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ifo DSGE Model 2.0

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Abstract

This documentation concisely describes the dynamic stochastic general-equilibrium model that the ifo Institute currently uses for simulations and business-cycle analysis. The model consists of three countries and contains a wide range of rigidities. The model is regularly estimated by quarterly macroeconomic data.

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1 Overview

The ifo DSGE model 2.0 represents a large-scale dynamic stochastic general-equilibrium model, which consists of three fully-microfounded countries. The first country stands for Germany; the second country corresponds to the rest of the euro area; the third country captures the rest of the world. Like in reality, the first and the second country constitute a currency union with a common monetary policy. The third country, in contrast, follows its own specific central-bank policy. The model distinguishes in each country between a tradable and a non-tradable sector. Additionally, the model features a series of real and nominal frictions that are usual in quantitative macroeconomic models. Consumption exhibits habit formation; investment suffers from adjustment costs; a utilization rate makes predetermined capital stock variable; prices and nominal wages are sticky. In the recent past, the model has been extensively used to analyze the business cycle (Gemeinschaftsdiagnose, 2019; Wollmershäuser et al., 2020*b*, 2021), to quantify the effects of fiscal policy (Wollmershäuser et al., 2018, 2020*a*; Gemeinschaftsdiagnose, 2020), or to investigate the impact of trade barriers (Gemeinschaftsdiagnose, 2018; Grimme, Šauer and Wollmershäuser, 2019).

2 Data and Estimation

Every quarter, the model is estimated by 19 observables. The model obtains information on the German economy from eight observables: GDP, private consumption, private investment, export, import, hours, hourly wage, and harmonized CPI inflation. Information on the rest of the euro area comes from six observables: GDP, private consumption, investment, hours, hourly wage, and harmonized CPI inflation. About the rest of the world, the model is informed by three observables: GDP, CPI inflation, and the central bank's policy rate. The remaining two observables that the model takes advantage of are the policy rate of the ECB and the effective nominal exchange rate between the euro area and the rest of the world.

The 19 observables form a data set that covers quarters from 1999:Q2 onward. The data that is necessary for constructing the German observables can be easily accessed through Destatis. In comparison, the construction of the observables for the rest of the euro area and the rest of the world shows more complexity. The rest of the euro area is approximated by the data of France, Italy, and Spain, which together make up about two thirds of the rest of the euro area. The data for the rest of the world is approximated by the most important trading partners of Germany that lie outside of the euro area: China, the United Kingdom, and the United States. Because the observables for the rest of the euro area rely directly on national data, a new vintage of the data set can be constructed already in t+60. One doesn't need to wait for the official data of the euro area, which Eurostat publishes in t+65. The gain of five working days brings a crucial time advantage in the busy process of forecasting and business-cycle analysis.

The majority of the officially published time series have to be transformed to match the model observables. Whenever meaningful, the time series are expressed in real per-capita terms, and quarter-on-quarter growth rates are computed by log differencing. The proxies for the rest of the euro area and the rest of the world are built as averages of the transformed national time series. The proxies are weighted by GDP. If a time series with seasonality is published without an official seasonal adjustment, the X-13ARIMA-SEATS program is applied. Table 1 lists the time series that one needs for the construction of the data set. The table also mentions the original data sources and the transformations that the series have to undergo to be in line with the observables. All time series are retrieved from Macrobond.

