

CES ifo

**12987
2026**

September 2026

Working Papers

Optimal Taxation with Migration and Regional Externalities

Eren Güler

CES ifo

Imprint:

CESifo Working Papers

ISSN 2364-1428 (digital)

Publisher and distributor: Munich Society for the Promotion
of Economic Research - CESifo GmbH

Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740

Email office@cesifo.de
<https://www.cesifo.org>

Editor: Clemens Fuest

An electronic version of the paper may be downloaded free of charge

- from the CESifo website: www.ifo.de/en/cesifo/publications/cesifo-working-papers
- from the SSRN website: www.ssrn.com/index.cfm/en/cesifo/
- from the RePEc website: <https://ideas.repec.org/s/ces/ceswps.html>

Optimal Taxation with Migration and Regional Externalities

Eren Gürer^{*†}

September 12, 2026

Abstract

This paper studies optimal taxation when marginal contributions to an externality differ between urban and rural regions (*non-atmospheric externalities*), and individuals can migrate from rural to urban regions to earn higher wages. I show that when the government is constrained to set a uniform commodity tax across regions, the presence of externalities alters the structure of optimal redistribution. In particular, if, for example, urban residents impose higher marginal external damages, the optimal policy features more redistribution, relative to a benchmark without externalities. This additional redistribution reduces the attractiveness of higher urban wages and thus discourages migration into the urban region. These findings highlight a potential role for redistributive policy in internalizing externalities.

JEL Classification: H21, H23, R23

Keywords: taxation, redistribution, regional externality, migration

^{*}Middle East Technical University. ORCID ID: 0000-0001-8238-1967. E-mail: egurer@metu.edu.tr.

[†]I would like to thank Çiğdem Ekiz, Matthias Kalkuhl, Osman Küçükşen, Lennart Stern, Elena Del Rey and Ebru Voyvoda as well as the participants of 2026 Annual Congress of the International Institute of Public Finance and Spring 2026 Meeting of the Turkish Economic Association for the valuable comments and discussions. I gratefully acknowledge financial support from the European Union's Horizon Europe Programme through the Marie Skłodowska-Curie Actions (MSCA) project *Finance4Green* (Grant Agreement No. 101183099).

1 Introduction

Urbanization is associated with increased air pollution exposure (Borck and Schrauth, 2021; Carozzi and Roth, 2023; Borck and Schrauth, 2024), higher crime rates (Glaeser and Sacerdote, 1999) and greater congestion (Chang et al., 2017), which may reduce overall productivity (Zivin and Neidell, 2012) and place additional burdens on the fiscal system through increased health and safety expenditures. At the same time, urbanization gives rise to agglomeration forces that benefit firms and raise overall productivity by clustering economic activity, facilitating knowledge spillovers, improving worker–firm matching, and providing easier access to industry-specific services (see, e.g., Ciccone and Hall (1996), Paul and Siegel (1999) and Combes and Gobillon (2015)). Because individuals consider themselves “atomic”, neither the aggregate productivity gains nor the social costs arising from their location choices are internalized in their migration decisions.

The trade-off of urbanization can be cast within the framework of *non-atmospheric consumption externalities* with respect to region, meaning that the marginal contribution to the externality depends on the location in which an individual resides. In particular, living and working in an urban region requires, for example, consumption of urban housing and infrastructure. When the external costs of urbanization exceed the external benefits, urban consumption (or urbanization) contributes to a net negative externality; when benefits dominate, it generates a net positive externality. Although the relative strengths of these forces are ultimately an empirical matter,¹ the internalization of (net) externalities is typically a task assigned to tax policy.

The optimal taxation of non-atmospheric consumption externalities is studied theoretically in Micheletto (2008). However, despite its important implication, namely, the breakdown of the well-known “additivity property” of Sandmo (1975) (discussed further under Related Literature), practical applications of this framework are scarce.² This paper contributes by applying the concept of non-atmospheric externalities to a regional setting in order to derive concrete policy implications. In particular, it studies the design of optimal tax policy in an economy with non-atmospheric externalities across regions

¹See Ahlfeldt and Pietrostefani (2019) for a comparison of the benefits and costs of density.

²An exception is the literature examining the implications of relative consumption concerns for optimal tax policy (Aronsson and Johansson-Stenman, 2010; Eckerstorfer and Wendner, 2013; Wendner, 2014; Eckerstorfer, 2014).

and wage-increasing migration from the rural to the urban region.

