects that the consumption tax must also be paid for electricity from renewables and for electricity that goes through the storage. For renewables, the subsidy component $\rho\chi^{\#}\int_{t_0}^{T}\alpha(t)\,dt$ compensates the net present value of losses from the lower equilibrium price. By contrast, for storage firms $p^{\#}(t) < p^*(t)$ implies that they pay and receive a lower price during storage and destorage, respectively. As the level of stored electricity exceeds that of destored electricity due to conversion losses, this is beneficial for storage firms. To compensate this requires a tax, which explains the subsidy component $\rho\chi^{\#}\left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right) < 0$. In particular, $\frac{1}{\eta_s} - \frac{1}{\eta_d}$ represents conversion losses over a storage cycle that determine the net benefit from the lower electricity price.[16]

Next, consider the situation with excess capacities of renewables ($\alpha_s Q_r = x(0+\chi)$). This does not affect the above analysis with one exception. From Table 1, overall supply during storage is now $\sum_j Y_j(t) = x(0+\chi)$ and, thus, lower with a consumption tax $\chi > 0$ than with a Pigouvian tax. Hence, less renewable capacities—which are now in excess supply—are needed to satisfy the lower demand. Therefore, consumption and production decisions would be distorted and the social optimum cannot be attained, even at first-best capacity levels. The proposition summarizes these results.

Proposition 2. *Suppose that we have an incomplete carbon tax $\tau < \delta$.*

(a) If there are no excess capacities of renewables ($\alpha_s Q_r < x(0+\chi)$), then the social optimum can still be implemented by a tax on electricity consumption, $\chi^{\#} = \delta - \tau$, as well as subsidies for renewable and storage capacities, $\sigma_r^{\#} = -Q_r^*\frac{\partial c_r}{\partial Q_r} + \rho(\delta - \tau)\int_{t_0}^{T}\alpha(t)\,dt$ and $\sigma_s^{\#} = -Q_s^*\frac{\partial c_s}{\partial Q_s} + \rho\chi^{\#}\left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right)$, where $\frac{1}{\eta_d} - \frac{1}{\eta_s} < 0$.

(b) With excess capacities of renewables ($\alpha_s Q_r = x(0+\chi)$), no policy mix of consumption taxes and capacity subsidies can attain the social optimum.

Accordingly, for a carbon tax below the Pigouvian level, $\tau < \delta$, a complementary consumption tax $\chi^{\#} = \delta - \tau$ closes the gap and achieves that the carbon externality is fully internalised. However, the consumption tax must also be paid for "clean" electricity from renewables, and for electricity that has gone through a storage cycle with associated conversion losses. This creates distortions that must be corrected by subsidising renewable capacities and—unless the cost externality $Q_s^*\frac{\partial c_s}{\partial Q_s}$ dominates—taxing storage capacities. This is an awkward policy. It results in Pigouvian taxation of fossils, but as a by-product it also leads to unintended taxes for renewables and storage, which in turn must be compensated by another round of subsidies and taxes. Moreover, in the real world these tax/subsidy streams lead to costs that we have ignored in our simple model, and the policy fails for excess capacities of renewables that we occasionally observe already now in some countries. Therefore, it is hard to see any advantage over a simple Pigouvian tax, even if one accounts for political economy issues that impede their implementation.

[16] Remember that $\eta_d \geq 1 \geq \eta_s$. $\frac{1}{\eta_s}$ is the quantity of electricity needed to fill the storage by one unit so that $\frac{1}{\eta_s} - 1$ is electricity lost during storage. Similarly, $\frac{1}{\eta_d}$ is the quantity of electricity that arrives at the market for each unit taken from the storage so that $1 - \frac{1}{\eta_d}$ is electricity lost during destorage. Summing up yields total conversion losses $\frac{1}{\eta_s} - \frac{1}{\eta_d}$.

5.3 Renewable and Storage Subsidies

In this section, we assume that it is not possible to implement a tax that fully internalises the carbon externality, neither directly by a Pigouvian tax nor through the backdoor by a consumption tax. Instead, noting that subsidies for renewable energies and, more recently, also for storage capacities have been the dominating policy instrument in many countries, we examine their optimal levels for a given imperfect carbon tax and no tax on electricity consumption.

For $\chi = 0, \tau < \delta$ given, the first-order conditions (35) for optimal subsidy levels become

$$\left(\sigma_r + Q_r \frac{\partial c_r}{\partial Q_r}\right) \frac{dQ_r}{d\sigma_r} + \left(\sigma_s + Q_s \frac{\partial c_s}{\partial Q_s}\right) \frac{dQ_s}{d\sigma_s} = \rho\left(\tau - \delta\right) \int_{t_0}^{T} \frac{dY_f\left(t, \mathbf{Q}\right)}{d\sigma_i} dt,\ i = r, s, \tag{40}$$

where $\frac{dY_f(t,\mathbf{Q})}{d\sigma_i} = \sum_{j=f,r,s} \frac{\partial Y_f}{\partial Q_j} \frac{dQ_j}{d\sigma_i}$ because subsidies have no direct effect on fossil production, i.e., $\frac{\partial Y_f}{\partial \sigma_i} = 0$. To evaluate this, we need a closer inspection of the effects of subsidies on the non-internalised carbon externality from fossils, $(\tau - \delta) \frac{dY_f}{d\sigma_i}$. These follow from Table 1 and depend on which of the intermediate cases 1 to 3 obtain in equilibrium.

For full usage of fossils during storage ($\alpha_s \leq \alpha_1$), only the intermediate case 1 obtains and fossil always operate at full capacity so that (subscripts to the integral sign denote the periods over which the integration applies)

$$\int_{t_0}^{T} \frac{dY_f\left(t, \mathbf{Q}\right)}{d\sigma_i} dt = \int_{d,1,s} dt \frac{dQ_f}{d\sigma_i},\ i = r, s. \tag{41}$$

For partial usage of fossils during storage ($\alpha_1 < \alpha_s \leq \alpha_2$), fossil capacities are only partly used during case 2 of the intermediate period and during storage because renewables are increasingly substituting them in these periods. Therefore, using Lemma 1, we obtain

$$\begin{aligned} \int_{t_0}^{T} \frac{dY_f\left(t, \mathbf{Q}\right)}{d\sigma_i} dt &= \int_{d,1} dt \frac{dQ_f}{d\sigma_i} - \int_{2} \alpha\left(t\right) dt \frac{dQ_r}{d\sigma_i} - \int_{s} dt \frac{d\left(\alpha_s Q_r\right)}{d\sigma_i} \\ &= \int_{d,1} dt \frac{dQ_f}{d\sigma_i} - \int_{2} \alpha\left(t\right) dt \frac{dQ_r}{d\sigma_i} - \int_{s} \alpha\left(t\right) dt \frac{dQ_r}{d\sigma_i} + \frac{1}{\eta_s} \frac{dQ_s}{d\sigma_i},\ i = r, s, \end{aligned} \tag{42}$$

where the second line follows from Lemma 2.[17]

For no usage of fossils during storage ($\alpha_2 < \alpha_s$), the intermediate period extends to case 3, but fossils do not produce in this case, nor in the storage period that follows them. Therefore, subsidies have no effects on fossil production during these periods and expression (42) simplifies to

$$\int_{t_0}^{T} \frac{dY_f\left(t, \mathbf{Q}\right)}{d\sigma_i} dt = \int_{d,1} dt \frac{dQ_f}{d\sigma_i} - \int_{2} \alpha\left(t\right) dt \frac{dQ_r}{d\sigma_i},\ i = r, s. \tag{43}$$

Intuitively, subsidies for renewables and storage affect fossil capacities only indirectly, via changing the incentives to invest in fossils if the level of renewable and storage capacities changes. Specifically, in Appendix F we show that for $j = r, s$:

$$\frac{dQ_f}{d\sigma_j} = \frac{\partial Q_f}{\partial Q_r} \frac{dQ_r}{d\sigma_j} + \frac{\partial Q_f}{\partial Q_s} \frac{dQ_s}{d\sigma_j}, \tag{44}$$