Country	Series	Source	Transformations	Observable
DE	total population, SA	Destatis	none	auxiliary
	real GDP, SA, CA	Destatis	per capita, Q/Q	$\widetilde{X}_t^a$
	real consumption, households & NPISH, SA, CA	Destatis	per capita, Q/Q	$\widetilde{C}_t^a$
	real gross fixed-capital formation, non-government, SA, CA	Destatis	per capita, Q/Q	$\widetilde{I}_t^a$
	real export, SA, CA	Destatis	per capita, Q/Q	$\widetilde{EX}_t^a$
	real import, SA, CA	Destatis	per capita, Q/Q	$\widetilde{IM}_t^a$
	harmonized CPI, total	Destatis	quarterly average, SA, Q/Q	$\widetilde{\pi}_t^a$
	hours worked, employees, domestic concept, SA	Destatis	per capita, Q/Q	$\widetilde{N}_t^a$
	nominal compensation of employees, domestic concept, SA	Destatis	per hour, CPI deflated, Q/Q	$\widetilde{w}_t^a$
FR	nominal GDP, SA, CA	Insee	none	auxiliary
	total population	Insee	quarterly average, SA	auxiliary
	real GDP, SA, CA	Insee	per capita, Q/Q	$\widetilde{X}_t^b$
	real consumption, households, SA, CA	Insee	per capita, Q/Q	$\widetilde{C}_t^b$
	real gross fixed-capital formation, total, SA, CA	Insee	per capita, Q/Q	$\widetilde{I}_t^b$
	harmonized CPI, total	Insee	quarterly average, SA, Q/Q	$\widetilde{\pi}_t^b$
	employees, all branches, SA	Insee	none	auxiliary
	average weekly hours worked, all non-agricultural sectors	MdT	SA, multiplied by employment, per capita, Q/Q	$\widetilde{N}_t^b$
	nominal compensation of employees, all branches, SA	Insee	per hour, CPI deflated, Q/Q	$\widetilde{w}_t^b$
IT	nominal GDP, SA, CA	Istat	none	auxiliary
	total population	Eurostat	SA	auxiliary
	real GDP, SA, CA	Istat	per capita, Q/Q	$\widetilde{X}_t^b$
	real consumption, households & NPISH, SA, CA	Istat	per capita, Q/Q	$\widetilde{C}_t^b$
	real gross fixed-capital formation, total, SA	Istat	per capita, Q/Q	$\widetilde{I}_t^b$
	harmonized CPI, total	Istat	quarterly average, SA, Q/Q	$\widetilde{\pi}_t^b$
	hours worked, total, SA	Istat	per capita, Q/Q	$\widetilde{N}_t^b$
	nominal compensation of employees, domestic concept, SA, CA	Istat	per hour, CPI deflated, Q/Q	$\widetilde{w}_t^b$
ES	total population, SA	Eurostat	none	auxiliary
	GDP deflator, SA	MINECO	none	auxiliary
	nominal GDP, SA, CA	INE	deflated, per capita, Q/Q	$\widetilde{X}_t^b$
	consumption deflator, households & NPISH, SA	MINECO	none	auxiliary
	nominal consumption, households & NPISH, SA, CA	INE	deflated, per capita, Q/Q	$\widetilde{C}_t^b$
	deflator of gross fixed-capital formation, total, SA	MINECO	none	auxiliary
	nominal gross fixed-capital formation, total, SA, CA	INE	deflated, per capita, Q/Q	$\widetilde{I}_t^b$
	harmonized CPI, total	INE	quarterly average, SA, Q/Q	$\widetilde{\pi}_t^b$
	hours worked, employees, SA, CA	INE	per capita, Q/Q	$\widetilde{N}_t^b$
	nominal compensation of employees, SA, CA	INE	per hour, CPI deflated, Q/Q	$\widetilde{w}_t^b$
EA	main-refinancing-operations rate	ECB	quarterly average	$\widetilde{i}_t^{mu}$
CN	nominal exchange rate USD/CNY, quarterly average	IMF	none	auxiliary
	nominal exchange rate CNY/EUR, quarterly average	IMF	Q/Q	$\widetilde{s}_t^{ca}$
	nominal GDP	NBS	SA, converted to USD	auxiliary

Continued from previous page

Country	Series	Source	Transformations	Observable
	total population, annual	NBS	linearly interpolated	auxiliary
	real GDP, SA	NBS	per capita, Q/Q	$\widetilde{X}_t^c$
	CPI, total	NBS	quarterly average, SA, Q/Q	$\widetilde{\pi}_t^c$
	one-year policy lending rate	PBC	quarterly average	$\widetilde{i}_t^c$
UK	nominal exchange rate USD/GBP, quarterly average	IMF	none	auxiliary
	nominal exchange rate GBP/EUR, quarterly average	IMF	Q/Q	$\widetilde{s}_t^{ca}$
	nominal GDP, SA	ONS	converted to USD	auxiliary
	resident population, SA	ONS	none	auxiliary
	real GDP, SA	ONS	per capita, Q/Q	$\widetilde{X}_t^c$
	CPI, total	ONS	quarterly average, SA, Q/Q	$\widetilde{\pi}_t^c$
	official bank rate	BOE	quarterly average	$\widetilde{i}_t^c$
US	nominal exchange rate USD/EUR, quarterly average	IMF	Q/Q	$\widetilde{s}_t^{ca}$
	nominal GDP, SA	BEA	none	auxiliary
	total population, SA	BEA	none	auxiliary
	real GDP, SA	BEA	per capita, Q/Q	$\widetilde{X}_t^c$
	CPI, all urban consumers, all items	BLS	quarterly average, SA, Q/Q	$\widetilde{\pi}_t^c$
	effective federal-funds rate	FRBNY	quarterly average	$\widetilde{i}_t^c$