For this purpose, I develop a model with two wage/productivity types, high- and low-type individuals, who may reside in one of two regions, urban or rural. Individuals choose labor supply at the intensive margin and determine their consumption levels. Consumption goods fall into two categories: externality-generating goods and normal goods. Externality-generating goods are best understood as region-specific goods that must be supplied from the location of residence, most notably housing, local infrastructure services (e.g., water and energy supply), and local transportation. The marginal contribution of consuming these goods to the aggregate externality differs across regions, and the resulting aggregate externality affects the utility of all individuals in the economy.

Individuals may be either mobile or immobile. Each region contains both high- and low-type individuals who are immobile and therefore remain in their respective regions. In addition, a mass of low-type individuals initially located in the rural region is mobile. These individuals may choose to incur a fixed utility cost to migrate to the urban region. Upon migrating, they become high-type with a certain probability; otherwise, they remain low-type. Hence, mobile individuals compare their expected utility in the urban region net of the fixed migration cost to the utility associated with remaining in the rural region as low-type individuals.

A utilitarian government maximizes social welfare. Its policy instruments are nonlinear income taxes and commodity taxes that are uniform across regions. This assumption of uniform commodity taxation is central to the main result of the paper and is discussed in detail below. Overall, the government's purpose is to design a tax system that simultaneously achieves optimal redistribution and efficient internalization of externalities while respecting its budgetary restrictions.

The main finding of the paper is that when commodity taxes cannot be differentiated across regions, and when migration can raise wages ("wage-increasing migration"), the level of optimal redistribution is shaped in a specific way by non-atmospheric externalities. Suppose that the marginal contribution of urban consumption to the negative externality is larger in the urban region than in the rural region. I show analytically for this scenario that the optimal uniform commodity tax rate on the externality-generating good lies below the level necessary to fully internalize the externalities generated by urban residents. As a result, a uniform commodity tax induces excessive migration to the urban region.

To counteract this excess migration/urban consumption, the government turns to income taxation. Migration occurs because individuals expect to earn higher wages in the urban region. By taxing high incomes more heavily and redistributing more (e.g., relative to a case with no externalities), the government can reduce the attractiveness of migrating and prevent excess inflows into the urban region. In effect, the government “bribes” mobile low-type individuals to remain in the rural region, in order to limit the external costs of urban consumption, by lowering the relative payoff from becoming high-type. Conversely, if urbanization (or urban consumption) generates a net positive externality, the logic reverses. In that case, the government optimally reduces the extent of redistribution in order to encourage additional migration to the urban region, expanding the positive spillovers associated with urban activity.

As mentioned earlier, the restriction to uniform commodity tax rates is crucial for the results above. According to the literature on the optimal taxation of non-atmospheric externalities ([Micheletto, 2008](#); [Aronsson and Sjögren, 2018](#)), the first-best strategy is to differentiate taxes across externality-generating units according to their marginal contributions to the externality. For instance, if the urban region contributes more at the margin to a negative externality than the rural region, the associated externality-generating good(s) should be taxed at a higher rate in the urban region.

While technically feasible, differential taxation of the same good across regions is often politically and administratively challenging. One option is a direct tax on individuals entering the urban region, but this is politically difficult to implement since it resembles restricting freedom of movement.

An alternative is to differentiate commodity taxes on housing, infrastructure and transportation across regions. However, this is administratively demanding for two reasons. First, it requires determining which goods to tax and at what rate in each region, a task complicated by the heterogeneity of externality profiles. Second, it would require centralizing the authority to set tax rates across regions; local governments, for example, would need to give up some of their existing taxation powers (such as over property taxes). Beyond these administrative hurdles, taxing identical goods at different rates across regions also violates the principle of horizontal equity (the idea that individuals in similar circumstances should be treated similarly), which many consider a core fairness criterion in tax design (see, e.g., [Musgrave and Musgrave \(1989, p. 223\)](#) or [Mirrlees](#)

(2011, p. 69–70)). Although externalities provide a theoretically valid justification for such departures, this may still face political resistance in practice. These political and administrative challenges make first-best regional tax differentiation difficult to achieve in practice.

Overall, this paper proposes progressive income taxation as an instrument for addressing regional externalities. Although it is second-best to regionally differentiated commodity taxes in an efficiency sense, it has the practical advantages of greater political acceptability, administrative simplicity, and consistency with horizontal and vertical equity.