[17] In particular, $\frac{\partial \alpha_s}{\partial Q_f} = 0$ so that total differentiation of $\alpha_s Q_r$ and multiplication by $\int_s dt$ gives

$$\begin{aligned} d\left(\alpha_s Q_r\right) \int_s dt &= \left[\left(\frac{\partial \alpha_s}{\partial Q_r} Q_r + \alpha_s\right) dQ_r + \frac{\partial \alpha_s}{\partial Q_s} Q_r dQ_s\right] \int_s dt \\ &= \left(-\int_s \left(\alpha_s - \alpha\left(t\right)\right) dt + \alpha_s \int_s dt\right) dQ_r - \frac{1}{\eta_s} dQ_s. \end{aligned}$$

where, using $\mathbf{1}_{\alpha_s \leq \alpha_1}$ to denote the indicator function that takes the value 1 if $\alpha_s \leq \alpha_1$ and 0 otherwise,

$$\frac{\partial Q_f}{\partial Q_r} = -\frac{\int_1 \frac{\partial p(t)}{\partial x(t)} \alpha(t) dt + \frac{\partial p_d}{\partial x_d} \int_d \alpha(t)\, dt + \mathbf{1}_{\alpha_s \leq \alpha_1} \cdot \frac{\partial p_s}{\partial x_s} \int_s \alpha(t)\, dt}{\int_1 \frac{\partial p(t)}{\partial x(t)} dt + \frac{\partial p_d}{\partial x_d} \int_d dt + \mathbf{1}_{\alpha_s \leq \alpha_1} \cdot \frac{\partial p_s}{\partial x_s} \int_s dt}, \tag{45}$$

$$\frac{\partial Q_f}{\partial Q_s} = -\frac{\frac{\partial p_d}{\partial x_d} \frac{1}{\eta_d} - \mathbf{1}_{\alpha_s \leq \alpha_1} \cdot \frac{\partial p_s}{\partial x_s} \frac{1}{\eta_s}}{\int_1 \frac{\partial p(t)}{\partial x(t)} dt + \frac{\partial p_d}{\partial x_d} \int_d dt + \mathbf{1}_{\alpha_s \leq \alpha_1} \cdot \frac{\partial p_s}{\partial x_s} \int_s dt}. \tag{46}$$

The case distinction captures that for $\alpha_s \leq \alpha_1$ storage starts during case 1 so that fossils are always fully used. By contrast, for $\alpha_s > \alpha_1$ fossils make no marginal profits because the equilibrium price equals their variable costs (if $\alpha_1 < \alpha_s \leq \alpha_2$), or they do not produce during storage (if $\alpha_2 < \alpha_s$). Hence their profitability and, thus, capacities are not affected by changes in renewable and storage capacities so that all terms concerning the storage period are dropped.

We can now use these expressions to evaluate the above three cases of full, partial, and no usage of the fossil technology during storage. In particular, substituting the term $\int_{t_0}^{T} \frac{dY_f(t,\mathbf{Q})}{d\theta} dt$ for these respective cases (i.e., (41), (42), and (43)) into the first-order conditions (40) for optimal subsidies, thereby using (44) together with (45) and (46), yields after some transformations (see Appendix G) the following result.

Proposition 3. *For an incomplete carbon and no consumption tax ($\tau < \delta, \chi = 0$), optimal subsidies of renewable and storage capacities for the respective cases of full ($\alpha_s \leq \alpha_1$), partial ($\alpha_s \in (\alpha_1, \alpha_2]$) and no ($\alpha_s > \alpha_2$) usage of the fossil technology during the storage period are*

$$\sigma_r^* = -Q_r(\boldsymbol{\theta}) \frac{\partial c_r}{\partial Q_r} + \rho(\delta - \tau) \begin{cases} \left(\int_{d,1,s} dt \frac{\int_{d,1,s} \frac{\partial p(t)}{\partial x(t)} \alpha(t) dt}{\int_{d,1,s} \frac{\partial p(t)}{\partial x(t)} dt} \right) & \text{if} \quad \alpha_s \leq \alpha_1, \\ \left(\int_{d,1} dt \frac{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} \alpha(t) dt}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt} + \int_{2,s} \alpha(t)\, dt \right) & \text{if} \quad \alpha_s \in (\alpha_1, \alpha_2], \\ \left(\int_{d,1} dt \frac{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} \alpha(t) dt}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt} + \int_{2} \alpha(t)\, dt \right) & \text{if} \quad \alpha_s > \alpha_2, \end{cases}$$

and

$$\sigma_s^* = -Q_s(\boldsymbol{\theta}) \frac{\partial c_s}{\partial Q_s} + \rho(\delta - \tau) \begin{cases} \left(\int_{d,1,s} dt \frac{\frac{\partial p_d}{\partial x_d} \frac{1}{\eta_d} - \frac{\partial p_s}{\partial x_s} \frac{1}{\eta_s}}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt + \frac{\partial p_s}{\partial x_s} \int_s dt} \right) & \text{if} \quad \alpha_s \leq \alpha_1, \\ \left(\int_{d,1} dt \frac{\frac{\partial p_d}{\partial x_d} \frac{1}{\eta_d}}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt} - \frac{1}{\eta_s} \right) & \text{if} \quad \alpha_s \in (\alpha_1, \alpha_2], \\ \left(\int_{d,1} dt \frac{\frac{\partial p_d}{\partial x_d} \frac{1}{\eta_d}}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt} \right) & \text{if} \quad \alpha_s > \alpha_2. \end{cases}$$

All expressions have the same intuitive structure: Optimal subsidies account for the cost externality, $Q_i(\boldsymbol{\theta}) \frac{\partial c_i}{\partial Q_i}$, and for the net present value of non-internalised avoided damages, $\rho(\delta - \tau)$, from the marginal effect of renewable respectively storage capacities on fossil production (the terms after the curly bracket). Obviously, if economies of scale or learning reduce unit costs of renewable and/or storage capacities so that $\frac{\partial c_i}{\partial Q_i} < 0$, then the optimal subsidy rises. The opposite result obtains if $\frac{\partial c_i}{\partial Q_i} > 0$. We now discuss the terms after the curly bracket.

Start with the *renewable subsidy* σ_r^*. From (45), the fractions represent the marginal effect of renewable capacities on fossils, $-\frac{\partial Q_f}{\partial Q_r}$, during the periods where fossils are fully used. Intuitively, this effect and, thus, σ_r is larger when the availability of renewables, $\alpha(t)$, is large. The weighting by $\frac{\partial p(t)}{\partial x(t)}$ captures that this availability is more relevant at times where it reduces the electricity price more strongly because this makes fossils less attractive. The additional term in the second and third line represent the marginal effect of renewable capacities on fossils when the latter are only partly used, i.e., during case 2 and storage if $\alpha_s \in (\alpha_1, \alpha_2]$ and during case 2 if $\alpha_s > \alpha_2$. Due to the constant equilibrium price $p(t) = k_f + \tau$ in these periods, there are no price effects. Hence a marginal increase in renewable capacities simply raises renewable production by $\alpha(t)$ and replaces fossil production by this amount.

Next, consider the *storage subsidy* σ_s^*. The fractions now represent the marginal effect of storage capacities on fossils, $-\frac{\partial Q_f}{\partial Q_s}$. A higher storage capacity reduces the price of the destorage period as more electricity from the storage is fed into the market ($\int_d \frac{\partial p_d}{\partial Q_s} dt = \frac{\partial p_d}{\partial x_d} \frac{1}{\eta_d} < 0$). This makes investment in fossils less attractive and provides an argument for subsidizing storage. Moreover, the effect is smaller if efficiency losses are large (high η_d) so that only a small share of electricity from the storage arrives in the market.

However, as long as fossils contribute to filling the storage—i.e., for $\alpha_s \leq \alpha_2$—there is the following countervailing effect: First, if fossils are fully used during storage ($\alpha_s \leq \alpha_1$), a higher Q_s raises the price of the storage period ($\int_s \frac{\partial p_s}{\partial Q_s} dt = -\frac{\partial p_s}{\partial x_s} \frac{1}{\eta_s} > 0$) and, thus, the profitability of investment in fossils. This provides an argument for taxing storage, especially, when efficiency losses are large (low η_s) so that more electricity has to be taken from the market to fill the storage. Moreover, one would expect that demand is more price responsive when prices are high, that is, during destorage, than at the low prices during storage (see Faruqui and Sergici, 2010). This implies that $|\frac{\partial x_d}{\partial p_d}| > |\frac{\partial x_s}{\partial p_s}| \iff |\frac{\partial p_d}{\partial x_d}| < |\frac{\partial p_s}{\partial x_s}|$, further supporting the rationale for taxing storage capacities. Second, no such price effects obtain if fossils are only partly used during storage ($\alpha_s \in (\alpha_1, \alpha_2]$) because the storage price is constant at $p_s = k_f + \tau$ in this case. However, now a marginal unit of storage capacities raises fossil production by $\int_s \frac{\partial Y_f}{\partial Q_s} dt = \int_s \left(-\frac{\partial \alpha_s}{\partial Q_s} Q_r\right) dt = \frac{1}{\eta_s}$ (from Lemma 2, see also (42)) because more fossils are needed to supplement renewables in filling the larger storage. This provides another reason for taxing storage, which again increases for higher efficiency losses during storage (low η_s).

Finally, if renewable capacities are large enough so that fossils not longer contribute to filling the storage ($\alpha_s > \alpha_2$), the above effects and, thus, also the puzzling optimality of taxing storage vanish.

5.3.1 The case of no cost externality

Ignoring the cost externality, in all of the above cases the optimal subsidy σ_s^* only depends on its indirect effects on fossil capacities. In particular, the negative effect of tax-induced lower storage capacities on investments in renewables turns out to be irrelevant. The reason is simply that the second policy instrument—the subsidy for renewables—is used to compensate this effect. The following corollary summarises the above results regarding the sign of the subsidies.