Table 1: Time Series That Are Needed for Constructing the Data Set of Observables

Several parameters of the model (elasticities, discount factors, depreciation rates, capital shares, aggregator weights, population sizes) are calibrated such that the steady state of the model replicates markups, great ratios, and trade flows that are documented in the data. Remaining parameters receive independent priors and are estimated by Bayesian methods.

3 Model Equations

This section reports equilibrium conditions that characterize the behavior of the German economy. Analogous equations hold for the rest of the euro area and the rest of the world. Germany is denoted as country a, the rest of the euro area as country b, and the rest of the world as country c.

Gross domestic product Y_t^a:

$$Y_t^a = R_t^{Taa} R_t^{Ta} Y_t^{Ta} + R_t^{Na} Y_t^{Na}$$

Output of the tradable sector Y_t^{Ta}:

$$Y_t^{Ta} \Delta_t^{Ta} = \left(A_t^a A_t^{Ta}\right)^{1-\alpha_a^T} \left(K_t^{Ta}\right)^{\alpha_a^T} \left(N_t^{Ta}\right)^{1-\alpha_a^T}$$

Output of the non-tradable sector Y_t^{Na}:

$$Y_t^{Na} \Delta_t^{Na} = \left(A_t^a A_t^{Na}\right)^{1-\alpha_a^N} \left(K_t^{Na}\right)^{\alpha_a^N} \left(N_t^{Na}\right)^{1-\alpha_a^N}$$

Price dispersion in the tradable sector Δ_t^{Ta}:

$$\Delta_t^{Ta} = \gamma_a^T \left[\frac{(\pi^a)^{\xi_a^p} \left(\pi_{t-1}^a\right)^{1-\xi_a^p}}{\pi_t^{Taa}} \right]^{-\theta_a^T} \Delta_{t-1}^{Ta} + \left(1 - \gamma_a^T\right) \left(\frac{R_t^{*Taa}}{R_t^{Taa}} \right)^{-\theta_a^T}$$

Price dispersion in the non-tradable sector Δ_t^{Na}:

$$\Delta_t^{Na} = \gamma_a^N \left[\frac{(\pi^a)^{\xi_a^p} \left(\pi_{t-1}^a\right)^{1-\xi_a^p}}{\pi_t^{Na}} \right]^{-\theta_a^N} \Delta_{t-1}^{Na} + \left(1 - \gamma_a^N\right) \left(\frac{R_t^{*Na}}{R_t^{Na}} \right)^{-\theta_a^N}$$

Labor-capital ratio in the tradable sector N_t^{Ta}/K_t^{Ta}:

$$\frac{r_t^{ka}}{w_t^a} = \frac{\alpha_a^T}{1-\alpha_a^T} \frac{N_t^{Ta}}{K_t^{Ta}}$$

Labor-capital ratio in the non-tradable sector N_t^{Na}/K_t^{Na}:

$$\frac{r_t^{ka}}{w_t^a} = \frac{\alpha_a^N}{1-\alpha_a^N} \frac{N_t^{Na}}{K_t^{Na}}$$

Marginal costs in the tradable sector mc_t^{Ta}:

$$mc_t^{Ta} = \frac{\left(r_t^{ka}\right)^{\alpha_a^T} \left(w_t^a\right)^{1-\alpha_a^T}}{\left(A_t^a A_t^{Ta}\right)^{1-\alpha_a^T} \left(\alpha_a^T\right)^{\alpha_a^T} \left(1-\alpha_a^T\right)^{1-\alpha_a^T}}$$

Marginal costs in the non-tradable sector mc_t^{Na}:

$$mc_t^{Na} = \frac{\left(r_t^{ka}\right)^{\alpha_a^N} \left(w_t^a\right)^{1-\alpha_a^N}}{\left(A_t^a A_t^{Na}\right)^{1-\alpha_a^N} \left(\alpha_a^N\right)^{\alpha_a^N} \left(1-\alpha_a^N\right)^{1-\alpha_a^N}}$$