Related literature. This paper contributes to the broader literature on the optimal taxation of externalities. In his seminal work, [Sandmo \(1975\)](#) shows that a Pigouvian tax on the externality-generating good is sufficient for effectively internalizing externalities. Other taxes need not be affected by externality considerations (the “additivity property”). This insight forms a starting point for the present study, which highlights a specific context in which additivity does not hold. A related strand of the literature examines the possibility of a double dividend from externality taxation ([Bovenberg and van der Ploeg, 1994](#); [Bovenberg and de Mooij, 1994](#); [Schneider, 1997](#); [Bovenberg and Van Der Ploeg, 1998](#)). While some results of this paper resonate with the double-dividend perspective, it is not a central aspect of this study.

More narrowly, the present study falls within the literature that examines the taxation of externalities alongside the problem of optimal redistribution à la [Mirrlees \(1971\)](#). This literature establishes that, even in the presence of pre-existing distortions from distortionary income taxation, the additivity property of [Sandmo \(1975\)](#) continues to hold ([Pirttilä and Tuomala, 1997](#); [Cremer et al., 1998](#)). In fact, this literature shows that, when consumption and labor supply are weakly separable and commodity taxes are restricted to be uniform, optimal externality taxes should be set at their Pigovian level; that is, they should equal the marginal external harm (or benefit) generated by the externality. Contemporary work in this field continues to investigate the structure of optimal externality taxes under various assumptions ([Jacobs and de Mooij, 2015](#); [Jacobs and van der Ploeg, 2019](#)), as well as their optimal levels ([Aigner, 2014](#)).

The studies discussed thus far, however, are confined to externalities of the atmospheric type, where each externality-generating unit contributes identically at the margin. Within the literature on optimal externality taxation and redistribution, this paper is most closely

related to studies that examine externalities of the non-atmospheric type. [Micheletto \(2008\)](#) shows that the additivity property no longer holds when the externality is non-atmospheric. In their helpful review, [Aronsson and Sjögren \(2018\)](#) similarly demonstrate, within a simpler framework, how the additivity property breaks down in the presence of non-atmospheric externalities. A series of papers analyzing the implications of relative consumption concerns for optimal commodity and income taxation can be viewed as studying optimal tax design under non-atmospheric consumption externalities ([Aronsson and Johansson-Stenman, 2010](#); [Eckerstorfer and Wendner, 2013](#); [Wendner, 2014](#); [Eckerstorfer, 2014](#)).

The additivity property of [Sandmo \(1975\)](#) is a cornerstone of optimal tax analyses of externalities and implicitly guides how policymakers design corrective taxes. Against this background, the result in [Micheletto \(2008\)](#), that additivity breaks down in the presence of non-atmospheric externalities, is important. While this theoretical insight forms a starting point, applying it in specific settings is necessary to generate concrete policy implications. To the best of my knowledge, apart from work on relative consumption concerns, there are no applied studies examining non-atmospheric externalities. This paper fills this gap by introducing non-atmospheric externalities into a regional context and examining the resulting implications for tax policy design.

The remainder of the paper is structured as follows. Section 2 introduces the model, describing individuals and the government. Section 3 presents the optimal policy rules. Section 4 provides illustrative simulations that reinforce the analytical results of Section 3. Section 5 concludes.

2 The Model

2.1 Individuals

2.1.1 Basics

There are individuals who may be of high (H) or low (L) wage/productivity types, and who can reside in either urban (U) or rural (R) areas. Individuals of type $n \in \{H, L\}$ earn a wage w_n per unit of labor supplied, irrespective of their location of residence. Each individual supplies labor l_n , generating gross labor income $y_n = w_n l_n$. The government imposes a nonlinear income tax $T(y_n)$ on labor income, leading to net income $z_n =$

$y_n - T(y_n)$. Individuals allocate their disposable income across three consumption goods: a normal good c_n , an externality-generating good q_n , and a numéraire good x_n . The externality-generating good q_n represents a bundle of region-specific goods (such as housing, local infrastructure and transportation) that must be sourced where individuals live. Thus, as the urban or rural population grows, total consumption of the region-specific bundle rises, amplifying the associated externality. The aggregate external effect generated by the externality-generating good is denoted by E , as defined more precisely later.

It is important to note that the same wage types residing in different regions (urban versus rural) are otherwise identical. Their labor supply (l_n), income levels (y_n, z_n), and, if prices do not differ across regions, consumption bundles (c_n, q_n, x_n) coincide.

Let a fraction α^M of individuals be born in the rural area but be mobile. Mobile individuals are heterogeneous with respect to their fixed utility cost of migration δ , which is distributed according to a probability density function $g(\delta)$ with support $[\underline{\delta}, \bar{\delta}]$ and corresponding cumulative distribution function $G(\delta)$.³ Mobile individuals can pay this fixed cost δ to migrate to the urban area. This approach for modeling migration decisions is simple yet standard. For instance, [Kessing et al. \(2020\)](#) model migration decisions, while [Jacquet et al. \(2013\)](#) model labor market participation decisions using a similar approach.