Corollary 1. *Suppose that the cost externality does not dominate by assuming $\frac{\partial c_r}{\partial Q_r} = \frac{\partial c_s}{\partial Q_s} = 0$. Then the optimal renewable subsidy, σ_r^*, is always strictly positive. By contrast, the optimal storage subsidy, σ_s^*, is*

- *for $\alpha_s \leq \alpha_1$ negative iff $\frac{1}{\eta_d} \left|\frac{\partial p_d}{\partial x_d}\right| < \frac{1}{\eta_s} \left|\frac{\partial p_s}{\partial x_s}\right|$,*
- *for $\alpha_s \in (\alpha_1, \alpha_2]$ negative iff $\frac{1}{\eta_d} \left|\frac{\partial p_d}{\partial x_d}\right| \int_{d,1} dt < \frac{1}{\eta_s} \int_{d,1} \left|\frac{\partial p(t)}{\partial x(t)}\right| dt$,*
- *for $\alpha_s \leq \alpha_2$ strictly positive.*

5.3.2 The case of linear demand

Price effects featured prominently in the above elaborations. If we simplify these by assuming linear demand, the optimal subsidies have a very simple structure.

Corollary 2. *For an incomplete carbon and no consumption tax ($\tau < \delta, \chi = 0$), linear demand, $\frac{\partial^2 p}{\partial x^2} = 0$, and constant unit costs, $\frac{\partial c_r}{\partial Q_r}, \frac{\partial c_s}{\partial Q_s} = 0$, optimal subsidies are*

$$\sigma_r^* = \rho (\delta - \tau) \begin{cases} \int_{t_0}^T \alpha(t)\, dt > 0 & \textit{if} \quad \alpha_s \le \alpha_1, \\ \int_{t_0}^T \alpha(t)\, dt > 0 & \textit{if} \quad \alpha_s \in (\alpha_1, \alpha_2], \\ \int_{d,1,2} \alpha(t)\, dt > 0 & \textit{if} \quad \alpha_s > \alpha_2, \end{cases}$$

and

$$\sigma_s^* = \rho (\delta - \tau) \begin{cases} \left(\frac{1}{\eta_d} - \frac{1}{\eta_s} \right) \le 0 & \textit{if} \quad \alpha_s \le \alpha_1, \\ \left(\frac{1}{\eta_d} - \frac{1}{\eta_s} \right) \le 0 & \textit{if} \quad \alpha_s \in (\alpha_1, \alpha_2], \\ \frac{1}{\eta_d} > 0 & \textit{if} \quad \alpha_s > \alpha_2, \end{cases}$$

where the respective inequalities are strict if there are conversion losses of storage.

Intuitively, the renewable subsidy σ_r^* reflects that an additional unit of renewable capacities displaces fossil production by $\alpha(t)$ as long as the latter are used. Accordingly, it is constant in the first two lines where fossils produce for all t. By contrast, the period $\int_{d,1,2} dt$ in the third line is shorter and decreasing in Q_r because the availability of renewables for which case 2 ends, $\alpha_2 = x(b_f + \chi)/Q_r$, is falling in Q_r. Hence fossil capacities are used less often so that there is less reason to subsidise their replacement by renewable capacities.

Turning to σ_s^*, storage capacities should be taxed as long as their price increasing effect during storage dominates their price reducing effect during destorage due to conversion losses. This reflects that fossil capacities are more profitable if electricity prices are higher. For $\alpha_s > \alpha_2$, storage capacities only contribute to destorage and, therefore, σ_s^* turns strictly positive.

6 Numerical Illustration of Optimal Subsidies and Discussion

Figure 2 presents results of a numerical simulation of the optimal subsidy scheme with linear demand, constant unit costs and no consumption tax, as in the Corollaries. The parameters are loosely calibrated to German data (see Appendix H for details). Values for optimal unit subsidies are depicted on the right axis (in €/MW and €/MWh, respectively); capacities (in MW and MWh) and total subsidy payments, $\Sigma := \sigma_r Q_r + \sigma_s Q_s$, on the left axis.[18] The (small) diamonds show efficient capacities (Q_r^*, Q_s^*, Q_f^*) that would occur with an efficient Pigouvian tax $\tau^* = \delta$. As shown in Section 5.2, the same capacities would also obtain for an imperfect carbon tax if it is complemented by a consumption tax $\chi^\# = \delta - \tau$, a renewable subsidy $\sigma_r^\# = \rho(\delta - \tau) \int_{t_0}^T \alpha(t)\, dt = 136,600$ €/MW, and a storage tax $\sigma_s^\# = \rho \chi^\# \left(\frac{1}{\eta_d} - \frac{1}{\eta_s} \right) = -7,490$ €/MWh (not depicted).

The figure should be read from the right to the left. Then all values are depicted as a function of unit capacity costs of renewables (c_r) and storage (c_s) that are falling at the same rate. Whereas the preceding analysis was restricted to the case where positive quantities of all three technologies are installed in equilibrium, we now consider a broader cost range. It also includes the situations where renewables and storage enter the market, and where they have fully captured it.

Intuitively, renewable capacities enter the market first, supported by a subsidy that is constant at the level 136,600 €/MW as long as fossils produce for all t (in line with Corollary 2). Storage capacities follow once the resulting volume of intermittent supply is large enough to make buffering electricity economically viable. Due to relatively high capacity costs and conversion losses (19 per cent in our calibration), this only happens when renewables have reached a capacity of 224 GW. This is large enough to completely satisfy electricity demand at times of high availability during which storage takes place. Therefore, the situations where fossils profited from the higher electricity prices

[18] Note that storage capacity is measured in MWh. This distinction between power and energy was irrelevant in the theoretical model, but is important now.

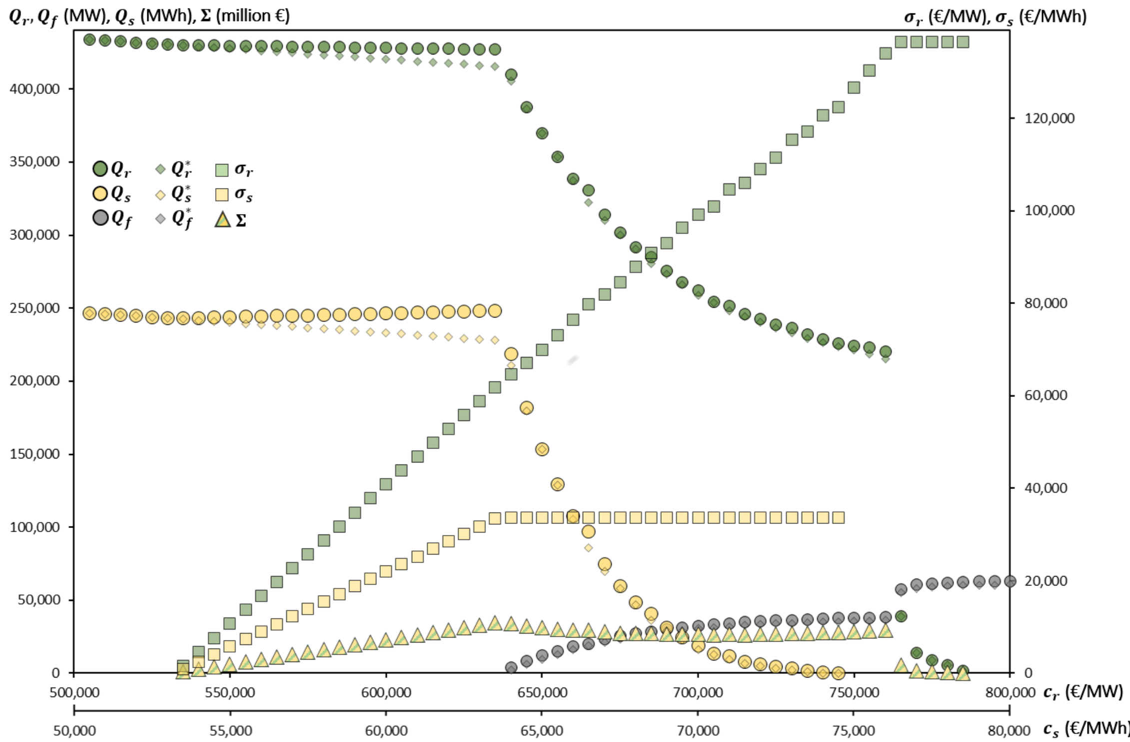

Fig. 2: Market diffusion and optimal subsidies

due to storage, which provided the rationale for taxing storage capacities, are leapfrogged.[19] Once storage capacities enter the market, they receive a constant subsidy, whereas the renewable subsidy is gradually decreasing (both values are exactly those of Corollary 2 for $\alpha_s > \alpha_2$). However, the lower rate is paid for larger capacities so that total subsidy payments, Σ, are even slightly increasing until fossils are completely driven out of the market. Even thereafter, falling but still substantial subsidies are required to prevent fossils from re-entering the market.