Price of the tradable output R_t^{Taa}:

$$1 = \gamma_a^T \left[\frac{(\pi^a)^{\xi_a^p} \left(\pi_{t-1}^a\right)^{1-\xi_a^p}}{\pi_t^{Taa}} \right]^{1-\theta_a^T} + \left(1 - \gamma_a^T\right) \left(\frac{R_t^{*Taa}}{R_t^{Taa}} \right)^{1-\theta_a^T}$$

Price of the German tradable output in the rest of the euro area R_t^{Tab}:

$$R_t^{Tab} = R_t^{Taa} \mathcal{E}_t^{ba} \frac{R_t^{Ta}}{R_t^{Tb}}$$

Price of the German tradable output in the rest of the world R_t^{Tac}:

$$R_t^{Tac} = R_t^{Taa} \mathcal{E}_t^{ca} \frac{R_t^{Ta}}{R_t^{Tc}}$$

Price of the non-tradable output R_t^{Na}:

$$1 = \gamma_a^N \left[\frac{(\pi^a)^{\xi_a^p} \left(\pi_{t-1}^a\right)^{1-\xi_a^p}}{\pi_t^{Na}} \right]^{1-\theta_a^N} + \left(1-\gamma_a^N\right) \left(\frac{R_t^{*Na}}{R_t^{Na}} \right)^{1-\theta_a^N}$$

Optimal price in the tradable sector R_t^{*Taa}:

$$R_t^{*Taa} R_t^{Ta} = \frac{\theta_a^T}{\theta_a^T - 1} \frac{KN_t^{Ta}}{KD_t^{Ta}}$$

Optimal price in the non-tradable sector R_t^{*Na}:

$$R_t^{*Na} = \frac{\theta_a^N}{\theta_a^N - 1} \frac{KN_t^{Na}}{KD_t^{Na}}$$

Auxiliary variable for the optimal price in the tradable sector KN_t^{Ta}:

$$KN_t^{Ta} = \lambda_t^a mc_t^{Ta} Y_t^{Ta} + \gamma_a^T \beta_a E_t \left(\frac{\pi_{t+1}^{Taa}}{(\pi^a)^{\xi_a^p} \left(\pi_t^a\right)^{1-\xi_a^p}} \right)^{\theta_a^T} KN_{t+1}^{Ta}$$

Auxiliary variable for the optimal price in the tradable sector KD_t^{Ta}:

$$KD_t^{Ta} = \lambda_t^a \left(1 - \tau_{at}^{Tv}\right) Y_t^{Ta} + \gamma_a^T \beta_a E_t \left(\frac{\pi_{t+1}^{Taa}}{(\pi^a)^{\xi_a^p} \left(\pi_t^a\right)^{1-\xi_a^p}} \right)^{\theta_a^T} \frac{(\pi^a)^{\xi_a^p} \left(\pi_t^a\right)^{1-\xi_a^p}}{\pi_{t+1}^a} KD_{t+1}^{Ta}$$

Auxiliary variable for the optimal price in the non-tradable sector KN_t^{Na}:

$$KN_t^{Na} = \lambda_t^a mc_t^{Na} Y_t^{Na} + \gamma_a^N \beta_a E_t \left(\frac{\pi_{t+1}^{Na}}{(\pi^a)^{\xi_a^p} \left(\pi_t^a\right)^{1-\xi_a^p}} \right)^{\theta_a^N} KN_{t+1}^{Na}$$

Auxiliary variable for the optimal price in the non-tradable sector KD_t^{Na}:

$$KD_t^{Na} = \lambda_t^a \left(1 - \tau_{at}^{Nv}\right) Y_t^{Na} + \gamma_a^N \beta_a E_t \left(\frac{\pi_{t+1}^{Na}}{(\pi^a)^{\xi_a^p} \left(\pi_t^a\right)^{1-\xi_a^p}} \right)^{\theta_a^N} \frac{(\pi^a)^{\xi_a^p} \left(\pi_t^a\right)^{1-\xi_a^p}}{\pi_{t+1}^a} KD_{t+1}^{Na}$$