Let δ^T denote the threshold level of the fixed migration cost, whose expression is introduced in the next subsection. Mobile individuals with $\delta \leq \delta^T$ choose to migrate, while those with $\delta > \delta^T$ remain in the rural region. Consequently, $G(\delta^T)$ represents the fraction of mobile individuals who opt to migrate. The motivation for migrating from the rural area to the urban area is to earn higher wages, i.e., to become high-type. I assume that the probability of becoming high-type after moving to the urban area is β . Hence, a fraction β of individuals who migrate to the urban area become H -type, while the remaining fraction $(1 - \beta)$ remain L -type.⁴

³I assume that δ is separable from consumption and labor supply choices, meaning that, conditional on wage type, individuals make identical consumption and labor supply decisions regardless of their specific value of δ .

⁴An equivalent formulation would be to assume that all mobile individuals are initially born in the urban area, where residents must incur a fixed cost δ (e.g., extra housing costs or the psychological cost of living in a dense environment). These individuals could then move to rural areas, thereby avoiding the fixed cost δ but foregoing the opportunity to become high-types. This alternative formulation is inconsequential for our main results, since δ is a constant and does not affect the optimal policy rules.

Mobile individuals, who represent a fraction α^M of the population, can be further decomposed. This decomposition will be useful later when constructing the government's program: (i) $\alpha^M G(\delta^T)\beta$: the fraction who migrate from the rural to the urban area and become H -type; (ii) $\alpha^M G(\delta^T)(1 - \beta)$: the fraction who migrate from the rural to the urban area but remain L -type; and (iii) $\alpha^M(1 - G(\delta^T))$: the fraction who choose not to migrate despite being mobile.

In addition to mobile, there are immobile individuals. Let α_n^U and α_n^R , with $n \in \{H, L\}$, denote the fractions of immobile high- and low-type individuals residing respectively in the urban and rural areas. Overall, mobile and immobile individuals constitute the entire population, so that the sum of their fractions can be assumed to equal one:

$$\alpha^M + \sum_{n \in \{H, L\}} (\alpha_n^U + \alpha_n^R) = 1.$$

Finally, individual utility is given by

$$u^n(c_n, q_n, x_n, l_n, E) - m\delta \tag{1}$$

where m is an indicator variable that takes the value one if an individual is mobile and chooses to migrate, and zero otherwise. The utility function is increasing, $u_c^n > 0$, and concave, $u_{cc}^n < 0$, with respect to all three consumption goods.⁵ It also satisfies $u_l^n < 0$, $u_{ll}^n > 0$ and $u_E^n < 0$.

2.1.2 Utility Maximization

The utility maximization problem can be represented as a three-stage process for mobile individuals and a two-stage process for immobile individuals. In the first stage, mobile individuals choose their location of residence. In the second stage, they decide on their optimal labor supply and determine their gross labor income. Given the nonlinear income tax schedule set by the government, this decision pins down their net income. In the third and final stage, individuals allocate their net income among three consumption goods, taking prices and other relevant variables as given. Immobile individuals skip the first stage, as their location is fixed, but follow the same second and third stages.

I solve this problem by backward induction, with the solution steps outlined below, starting from the third stage, which is the first step of the solution procedure.

⁵Subscripts denote partial derivatives with respect to the corresponding variable.

Third stage. In this final stage of their optimization problem, individuals maximize utility (1) subject to the budget constraint $p_c c_n + p_q q_n + p_x x_n = z_n$, for a given net income z_n . Consumer prices are given by $p_c = P_c + \tau_c$, $p_q = P_q + \tau_q$, $p_x = P_x$ where P_c , P_q and P_x represent base prices, and τ_c and τ_q are the specific commodity taxes imposed by the government on the normal and externality-generating goods. The numeraire good, x , is untaxed. Solving this stage yields the indirect utility function $\mathcal{U}^n(p_c, p_q, p_x, z_n, l_n, E) - m\delta$, as well as the conditional demand functions $c_n(p_c, p_q, p_x, z_n, l_n, E)$, $q_n(p_c, p_q, p_x, z_n, l_n, E)$ and $x_n(p_c, p_q, p_x, z_n, l_n, E)$.