In the analytical model, this boundary case where subsidies are chosen such that they are just sufficient to keep fossil firms out requires that their first-order condition (30) is satisfied at $Q_f = 0$. In particular, noting that a re-entering fossil firm would produce at full capacity during destorage and case 1, the first-order condition becomes (during case 2 fossils make zero profits)

$$\pi_{ff} = \rho \int_{d,1} \left(p\left(t, \mathbf{Q}\right) - k_f - \tau\right) dt - c_f = 0, \text{ where } \mathbf{Q} = \left(0, Q_r, Q_s\right). \tag{47}$$

Obviously, this condition would be met if renewable and storage capacities, Q_r, Q_s, were kept constant at the level that solves this equation, which is roughly the case in our numerical simulations. From the first-order conditions (31) and (32) for capacity choices of renewable and storage firms, this requires that falling capacity costs are balanced by lower subsidies; as in Figure 2. By contrast, when renewable and storage capacities have become cheap enough to defend the market without subsidies, both rise in response to further falling costs.

Finally, consider the evolution of capacities in the initial stages. The market diffusion of renewables is slow in the beginning, then accelerates rapidly, and thereafter slows down again. The evolution of fossil capacities matches this pattern in opposite direction. This is in line with the result in Helm

[19] Results of a model calibration that leads to an earlier build-up of storage capacities and, thus, taxes in the initial stages are available upon request. In a nutshell, they require lower costs and lower conversion losses of storages.

and Mier (2019), but the additional storage technology accelerates the build-up of renewables and the phase out of fossils. Storage capacities are increasing exponentially despite constant subsidies. The reason is their rising market value as there are more variable renewables and less reliable fossils.

7 Concluding Remarks

Countries often fail to implement a Pigouvian tax that fully internalises the carbon externality. We showed that a consumption tax of the same size imposes the same incentives on fossil producers because the tax incidence is independent of who pays the tax. However, this result does not extend to a model with several fossil technologies with different carbon intensities, as this would require differentiated Pigouvian and, therefore, differentiated consumption taxes for the homogeneous good electricity (see also Abrell et al. (2019)). Moreover, unlike the Pigouvian tax the consumption tax must also be paid for electricity that comes from renewables and from the storage. This distorts investment decisions that must be corrected by subsidising renewables and taxing storage capacities.

Given that an optimal consumption tax is basically a Pigouvian tax through the back-door, we then analysed second-best subsidies for installing capacities of intermittent renewable energies and storage. Renewables reduce the profitability of fossil investments by lowering expected prices and by displacing fossil production at times of high availability. This provides a rationale for the subsidisation of renewable energies. Storage capacities raise the electricity price when the storage is filled and lower it during destorage. This has countervailing effects on the profitability of fossils, and the storage subsidy is chosen on the basis of the relative strength of these effects. In particular, if fossil energies produce at times when storage drives up electricity prices, it is usually optimal to tax storage. In a stylised way, our analysis also accounts for a cost externality, e.g., from learning or economy-wide economies of scale. Given the current technological progress in storage technologies such as batteries and power-to-gas, these externalities may well provide an overriding argument for subsidising storage.

Under the stylised calibration of the model, storage capacities only enter the market when renewables capacities are sufficient to satisfy the whole electricity demand at times of high availability, during which storage takes place. Hence it is always optimal to subsidise storage. However, in the real world fossil capacities nearly always produce at times of electricity storage and, thus, benefit from the price increase of the resulting higher electricity demand.[20] Similarly, electric vehicles, which are powered by stored electricity, are usually charged to a substantial extent with electricity from burning fossil fuels. Even worse, currently the batteries in electric vehicles are almost never used for destorage. Hence the price dampening effect, which hampers the competitiveness of fossil firms, does not occur. Both effects weaken the case for subsidising electric vehicles to reduce CO_2 emissions; although other reasons such as reducing local air pollution may still justify subsidies (see Holland, Mansur, Muller, and Yates, 2016). Moreover, real world availability patterns of renewables are more heterogeneous than in our model. Therefore, even with a relatively large share of renewables in the energy system there may be periods where fossils contribute to filling the storage. In conclusion, the rationale for taxing storage that resulted from the analytical model may actually be stronger than the numerical simulation suggests.

The subsidy scheme is only second-best and substantially more complex than a first-best Pigouvian tax. In particular, optimal subsidies constitute a moving target because they vary, sometimes even in their sign, depending on the relative shares of the three technologies in the electricity system. They also require substantially more knowledge about the electricity market—such as demand sensitivity—than does a Pigouvian tax, which, in our model, simply equals the environmental unit costs of fossil production. Hence one should read the paper not so much as a call for taxing storage, but as a lesson in the complexity of second-best policies that strengthens the case for directly addressing the externality with a price on carbon.

Our results show that accounting for intermittency of renewables—an aspect that is still often neglected in the analytical literature—has substantial implications for the design of policy instruments.

[20] Note that the largest share of storage is done by commercial customers that face variable prices, as in our model.

Moreover, given the substantial public funds that currently subsidise renewables, and, increasingly, storage, this paper is of high policy relevance. At least under the simplifying assumptions in the corollaries, it provides an argument for gradually reducing the subsidy for renewables as their market penetration rises, and raising the subsidy for storage instead. However, the latter is not targeted at supporting the rising share of renewables, because the market provides sufficient incentives to build storage capacities if there is more fluctuating electricity from renewables. Rather, storage of electricity is subsidised because it substitutes fossil production when the availability of renewables is low.

Let us now address some further limitations and potential extensions of the above analysis that need be taken into account when drawing policy recommendations. First, we have ignored effects that may result from the interaction with existing overlapping instruments. Most importantly, an increasing number of countries are implementing cap-and-trade systems that substantially impact the effectiveness of complementary subsidies for renewables (Jarke and Perino, 2017). Second, we only considered subsidies for capacities, whereas the dominating instruments for renewables have been feed-in-tariffs—that is, a subsidisation of electricity output. However, these are quite similar in that both are paid independently of the price that obtains on the market for electricity. Moreover, in our numerical simulation renewable capacities are always fully used due to a sufficient storage capacity. In this case, a subsidy per unit of output is equivalent to a subsidy per unit of capacity that is available on average.

Third, we have restricted the analysis to one renewable and one fossil technology. This could be extended relatively straightforwardly to several renewable technologies—e.g., PV, offshore and onshore wind—with technology specific availability factors, $\alpha_l(t)$.[21] There would then be a separate optimal subsidy rate for each renewable technology such that technologies which (on average) reduce electricity prices more strongly receive a higher subsidy rate.[22] With peak prices around midday, this would suggest a subsidy mark-up for solar power. By contrast, with high PV shares there may be consistently higher prices during the winter season, which would suggest a subsidy mark-up for more stable offshore wind power. If several fossil technologies were considered, then a different emission intensity of the marginal technology during storage and destorage periods would affect the analysis (see Carson and Novan, 2013). As destorage takes places during high price periods, one might expect that it displaces primarily gas with comparatively low emission intensity.

A related extension would be to include several storage technologies (e.g., batteries, pumped hydro storage, and power-to-gas). This would allow a more accurate distinction between the different storage needs that result from daily and seasonal variations in the availability of intermittent renewables (see Sinn (2017) and Zerrahn, Schill, and Kemfert (2018) for a discussion). Finally, the model could also be extended by including other market failures—e.g., non-reactive consumer demand, distortionary taxation, and imperfect competition—or further aspects of electricity markets, such as variable demand, trade, and the transmission grid. However, the first extension would make it more difficult to isolate the effects of the pollution externality that was the focus of this contribution, and the second would probably come at the cost of greater reliance on numerical simulations.

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[21] There would then be a profit maximisation problem for each type l of renewable firms (Eqs. (5) and (6)), and supply from renewables would be given by the sum over all technologies, $\sum_l \alpha_l(t)Q_{lr}$.

[22] For example, in Proposition 3 this would be captured by terms $\int_{t_0}^{T} \frac{\partial p(t)}{\partial x(t)} \alpha_l(t)\, dt$, where the availability parameter $\alpha_l(t)$ would now be technology specific.

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Appendix

A Proof of Lemma 1

As shown in the main text, the sufficient conditions for optimality are equations (3) and (4) for fossil firms, (7) and (8) for renewable firms, and (16) to (21) for storage firms. We now prove that the equilibrium values in Lemma 1 satisfy these conditions.