Inflation of the tradable output π_t^{Taa}:

$$\pi_t^{Taa} = \pi_t^{Ta} \frac{R_t^{Taa}}{R_{t-1}^{Taa}}$$

Inflation of the German tradable output in the rest of the euro area π_t^{Tab}:

$$\pi_t^{Tab} = \frac{\pi_t^{Taa}}{\Delta S_t^{ab}}$$

Inflation of the German tradable output in the rest of the world π_t^{Tac}:

$$\pi_t^{Tac} = \frac{\pi_t^{Taa}}{\Delta S_t^{ac}}$$

Inflation of the non-tradable output π_t^{Na}:

$$\pi_t^{Na} = \pi_t^{a} \frac{R_t^{Na}}{R_{t-1}^{Na}}$$

Supply of the tradable output Y_t^{Ta}:

$$\begin{aligned} Y_t^{Ta} = & C_t^{Taa} + I_t^{Taa} + G_t^{Taa} + \tilde{\Gamma}_t^{Taa} \\ & + \frac{\mathcal{P}_b}{\mathcal{P}_a} C_t^{Tab} + \frac{\mathcal{P}_b}{\mathcal{P}_a} I_t^{Tab} + \frac{\mathcal{P}_b}{\mathcal{P}_a} G_t^{Tab} + \frac{\mathcal{P}_b}{\mathcal{P}_a} \tilde{\Gamma}_t^{Tab} \\ & + \frac{\mathcal{P}_c}{\mathcal{P}_a} C_t^{Tac} + \frac{\mathcal{P}_c}{\mathcal{P}_a} I_t^{Tac} + \frac{\mathcal{P}_c}{\mathcal{P}_a} G_t^{Tac} + \frac{\mathcal{P}_c}{\mathcal{P}_a} \tilde{\Gamma}_t^{Tac} \end{aligned}$$

Supply of the non-tradable output Y_t^{Na}:

$$Y_t^{Na} = C_t^{Na} + I_t^{Na} + G_t^{Na} + \tilde{\Gamma}_t^{Na}$$

Effective capital stock $K_{t-1}^a u_t^a$:

$$K_{t-1}^a u_t^a = K_t^{Ta} + K_t^{Na}$$

Hours N_t^a:

$$N_t^a = N_t^{Ta} + N_t^{Na}$$

Average hourly wage w_t^a:

$$\left(w_t^a\right)^{1-\theta_a^w} = \left(1-\gamma_a^w\right)\left(w_t^{*a}\right)^{1-\theta_a^w} + \gamma_a^w \left[w_{t-1}^a \frac{\left(\pi_{t-1}^a\right)^{\xi_a^w} \left(\pi^a\right)^{1-\xi_a^w}}{\pi_t^a} \right]^{1-\theta_a^w}$$

Optimal hourly wage w_t^{*a}:

$$w_t^{*a} = \left(\frac{\theta_a^w}{\theta_a^w - 1} \frac{FW_t^a}{HW_t^a} \right)^{\frac{1}{1+\theta_a^w \psi_a}}$$

Auxiliary variable for the optimal wage FW_t^a:

$$FW_t^a = \exp\left(\varepsilon_t^{\beta_a} + \varepsilon_t^{N^a} \right) \kappa_a \left(N_t^a \right)^{1+\psi_a} \left(w_t^a \right)^{\theta_a^w (1+\psi_a)} + \beta_a \gamma_a^w E_t \left[\frac{\pi_{t+1}^a}{\left(\pi_t^a \right)^{\xi_a^w} \left(\pi^a \right)^{1-\xi_a^w}} \right]^{\theta_a^w (1+\psi_a)} FW_{t+1}^a$$

Auxiliary variable for the optimal wage HW_t^a:

$$HW_t^a = \left(1 - \tau_{at}^w \right) \left(w_t^a \right)^{\theta_a^w} N_t^a \lambda_t^a + \beta_a \gamma_a^w E_t \left[\frac{\pi_{t+1}^a}{\left(\pi_t^a \right)^{\xi_a^w} \left(\pi^a \right)^{1-\xi_a^w}} \right]^{\theta_a^w - 1} HW_{t+1}^a$$

Lagrange multiplier on the budget constraint λ_t^a:

$$\lambda_t^a = \left(C_t^a - h_a C_{t-1}^a \right)^{-\sigma_a} \exp\left(\varepsilon_t^{\beta_a} \right)$$

Private consumption C_t^a:

$$\lambda_t^a = E_t \beta_a \lambda_{t+1}^a \frac{1 + i_t^a}{\pi_{t+1}^a}$$

Private investment I_t^a:

$$\begin{aligned} 1 = Q_t^a & \left[1 - \frac{\upsilon_a}{2} \left(\frac{I_t^a}{I_{t-1}^a} - 1 \right)^2 - \upsilon_a \frac{I_t^a}{I_{t-1}^a} \left(\frac{I_t^a}{I_{t-1}^a} - 1 \right) \right] \exp\left(\varepsilon_t^{I^a} \right) \\ & + E_t \beta_a \frac{\lambda_{t+1}^a}{\lambda_t^a} Q_{t+1}^a \upsilon_a \left(\frac{I_{t+1}^a}{I_t^a} \right)^2 \left(\frac{I_{t+1}^a}{I_t^a} - 1 \right) \exp\left(\varepsilon_{t+1}^{I^a} \right) \end{aligned}$$

Tobin's Q Q_t^a:

$$Q_t^a = E_t \beta_a \frac{\lambda_{t+1}^a}{\lambda_t^a} \left[\left(1 - \delta_a \right) Q_{t+1}^a + r_{t+1}^{ka} u_{t+1}^a - \frac{\tilde{\Gamma}_{t+1}^a}{K_t^a} - \tau_{at+1}^k \left(r_{t+1}^{ka} u_{t+1}^a - \frac{\tilde{\Gamma}_{t+1}^a}{K_t^a} - \delta_a \right) \right]$$

Physical capital stock K_t^a:

$$K_t^a = (1 - \delta_a) K_{t-1}^a + I_t^a \left[1 - \frac{\upsilon_a}{2} \left(\frac{I_t^a}{I_{t-1}^a} - 1 \right)^2 \right] \exp\left(\varepsilon_t^{I^a} \right)$$

Utilization rate of capital u_t^a:

$$r_t^{ka} = r^{ka} \exp\left(\psi_a^k \left(u_t^a - 1 \right) \right)$$

Total costs of capital utilization $\tilde{\Gamma}_t^a$:

$$\tilde{\Gamma}_t^a = \frac{r^{ka}}{\psi_a^k} \left[\exp\left(\psi_a^k \left(u_t^a - 1\right)\right) - 1\right] K_{t-1}^a$$

Price of the tradable-goods bundle R_t^{Ta} and price of the non-tradable output R_t^{Na}:

$$1 = \mu_{at} \left(R_t^{Ta}\right)^{1-\theta_a} + \left(1 - \mu_{at}\right) \left(R_t^{Na}\right)^{1-\theta_a}$$

Inflation of the final bundle π_t^a:

$$\left(\pi_t^a\right)^{1-\theta_a} = \mu_{at} \left(R_{t-1}^{Ta} \pi_t^{Ta}\right)^{1-\theta_a} + \left(1 - \mu_{at}\right) \left(R_{t-1}^{Na} \pi_t^{Na}\right)^{1-\theta_a}$$

Tradable private consumption C_t^{Ta}:

$$C_t^{Ta} = \mu_{at} \left(R_t^{Ta}\right)^{-\theta_a} C_t^a$$

Non-tradable private consumption C_t^{Na}:

$$C_t^{Na} = \left(1 - \mu_{at}\right) \left(R_t^{Na}\right)^{-\theta_a} C_t^a$$

Tradable private investment I_t^{Ta}:

$$I_t^{Ta} = \mu_{at} \left(R_t^{Ta}\right)^{-\theta_a} I_t^a$$

Non-tradable private investment I_t^{Na}:

$$I_t^{Na} = \left(1 - \mu_{at}\right) \left(R_t^{Na}\right)^{-\theta_a} I_t^a$$

Tradable government consumption G_t^{Ta}:

$$G_t^{Ta} = \mu_{at} \left(R_t^{Ta}\right)^{-\theta_a} G_t^a$$

Non-tradable government consumption G_t^{Na}:

$$G_t^{Na} = \left(1 - \mu_{at}\right) \left(R_t^{Na}\right)^{-\theta_a} G_t^a$$

Tradable costs of capital utilization $\tilde{\Gamma}_t^{Ta}$:

$$\tilde{\Gamma}_t^{Ta} = \mu_{at} \left(R_t^{Ta}\right)^{-\theta_a} \tilde{\Gamma}_t^a$$