Starting with the *destorage period*, it is straightforward to see that the solution $y_r(t) = \alpha(t) q_r$ and $y_f(t) = q_f$ satisfies (3), (4), (7), and (8) for $\mu_r, \mu_f > 0$, and results in a price $p(t)$ above the total unit costs of fossils, b_f.[23] Turning to storage firms, conditions (16) and (18) to (20) have already been used at the beginning of Section 3.2 to derive the result of a constant price during destorage. The remaining condition (17) describes how storage is depleting ($\dot{s}(t) < 0$), where the length of the destorage period that follows from (23) ensures that destorage levels are consistent with the quantity of stored electricity.

Once the storage has run empty, we have $s(t) = 0$ so that $\varphi_d(t)$ turns (weakly) positive (see (20)). This initiates the *first intermediate period*, $t \in (t_d, t_s)$, during which $\dot{s}(t) = y_s(t) = 0$ (eqs. (17)). For renewables, (7) and (8) imply that $y_r(t) = \alpha(t) q_r$ for any $p(t) > 0$, whereas for $p(t) = 0$ any output $y_r(t)$ is profit maximizing due to marginal costs of zero. Moreover, the fact that $\alpha(t)$

[23] Note that firms are identical so that $y_r(t) = \alpha(t) q_r$ implies $Y_r(t) = n_r y_r(t) = \alpha(t) Q_r$, where n_r is the number of renewable firms. An equivalent argument applies to the other quantities in the proof.

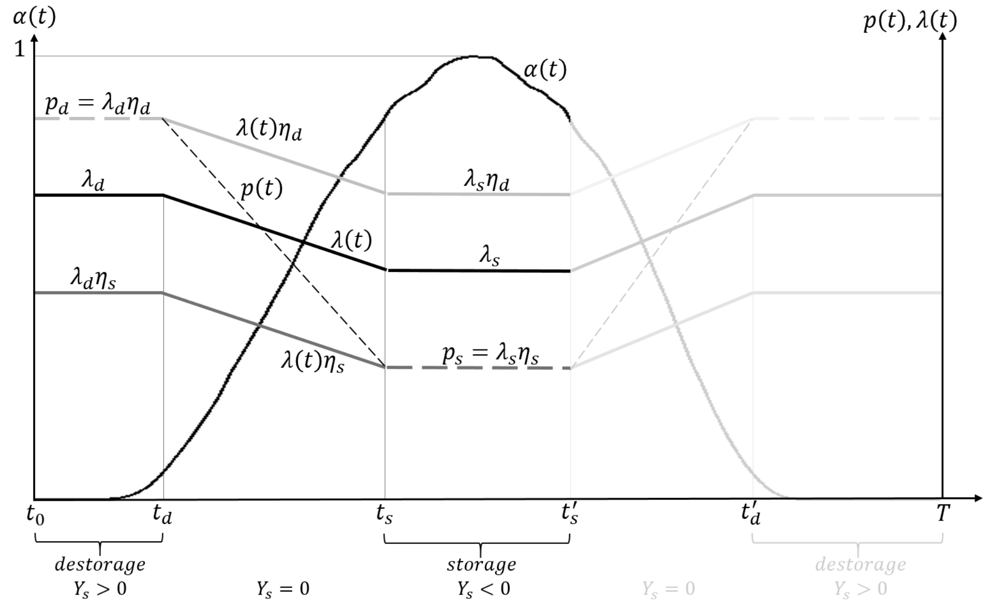

Fig. 3: Availability of renewables and resulting values of the adjoint variable

is increasing during the first intermediate period implies that, ceteris paribus, $p(t)$ is decreasing. As long as $p(t) > b_f$, we have $\mu_f(t) > 0$ from (3) so that $y_f(t) = q_f$ from (4). This *case 1* continues until full usage of fossil and available renewable capacities have lowered the price to the total unit costs of fossils, $p(t) = b_f$.

Noting that consumers may face a consumption tax χ, *case 2* starts when $x(b_f + \chi) = \alpha(t) Q_r + Q_f$ which yields $\alpha(t_1) = \alpha_1 = \frac{x(b_f+\chi)-Q_f}{Q_r}$. During case 2, fossils continue to be used so that (3) still binds. Hence, $p(t) = b_f$ implies $\mu_f(t) = 0$ from (3) and so that $0 < y_f(t) < q_f$ by the complementary slackness conditions in (4). Once available renewable capacities, $\alpha(t) Q_r$, are large enough to satisfy demand at consumers' after tax price $b_f + \chi$, we enter *case 3*. Accordingly, case 3 starts when $x(b_f + \chi) = \alpha(t) Q_r$ so that $\alpha(t_2) = \alpha_2 = \frac{x(b_f+\chi)}{Q_r}$. For $\alpha(t) > \alpha_2$, we have $p(t) - b_f < 0$ so that $y_f(t) = 0$ from (3).

Turning to *storage firms*, condition (19) is met for $\varphi_s(t) = 0$ because $q_s > s(t)$ during the first intermediate period, which implies $\dot{\lambda}(t) = -\varphi_d(t) \leq 0$ (from (18)). At the beginning of the preceding paragraph, we have already addressed conditions (20) and (17); hence it remains to show that $y_s(t) = 0$ maximizes $[p(t) - \lambda(t)\eta(y_s)]\, y_s(t)$ (eqs. (16)). Using $\dot{\lambda}(t) \leq 0$, $\eta_d \geq \eta_s$, and noting that continuity of $\lambda(t)$ was a precondition for (16) to (21) being sufficient, we obtain a situation as depicted by the bold lines in Figure 3. Moreover, during $t \in [t_d, t_s]$ the price $p(t)$ is monotonically decreasing from p_d to p_s, as represented by the dashed line in Figure 3. It is straightforward to see that the values of the multiplier $\varphi_d(t)$ which determines the course of $\lambda(t)$ can be chosen such that $\lambda(t)\eta_d > p(t) > \lambda(t)\eta_s$. Using this, $y_s(t) > 0$ would lead to $[p(t) - \lambda(t)\eta_d]\, y_s(t) < 0$, and $y_s(t) < 0$ to $[p(t) - \lambda(t)\eta_s]\, y_s(t) < 0$. Therefore, $y_s(t) = 0$ must be optimal.

The *storage period* ($y_s(t) < 0$) can start during either of the cases 1 to 3. As shown in the main text, it has a price that equals the one at the end of the intermediate period, i.e., $p_s = p(t_s)$. Hence, $Y_f = Y_f(t_s)$ from (3) and (4). Moreover, for $p_s > 0$ conditions (7) and (8) imply $Y_r = \alpha(t)Q_r$, whereas for $p_s = 0$ any Y_r is profit maximizing due to the assumption of no variable costs. Turning to storage firms, the argument parallels that during the destorage period: Conditions (16) and (18) to (20) have already been addressed and (17) now implies that $\dot{s}(t) > 0$. Moreover, the conditions (25) for $\alpha_s < \frac{x(0+\chi)}{Q_r}$ and (26) for $\alpha_s = \frac{x(0+\chi)}{Q_r}$ ensure that storage levels are consistent with the available storage capacity. Once the storage is completely filled, $s(t) = q_s$ and $\varphi_s(t)$ turns (weakly) positive (see (19)). Thereafter, the second intermediate and destorage periods follow. Their solution and the critical availabilities that distinguish these solutions are the same as discussed above for each respective $\alpha(t)$.

B Proof of Lemma 2

The statements in (c) that relate to the situation $\alpha_s Q_r = x(0+\chi)$ (excess capacities of renewables) follow immediately from implicit differentiation of this expression and the fact that demand is constant at $x(0+\chi)$. Hence we only need to prove the statements in (a) and (b) that concern the situation without excess capacities ($\alpha_s Q_r < x(0+\chi)$).

Conditions (23) and (25) that implicitly determine α_d and α_s can be written as

$$f_d := \eta_d \int_{t_0}^{t_d} (\alpha_d - \alpha(t)) Q_r dt + \eta_d \int_{t'_d}^{T} (\alpha_d - \alpha(t)) Q_r dt - Q_s = 0, \tag{48}$$

$$f_s := -\eta_s \int_{t_s}^{t'_s} (\alpha_s - \alpha(t)) Q_r dt - Q_s = 0. \tag{49}$$

The comparative static effects of a change in Q_f, Q_r, or Q_s, thereby taking the other capacities as given, follow from applying the implicit function theorem, i.e., $\frac{\partial \alpha_u}{\partial Q_j} = -\frac{\partial f_u}{\partial Q_j} / \frac{\partial f_u}{\partial \alpha_u}$ for $u = d, s$ and $j = f, r, s$. It follows that $\frac{\partial \alpha_s}{\partial Q_f} = \frac{\partial \alpha_d}{\partial Q_f} = 0$. Next, note that $\alpha_d = \alpha(t_d) = \alpha(t'_d)$ and $\alpha_s = \alpha(t_s) = \alpha(t'_s)$. This implies that the integral terms in (48) and (49) are zero if evaluated at the boundaries of the integral, t_d, t'_d and t_s, t'_s, respectively. Using this when applying the implicit function theorem yields the comparative statics $\frac{\partial \alpha_d}{\partial Q_r}, \frac{\partial \alpha_d}{\partial Q_s}, \frac{\partial \alpha_s}{\partial Q_r}$, and $\frac{\partial \alpha_s}{\partial Q_s}$ in Lemma 2.