Non-tradable costs of capital utilization $\tilde{\Gamma}_t^{Na}$:

$$\tilde{\Gamma}_t^{Na} = (1 - \mu_{at}) \left(R_t^{Na}\right)^{-\theta_a} \tilde{\Gamma}_t^{a}$$

German prices of the tradable output from Germany R_t^{Taa}, from the rest of the euro area R_t^{Tba}, and from the rest of the world R_t^{Tca}:

$$1 = n^{aa} \left(R_t^{Taa}\right)^{1-\eta_a} + n^{ba} \left(R_t^{Tba}\right)^{1-\eta_a} + n^{ca} \left(R_t^{Tca}\right)^{1-\eta_a}$$

Inflation of the tradable-goods bundle π_t^{Ta}:

$$\left(\pi_t^{Ta}\right)^{1-\eta_a} = n^{aa} \left(R_{t-1}^{Taa}\pi_t^{Taa}\right)^{1-\eta_a} + n^{ba} \left(R_{t-1}^{Tba}\pi_t^{Tba}\right)^{1-\eta_a} + n^{ca} \left(R_{t-1}^{Tca}\pi_t^{Tca}\right)^{1-\eta_a}$$

German tradable private consumption from Germany C_t^{Taa}:

$$C_t^{Taa} = n^{aa} \left(R_t^{Taa}\right)^{-\eta_a} C_t^{Ta}$$

German tradable private consumption from the rest of the euro area C_t^{Tba}:

$$C_t^{Tba} = n^{ba} \left(R_t^{Tba}\right)^{-\eta_a} C_t^{Ta}$$

German tradable private consumption from the rest of the world C_t^{Tca}:

$$C_t^{Tca} = n^{ca} \left(R_t^{Tca}\right)^{-\eta_a} C_t^{Ta}$$

German tradable private investment from Germany I_t^{Taa}:

$$I_t^{Taa} = n^{aa} \left(R_t^{Taa}\right)^{-\eta_a} I_t^{Ta}$$

German tradable private investment from the rest of the euro area I_t^{Tba}:

$$I_t^{Tba} = n^{ba} \left(R_t^{Tba}\right)^{-\eta_a} I_t^{Ta}$$

German tradable private investment from the rest of the world I_t^{Tca}:

$$I_t^{Tca} = n^{ca} \left(R_t^{Tca}\right)^{-\eta_a} I_t^{Ta}$$

German tradable government consumption from Germany G_t^{Taa}:

$$G_t^{Taa} = n^{aa} \left(R_t^{Taa}\right)^{-\eta_a} G_t^{Ta}$$

German tradable government consumption from the rest of the euro area G_t^{Tba}:

$$G_t^{Tba} = n^{ba} \left(R_t^{Tba}\right)^{-\eta_a} G_t^{Ta}$$

German tradable government consumption from the rest of the world G_t^{Tca}:

$$G_t^{Tca} = n^{ca} \left(R_t^{Tca}\right)^{-\eta_a} G_t^{Ta}$$

German tradable costs of capital utilization from Germany $\tilde{\Gamma}_t^{Taa}$:

$$\tilde{\Gamma}_t^{Taa} = n^{aa} \left(R_t^{Taa}\right)^{-\eta_a} \tilde{\Gamma}_t^{Ta}$$

German tradable costs of capital utilization from the rest of the euro area $\tilde{\Gamma}_t^{Tba}$:

$$\tilde{\Gamma}_t^{Tba} = n^{ba} \left(R_t^{Tba}\right)^{-\eta_a} \tilde{\Gamma}_t^{Ta}$$

German tradable costs of capital utilization from the rest of the world $\tilde{\Gamma}_t^{Tca}$:

$$\tilde{\Gamma}_t^{Tca} = n^{ca} \left(R_t^{Tca}\right)^{-\eta_a} \tilde{\Gamma}_t^{Ta}$$

Nominal exchange rate between Germany and the rest of the euro area ΔS_t^{ab}:

$$\Delta S_t^{ab} = 1$$

Real exchange rate between Germany and the rest of the euro area $\mathcal{E}_t^{ab}$:

$$\Delta S_t^{ab} = \frac{\mathcal{E}_t^{ab}}{\mathcal{E}_{t-1}^{ab}} \frac{\pi_t^a}{\pi_t^b}$$

Real exchange rate between Germany and the rest of the world $\mathcal{E}_t^{ca}$:

$$\Delta S_t^{ca} = \frac{\mathcal{E}_t^{ca}}{\mathcal{E}_{t-1}^{ca}} \frac{\pi_t^c}{\pi_t^a}$$

Policy rate of the ECB i_t^{mu}:

$$\ln\left(\frac{1+i_t^{mu}}{1+i^{mu}}\right) = \rho_{mu}^{i}\ln\left(\frac{1+i_{t-1}^{mu}}{1+i^{mu}}\right) + \left(1-\rho_{mu}^{i}\right)\phi_{mu}^{\pi}\left[\frac{\mathcal{P}_a}{\mathcal{P}_a+\mathcal{P}_b}\ln\left(\frac{\pi_t^a}{\pi^a}\right) + \frac{\mathcal{P}_b}{\mathcal{P}_a+\mathcal{P}_b}\ln\left(\frac{\pi_t^b}{\pi^b}\right)\right]$$
$$+\left(1-\rho_{mu}^{i}\right)\phi_{mu}^{y}\left[\frac{\mathcal{P}_a}{\mathcal{P}_a+\mathcal{P}_b}\ln\left(\frac{Y_t^a}{Y_{t-1}^a}\right) + \frac{\mathcal{P}_b}{\mathcal{P}_a+\mathcal{P}_b}\ln\left(\frac{Y_t^b}{Y_{t-1}^b}\right)\right] + \varepsilon_t^{i^{mu}}$$

Nominal interest rate in Germany i_t^a and in the rest of the euro area i_t^b:

$$\ln\left(\frac{1+i_t^{mu}}{1+i^{mu}}\right) = \frac{\mathcal{P}_a}{\mathcal{P}_a+\mathcal{P}_b}\ln\left(\frac{1+i_t^a}{1+i^a}\right) + \frac{\mathcal{P}_b}{\mathcal{P}_a+\mathcal{P}_b}\ln\left(\frac{1+i_t^b}{1+i^b}\right)$$

Nominal interest rate on bonds between Germany and the rest of the euro area i_t^{ab}:

$$1+i_t^{ab} = \left(1+i_t^b\right)\exp\left(-\omega^b\left[\mathcal{E}_t^{ab}b_t^{ab} - \mathcal{E}^{ab}b^{ab}\right] - \omega^s\left[E_t\Delta S_{t+1}^{ab}\Delta S_t^{ab} - \left(\Delta S^{ab}\right)^2\right]\right)$$

Nominal interest rate on bonds between Germany and the rest of the world i_t^{ca}:

$$1+i_t^{ca} = \left(1+i_t^a\right)\exp\left(-\omega^b\left[\mathcal{E}_t^{ca}b_t^{ca} - \mathcal{E}^{ca}b^{ca}\right] - \omega^s\left[E_t\Delta S_{t+1}^{ca}\Delta S_t^{ca} - \left(\Delta S^{ca}\right)^2\right]\right)$$

Bonds between Germany and the rest of the euro area b_t^{ab}:

$$\lambda_t^a = E_t\beta_a\lambda_{t+1}^a\frac{\left(1+i_t^{ab}\right)\Delta S_{t+1}^{ab}}{\pi_{t+1}^a}\exp\left(\varepsilon_t^{RP_{ab}}\right)$$

Nominal exchange rate between Germany and the rest of the world ΔS_t^{ca}:

$$\lambda_t^c = E_t\beta_c\lambda_{t+1}^c\frac{\left(1+i_t^{ca}\right)\Delta S_{t+1}^{ca}}{\pi_{t+1}^c}\exp\left(\varepsilon_t^{RP_{ca}}\right)$$

Bonds between Germany and the rest of the world b_t^{ca}:

$$Y_t^a + \mathcal{E}_t^{ab}\frac{1+i_{t-1}^{ab}}{\pi_t^b}\frac{\mathcal{P}_b}{\mathcal{P}_a}b_{t-1}^{ab} + b_t^{ca} = C_t^a + I_t^a + G_t^a + \tilde{\Gamma}_t^a + \mathcal{E}_t^{ab}\frac{\mathcal{P}_b}{\mathcal{P}_a}b_t^{ab} + \frac{1+i_{t-1}^{ca}}{\pi_t^a}b_{t-1}^{ca}$$

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