Demand during destorage, $x_d = \sum_j Y_j(t) = Q_f + \alpha_d Q_r$, follows straightforwardly from Lemma 1 and Table 1. Differentiation yields $\frac{\partial x_d}{\partial Q_r} = \alpha_d + \frac{\partial \alpha_d}{\partial Q_r} Q_r$ and $\frac{\partial x_d}{\partial Q_s} = \frac{\partial \alpha_d}{\partial Q_s} Q_r$. Substitution of $\frac{\partial \alpha_d}{\partial Q_r}, \frac{\partial \alpha_d}{\partial Q_s}$ yields the values in Lemma 2, where we have used

$$\frac{\partial x_d}{\partial Q_r} = \alpha_d - \frac{\int_d (\alpha_d - \alpha(t)) dt}{\int_d dt} = \alpha_d - \alpha_d \frac{\int_d dt}{\int_d dt} + \frac{\int_d \alpha(t) dt}{\int_d dt}. \tag{50}$$

For storage, demand depends on the case that obtains at the beginning of the storage period. From Table 1, $x_s = \sum_j Y_j(t) = Q_f + \alpha_s Q_r$ if it starts during case 1, and $x_s = \alpha_s Q_r$ if it starts during case 3. In both situations, $\frac{\partial x_s}{\partial Q_r} = \alpha_s + \frac{\partial \alpha_s}{\partial Q_r} Q_r$, and $\frac{\partial x_s}{\partial Q_s} = \frac{\partial \alpha_s}{\partial Q_s}$. The values in the Table in Lemma 2 follow again after substituting for $\frac{\partial \alpha_s}{\partial Q_r}, \frac{\partial \alpha_s}{\partial Q_s}$, thereby applying the same steps as in (50) to x_s. Finally, if storage starts during case 2, then $x_s = \sum_j Y_j(t) = x(b_f + \chi)$ so that $\frac{\partial x_s}{\partial Q_j} = 0$ for $j = f, r, s$.

C Calculation of Equation (35)

In this proof we use the compact notations $\sum_j$ and $\sum_i$ for summation over all three technologies $j, i = f, r, s$. Using $x(t, \chi, \tau, \mathbf{Q}) = \sum_j Y_j(t, \chi, \tau, \mathbf{Q})$, the first integrand term in (34) can be written as

$$\frac{dx(t, \chi, \tau, \mathbf{Q})}{d\theta} = \sum_j \left(\frac{\partial Y_j(t, \chi, \tau, \mathbf{Q})}{\partial \theta} + \sum_i \frac{\partial Y_j(t, \chi, \tau, \mathbf{Q})}{\partial Q_i} \frac{dQ_i}{d\theta} \right). \tag{51}$$

The first term represents the direct effects of policy instruments θ on production (it is zero for subsidies), and the second the indirect effects via capacity choices. Moreover, this latter effect can be written out as (dropping the arguments)

$$\sum_j \left(\sum_i \frac{\partial Y_j}{\partial Q_i} \frac{dQ_i}{d\theta} \right) = \sum_j \frac{\partial Y_j}{\partial Q_j} \frac{dQ_j}{d\theta} + \frac{\partial \left(Y_r + Y_s\right)}{\partial Q_f} \frac{dQ_f}{d\theta} + \frac{\partial \left(Y_f + Y_s\right)}{\partial Q_r} \frac{dQ_r}{d\theta} + \frac{\partial \left(Y_f + Y_r\right)}{\partial Q_s} \frac{dQ_s}{d\theta}, \quad (52)$$

where the first term sums up the effects of capacity on production within type-j firms, whereas the other terms summarise the cross effects. From Table 1, $\frac{\partial (Y_r(t)+Y_s(t))}{\partial Q_f} = 0$ for all t. Moreover, also $\int_{t_0}^{T} (p(t) + \chi) \frac{\partial Y_s(t)}{\partial Q_r} \frac{dQ_r}{d\theta} dt = 0$. To see this, note that $Y_s(t) = 0$ during the intermediate period and, by construction, the available storage capacity is completely filled during storage and completely emptied during destorage, independent of Q_r. Hence $\int_d (p(t) + \chi) \frac{\partial Y_s(t)}{\partial Q_r} \frac{dQ_r}{d\theta} dt = (p_d + \chi) \frac{dQ_r}{d\theta} \frac{\partial}{\partial Q_r} \left(\int_d Y_s(t) \, dt \right) = 0$ and equivalently for the storage period. These terms can be eliminated when substituting (51) together with (52) into (34). After rearranging terms, this yields

$$\begin{aligned} \frac{dW}{d\theta} &= \sum_j \left(\rho \int_{t_0}^{T} (p(t) + \chi) \frac{\partial Y_j(t)}{\partial Q_j} dt - c_j(Q_j) \right) \frac{dQ_j}{d\theta} - \rho \int_{t_0}^{T} (k_f + \delta) \frac{dY_f(t)}{d\theta} dt - \sum_{i=r,s} Q_i \frac{\partial c_i}{\partial Q_i} \frac{dQ_i}{d\theta} \\ &\quad + \rho \int_{t_0}^{T} (p(t) + \chi) \left(\sum_{i=r,s} \frac{\partial Y_f(t)}{\partial Q_i} \frac{dQ_i}{d\theta} + \frac{\partial Y_r(t)}{\partial Q_s} \frac{dQ_s}{d\theta} \right) dt + \rho \int_{t_0}^{T} (p(t) + \chi) \sum_j \frac{\partial Y_j(t)}{\partial \theta} dt, \end{aligned}$$

where the last integral sums up the direct effects. Noting that $\frac{\partial Y_j(t)}{\partial Q_j} = \frac{\partial y_j(t)}{\partial q_j}$, substitution from the first-order conditions (30) to (32) for firms' capacity choices and collecting terms gives

$$\begin{aligned} \frac{dW}{d\theta} &= \sum_{i=r,s} \left(\rho \int_{t_0}^{T} \chi \frac{\partial Y_i(t)}{\partial Q_i} dt - Q_i \frac{\partial c_i}{\partial Q_i} - \sigma_i \right) \frac{dQ_i}{d\theta} + \rho \int_{t_0}^{T} (\tau + k_f + \chi) \frac{\partial Y_f(t)}{\partial Q_f} \frac{dQ_f}{d\theta} dt - \rho \int_{t_0}^{T} (k_f + \delta) \frac{dY_f(t)}{d\theta} dt \\ &\quad + \rho \int_{t_0}^{T} (p(t) + \chi) \left(\sum_{i=r,s} \frac{\partial Y_f(t)}{\partial Q_i} \frac{dQ_i}{d\theta} + \frac{\partial Y_r(t)}{\partial Q_s} \frac{dQ_s}{d\theta} \right) dt + \rho \int_{t_0}^{T} (p(t) + \chi) \sum_j \frac{\partial Y_j(t)}{\partial \theta} dt. \end{aligned} \quad (53)$$

We can disaggregate $\frac{dY_f(t)}{d\theta}$ into a direct and indirect effect, $\frac{dY_f(t,\chi,\tau,\mathbf{Q})}{d\theta} = \frac{\partial Y_f(t)}{\partial \theta} + \sum_j \frac{\partial Y_f(t)}{\partial Q_j} \frac{dQ_j}{d\theta}$. Using this, we have

$$-(k_f + \delta) \frac{dY_f(t)}{d\theta} = (\tau - \delta + \chi) \frac{dY_f(t)}{d\theta} - (\tau + \chi + k_f) \left(\frac{\partial Y_f(t)}{\partial \theta} + \frac{\partial Y_f(t)}{\partial Q_f} \frac{dQ_f}{d\theta} + \sum_{i=r,s} \frac{\partial Y_f(t)}{\partial Q_i} \frac{dQ_i}{d\theta} \right).$$

Substitution and using this to cancel the second term in (53) yields

$$\begin{aligned} \frac{dW}{d\theta} &= \sum_{i=r,s} \left(\rho \int_{t_0}^{T} \chi \frac{\partial Y_i(t)}{\partial Q_i} dt - Q_i \frac{\partial c_i}{\partial Q_i} - \sigma_i \right) \frac{dQ_i}{d\theta} + \rho \int_{t_0}^{T} (\tau - \delta + \chi) \frac{dY_f(t)}{d\theta} dt \\ &\quad + \rho \int_{t_0}^{T} (p(t) - k_f - \tau) \sum_{i=r,s} \frac{\partial Y_f(t)}{\partial Q_i} \frac{dQ_i}{d\theta} dt + \rho \int_{t_0}^{T} (p(t) - \tau - k_f) \frac{\partial Y_f(t)}{\partial \theta} dt \\ &\quad + \rho \int_{t_0}^{T} (p(t) + \chi) \sum_{i=r,s} \frac{\partial Y_i(t)}{\partial \theta} dt + \rho \int_{t_0}^{T} (p(t) + \chi) \frac{\partial Y_r(t)}{\partial Q_s} \frac{dQ_s}{d\theta} dt. \end{aligned}$$

Note that $(p(t) - \tau - k_f) \frac{\partial Y_f}{\partial Q_i} = 0$, $i = r, s$, because $\frac{\partial Y_f}{\partial Q_i} = 0$ except during stage 2 (and during storage if the storage period starts in case 2) for which, however, $p(t) = \tau + k_f$. An equivalent argument yields $(p(t) - \tau - k_f) \frac{\partial Y_f}{\partial \theta} = 0$, so that the second line vanishes. Defining $z := \rho \int_{t_0}^{T} (p(t) + \chi) \left(\sum_{i=r,s} \frac{\partial Y_i(t)}{\partial \theta} + \frac{\partial Y_r(t)}{\partial Q_s} \frac{dQ_s}{d\theta} \right) dt$, we obtain the wanted expression in (35).

It remains to specify z for the different periods and policy instruments. From Table 1, we have $\frac{\partial Y_s(t)}{\partial \theta}, \frac{\partial Y_r(t)}{\partial \theta} = 0$ for $\theta = \sigma_r, \sigma_s, \tau$. If $\alpha_s Q_r < x(0+\chi)$, then also $\frac{\partial Y_r(t)}{\partial Q_s}, \frac{\partial Y_s(t)}{\partial \chi}, \frac{\partial Y_r(t)}{\partial \chi} = 0$. Therefore, z can only be non-zero if there are excess capacities of renewables ($\alpha_s Q_r = x(0+\chi)$). From Table 1, for this case $\sum_{i=r,s} Y_i(t) = x(0+\chi)$ for all $\alpha(t) > \alpha_s$—i.e., during storage—so that $\int_{t_0}^{T} \sum_{i=r,s} \frac{\partial Y_i(t)}{\partial \chi} dt = \int_{\alpha_s}^{\alpha_{\max}} \frac{\partial x(0+\chi)}{\partial \chi} d\alpha$, which takes into account that $\sum_{i=r,s} Y_i(t)$ does not *directly* depend on χ for $\alpha(t) \leq \alpha_s$. Moreover, $Y_r(t) = \alpha_c Q_r$ for all $\alpha(t) > \alpha_c$, and implicit differentiation of (26) at $\alpha_s = \frac{x(0+\chi)}{Q_r}$ gives $\frac{\partial a_c}{\partial Q_s} = \frac{1}{\eta_s \int_{\alpha_c}^{\alpha_{\max}} Q_r d\alpha} > 0$. This reflects that with a larger storage capacity renewable production is capped later. Using this, $\int_{t_0}^{T} \frac{\partial Y_r(t)}{\partial Q_s} dt = \int_{\alpha_c}^{\alpha_{\max}} \frac{\partial \alpha_c Q_r}{\partial Q_s} d\alpha = \int_{\alpha_c}^{\alpha_{\max}} \frac{1}{\eta_s \int_{\alpha_c}^{\alpha_{\max}} d\alpha} d\alpha = \frac{1}{\eta_s}$. Collecting terms we obtain the expression for z in (36).

D Proof of Proposition 1

It remains to show that the policy mix in the Proposition leads to first-best capacity levels, which we now determine. In equilibrium, demand equals supply so that $\frac{\partial x(t,\mathbf{Q})}{\partial Q_j} = \sum_{i=f,r,s} \frac{\partial Y_i(t)}{\partial Q_j}$. We have $\int_{t_0}^{T} \frac{\partial Y_s(t)}{\partial Q_r} dt = 0$ because the available storage capacity is completely filled (emptied) during storage (destorage), independent of Q_r. Moreover, from Table 1, $\frac{\partial Y_f(t)}{\partial Q_s} = 0$ and $\frac{\partial (Y_r(t)+Y_s(t))}{\partial Q_f} = 0$ for all t. Finally, $p(t) \frac{\partial Y_r(t,\mathbf{Q})}{\partial Q_s} = 0$ because $\frac{\partial Y_r(t,\mathbf{Q})}{\partial Q_s} = 0$ except for excess capacities of renewables during storing ($\alpha_s Q_r = x(0+\chi)$), for which however $p(t) = 0$. Substitution of this into the first-order conditions for welfare maximizing capacity choices (38) yields

$$\frac{\partial W}{\partial Q_f} = \rho \int_{t_0}^{T} (p^*(t) - \delta - k_f) \frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_f} dt - c_f = 0, \tag{54}$$

$$\frac{\partial W}{\partial Q_r} = \rho \int_{t_0}^{T} p^*(t) \frac{\partial Y_r^*(t,\mathbf{Q})}{\partial Q_r} dt + \int_{t_0}^{T} (p^*(t) - \delta - k_f) \frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_r} dt - c_r(Q_r) - \frac{\partial c_r}{\partial Q_r} Q_r = 0, \tag{55}$$

$$\frac{\partial W}{\partial Q_s} = \rho \int_{t_0}^{T} p^*(t) \frac{\partial Y_s^*(t,\mathbf{Q})}{\partial Q_s} dt + \int_{t_0}^{T} (p^*(t) - \delta - k_f) \frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_s} dt - c_s(Q_s) - \frac{\partial c_s}{\partial Q_s} Q_s = 0, \tag{56}$$

where superscript * indicates that the outcome at the production stage is first-best.

Note that $\frac{\partial Y_j(t)}{\partial Q_j} = \frac{\partial y_j(t)}{\partial q_j}$. Moreover, $\frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_r} = \frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_s} = 0$ except during case 2 and during storage if it starts during case 2, for which, however, $p^*(t) - \delta - k_f = 0$. Using this, the above conditions are exactly the same as the first-order conditions (30) to (32) for firms' capacity choices if all conditions are evaluated at the first-best policy instruments, $\tau^* = \delta, \chi^* = 0$, $\sigma_i^* = -Q_i(\boldsymbol{\theta}) \frac{\partial c_i}{\partial Q_i}$, $i = r, s$.

E Proof of Proposition 2

The optimal values $\chi^{\#}, \sigma_r^{\#}, \sigma_s^{\#}$ as stated under the first bullet point have already been determined in the main text. It remains to prove that they implement the social optimum for the situation of no excess capacities. Remember that production and consumption choices on competitive markets as analysed in Section 3.2 were only distorted by the pollution externality. Therefore, with a Pigouvian tax $\tau = \delta$ production levels as summarised in Table 1 and, thus, demand $x(t) = \sum_{j=f,r,s} Y_j(t)$ are obviously first-best, provided that the underlying capacity levels are first-best. In the main text we have shown that the combination of an incomplete carbon tax, $\tau < \delta$, and a correcting consumption tax $\chi^{\#} = \delta - \tau$ leads to exactly the same output and demand levels. Now we verify that for the policy vector in Proposition 2 also firms' capacity choices are first-best, i.e., satisfy conditions (54) to (56). In particular, upon substitution of $\sigma_r^{\#} = -Q_r^* \frac{\partial c_r}{\partial Q_r} + \rho(\delta - \tau) \int_{t_0}^{T} \alpha(t)\, dt$ and $\sigma_s^{\#} = -Q_s^* \frac{\partial c_s}{\partial Q_s} + \rho \chi^{\#} \left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right)$, the first-order conditions for firms' capacity choices are

$$\rho \int_{t_0}^{T} \left(p^{\#}(t) - \tau - k_f\right) \frac{\partial y_f^*(t, q_f)}{\partial q_f} dt - c_f = 0, \tag{57}$$

$$\rho \int_{t_0}^{T} p^{\#}(t) \frac{\partial y_r^*(t, q_r)}{\partial q_r} dt - c_r(Q_r) - Q_r^* \frac{\partial c_r}{\partial Q_r} + \rho(\delta - \tau) \int_{t_0}^{T} \alpha(t)\, dt = 0, \tag{58}$$

$$\rho \int_{t_0}^{T} p^{\#}(t) \frac{\partial y_s^*(t, q_s)}{\partial q_s} dt - c_s(Q_s) - Q_s^* \frac{\partial c_s}{\partial Q_s} + \rho \chi^{\#} \left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right) = 0, \tag{59}$$

where $p^{\#}(t) = p^*(t) - (\delta - \tau)$ (see main text). Substituting this into (57) to (59), and using $\frac{\partial y_j(t)}{\partial q_j} = \frac{\partial Y_j(t)}{\partial Q_j}$ as well as $\int_{t_0}^{T} \frac{\partial Y_r^*(t,q_r)}{\partial Q_r} dt = \int_{t_0}^{T} \alpha(t)\, dt$ and $\int_{t_0}^{T} (\delta - \tau) \frac{\partial Y_s^*(t,q_s)}{\partial Q_s} dt = \chi^{\#} \left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right)$, the conditions are the same as those for first-best capacity choices in (54) to (56).

F Calculation of Equation (44)

For each t, the equilibrium electricity price that obtains in Stage 3 is a function of capacities that are given at this stage, i.e., $p(t) = p(t, \mathbf{Q})$. Moreover, $\frac{\partial y_f^*(t)}{\partial q_f} = 1$ during destorage and case 1, but $\frac{\partial y_f^*(t)}{\partial q_f} = 0$ for cases 2 and 3 (see Table 1). Therefore, during storage $\frac{\partial y_f^*(t)}{\partial q_f}$ depends on the case during which it starts. Using this, total differentiation of firms' first-order condition (30) for fossil capacities yields

$$d\pi_{ff} = \begin{cases} \sum_{j=f,r,s} \rho \left(\int_1 \frac{\partial p(t)}{\partial Q_j} dt + \frac{\partial p_d}{\partial Q_j} \int_d dt + \frac{\partial p_s}{\partial Q_j} \int_s dt\right) dQ_j & \text{if } \alpha_s \leq \alpha_1, \\ \sum_{j=f,r,s} \rho \left(\int_1 \frac{\partial p(t)}{\partial Q_j} dt + \frac{\partial p_d}{\partial Q_j} \int_d dt\right) dQ_j & \text{if } \alpha_s > \alpha_1, \end{cases} \tag{60}$$

where p_d, p_s are the constant prices during destorage and storage. Storage capacities are not used during case 1 so that $\int_1 \frac{\partial p(t)}{\partial Q_s} dt = 0$. For the other derivatives in (60), applying the chain rule when partially differentiating equilibrium prices with respect to Q_j yields $\frac{\partial p(t)}{\partial Q_j} = \frac{\partial p(t)}{\partial x(t)} \frac{\partial x(t)}{\partial Q_j}$, where $\frac{\partial x(t)}{\partial Q_j}$ follows from Lemma 2. In particular, $\frac{\partial p_d}{\partial x_d} \frac{\partial x_d}{\partial Q_s} \int_d dt = \frac{1}{\eta_d}$ and $\frac{\partial p_s}{\partial x_s} \frac{\partial x_s}{\partial Q_s} \int_d dt = -\frac{1}{\eta_s}$. Using this, setting $d\pi_{ff} = 0$, dividing by $d\sigma_j$, and rearranging yields (44).

G Proof of Proposition 3

For $\alpha_s \leq \alpha_1$, substitution of (41) together with (44) into the first-order conditions (40) for optimal subsidies and collecting terms with $\frac{dQ_r}{d\sigma_i}$ and $\frac{dQ_s}{d\sigma_i}$ yields

$$\sum_{j=r,s} \left(\sigma_j + \rho(\delta - \tau) \int_{d,1,s} dt \frac{\partial Q_f}{\partial Q_j} + Q_j(\boldsymbol{\theta}) \frac{\partial c_j}{\partial Q_j}\right) \frac{dQ_r}{d\sigma_i} = 0, \; i = r, s. \tag{61}$$

In these two first-order conditions with respect to σ_r and σ_s, the term $\sum_{j=r,s}(\cdot)$ has the same value. Therefore, (61) is obviously satisfied if this term is equal to zero.[24] Substitution from (45) and (46) into (61) yields the optimal subsidies for $\alpha_s \leq \alpha_1$.

Similarly, substitution of (42) for $\alpha_s \in (\alpha_1, \alpha_2]$ and (43) for $\alpha_s > \alpha_2$ together with (44) into (61), and collecting terms with $\frac{dQ_r}{d\sigma_i}, \frac{dQ_s}{d\sigma_i}$ yields the following two conditions:

[24] For any given τ, this is the only solution if one abstracts from pathological cases.

$$\left(\sigma_r + \rho(\delta-\tau)\left(\int_{d,1} dt \frac{\partial Q_f}{\partial Q_r} - \int_{2,s} \alpha(t)\,dt\right) + Q_r(\boldsymbol{\theta})\frac{\partial c_r}{\partial Q_r}\right)\frac{dQ_r}{d\sigma_i} + \left(\sigma_s + \rho(\delta-\tau)\left(\int_{d,1} dt\,\frac{\partial Q_f}{\partial Q_s} + \frac{1}{\eta_s}\right) + Q_s(\boldsymbol{\theta})\frac{\partial c_s}{\partial Q_s}\right)\frac{dQ_s}{d\sigma_i} = 0,\ i = r, s, \tag{62}$$

$$\left(\sigma_r + \rho(\delta-\tau)\left(\int_{d,1} dt \frac{\partial Q_f}{\partial Q_r} - \int_{2} \alpha(t)\,dt\right) + Q_r(\boldsymbol{\theta})\frac{\partial c_r}{\partial Q_r}\right)\frac{dQ_r}{d\sigma_i} + \left(\sigma_s + \rho(\delta-\tau)\left(\int_{d,1} dt\,\frac{\partial Q_f}{\partial Q_s}\right) + Q_s(\boldsymbol{\theta})\frac{\partial c_s}{\partial Q_s}\right)\frac{dQ_s}{d\sigma_i} = 0,\ i = r, s. \tag{63}$$

The optimal subsidies for $\alpha_s \in (\alpha_1, \alpha_2]$ and $\alpha_s > \alpha_2$ follow after substitution from (45) and (46).

H Calibration for Section 6

We use German data to calibrate the model and make the following assumptions. 96 quarterly hours represent one cycle. Each representative quarterly hour depicts the weighted average of the hourly availability of solar PV (45%), wind offshore (5%), and wind onshore (50%) over an entire year ($\min\{\alpha(t)\} = 0.0976$, $\max\{\alpha(t)\} = 0.2918$), which roughly presents the German capacity mix. Demand is $x(t) = 75,000 - 375 \cdot p(t)$, where the sensitivity -375 reflects a price elasticity of demand of -0.25 at a reference price of 40 €/MWh (e.g., Thimmapuram and Kim, 2013).[25]

We assume that fossil firms use a gas turbine technology and thus pay $c_f = 500,000$ €/MW for capacity. Natural gas prices are around 7.5 €/MWh, the efficiency of the fossil technology is assumed to be 50%, so that $k_f = 0.5 \cdot 7.5 = 15$ €/MWh are private production costs. On July 2, 2019, the CO_2 price in the EU ETS peaked at 29 €/ton, and we take this as the carbon tax. The emission factor of natural gas is 0.2358 tons CO_2/MWh (here MWh refers to the heat value of natural gas), yielding a carbon tax of $\tau = 0.5 \cdot 0.2358 \cdot 29 = 13.68$ €/MWh. For the social costs of carbon we take a value that is 50% higher, i.e., $\delta = 1.5 \cdot \tau = 20.51$ €/MWh.

Capacity costs of renewables and storage firms fall at the same rate, starting at $c_r = 800,000$ €/MW and $c_s = 80,000$ €/MWh, respectively. Actual costs of renewables are around $1,000,000$ (solar PV), $2,500,000$ (wind offshore), and $1,200,000$ €/MW (wind onshore) (IEA, 2015; Schröder, Kunz, Meiss, Mendelevitch, and Von Hirschhausen, 2013). However, renewables costs are expected to fall further, and thus the depicted costs seem to be a good guess. Conversion losses of storage operations are $\eta_d = 1.1$ and $\eta_s = 0.9$, reflecting total losses of 19%. There is no particular storage technology that fits our synthetic one (see IRENA, 2017; Nykvist and Nilsson, 2015; Schmidt et al., 2017, for different cost estimates). A good guess is pumped hydro with similar capacity costs (5,000 to 100,000 €/MWh, mean around 60,000 €/MWh) and efficiency losses (15 to 30%). Battery storage systems cost around 200,000 €/MWh, but costs are expected to fall to 75,000 €/MWh (efficiency losses of only 3%). Power-to-gas technologies face lower capacity costs (around 40,000 €/MWh in the future), but fundamentally higher higher conversion losses (40 to 70%).

Finally, we use a discount rate of 3% and lifetimes of 30 years to calculate ρ, but deviate from the theoretical model by abstracting from within-year discounting for parsimony. This yields $\rho = 365 \cdot \left(\frac{1}{r} - \frac{1}{r(1+r)^{30}}\right) = 5,430$ with $r = 0.03$.

We set up the program in GAMS as a welfare maximization problem. The assumption of linear demand makes the program quadratic with non-convex constraints from the three zero-profit conditions of fossil, renewable, and storage firms. We therefore use the solver IPOPT, which is powerful in solving non-convex programs and finding local maxima, but cannot ensure the global maximum.

[25] See www.energy-charts.de/power_inst.htm for the German capacity mix and www.energy-charts.de/price.htm for load data.

We therefore assist the solver by giving him the (linear) first-order conditions of firms production and capacity choices, and by constraining the solution space. The code is available upon request.