Second stage. Next, I solve for optimal gross income (or labor supply) by maximizing the indirect utility function derived above, $\mathcal{U}^n(p_c, p_q, p_x, y_n - T(y_n), \frac{y_n}{w_n}, E) - m\delta$, with respect to y_n where we substituted $z_n = y_n - T(y_n)$ and $l_n = \frac{y_n}{w_n}$. The first-order condition from this problem defines the marginal income tax rate faced by individuals:

$$T'(y_n) = 1 + \frac{\frac{\partial \mathcal{U}^n}{\partial l_n} \frac{1}{w_n}}{\frac{\partial \mathcal{U}^n}{\partial z_n}}. \quad (2)$$

Equation (2) can also be viewed as an expression implicitly defining optimal labor supply.

First stage. This stage applies only to mobile individuals. They decide whether to migrate by comparing the expected utility from migration with the utility from staying. Recall that after migrating, a mobile individual becomes an H -type with probability β and remains an L -type with probability $(1 - \beta)$. If the individual does not migrate, they remain an L -type. Hence, migration occurs if the following condition holds:

$$\beta(\mathcal{U}^H - \delta) + (1 - \beta)(\mathcal{U}^L - \delta) \geq \mathcal{U}^L$$

Rearranging yields the threshold level of the fixed migration cost:

$$\delta^T = \beta(\mathcal{U}^H - \mathcal{U}^L) \quad (3)$$

Mobile individuals with fixed migration costs below δ^T choose to migrate, while those with higher costs do not. Consequently, as noted in the previous subsection, $G(\delta^T = \beta(\mathcal{U}^H - \mathcal{U}^L))$ represents the fraction of mobile individuals who migrate.

We assume that individuals behave atomistically in their utility maximization. That is, they treat their own contribution to the externality E as negligible and do not internalize the external effects of their consumption or labor supply decisions.

2.1.3 Externality

Consumption of the externality-generating good q gives rise to an external effect. The total external effect depends on aggregate consumption of q in the urban, Q^U , and in the rural areas, Q^R . Accordingly, the total externality is defined as

$$\begin{aligned} E &= f(Q^U, Q^R), \quad \text{where} \\ Q^U &= \sum_{n \in \{H, L\}} \alpha_n^U q_n + \alpha^M G(\delta^T) [\beta q_H + (1 - \beta) q_L], \\ Q^R &= \sum_{n \in \{H, L\}} \alpha_n^R q_n + \alpha^M (1 - G(\delta^T)) q_L. \end{aligned} \tag{4}$$

Let the marginal effect of q consumption on the externality be defined as $\partial f / \partial Q^U = f_u$ in the urban area and $\partial f / \partial Q^R = f_r$ in the rural area. The externality is said to be atmospheric with respect to location if the marginal contributions in the urban and rural regions are identical, that is, $f_u = f_r$. In contrast, non-atmospheric externalities or location-dependent externality generation arise when $f_u \neq f_r$, implying that the marginal external effect depends on the location of residence. In this case, a unit of q consumption contributes to the externality differently across regions.

To illustrate, suppose urbanization generates a net negative externality due to congestion or pollution. Then, consuming the region-specific bundle q in the urban area contributes more to the aggregate external cost than consuming the same bundle in the rural area, implying $f_u > f_r$.

Note also that a positive f_u or f_r corresponds to a negative externality, whereas a negative f_u or f_r corresponds to a positive externality, given that $u_E^n < 0$.

2.2 Government

The government is utilitarian and maximizes the sum of individual utilities. The government's objective function is given by:

$$\sum_{n \in \{H, L\}} (\alpha_n^U + \alpha_n^R) \mathcal{U}^n + \alpha^M \left\{ \beta \int_{\underline{\delta}}^{\delta^T} (\mathcal{U}^H - \delta) g(\delta) d\delta + (1 - \beta) \int_{\underline{\delta}}^{\delta^T} (\mathcal{U}^L - \delta) g(\delta) d\delta + (1 - G(\delta^T)) \mathcal{U}^L \right\}. \tag{5}$$

Equation (5) has two components. The summation term aggregates the utilities of immobile individuals, weighted by their population shares. The expression in curly brackets captures the contribution of mobile individuals. As described earlier, a fraction

β of individuals who choose to migrate become H -type and the remaining fraction $1 - \beta$ stay L -type. Accordingly, the first integral reflects the utility of those who migrate and become H -type, net of the fixed migration cost, up to the migration threshold δ^T . In a similar fashion, the second integral represents the utilities of those who migrate but remain L -type. The final term within curly brackets represents the utilities of mobile individuals who choose not to migrate and thus remain L -type.

The government must satisfy the following budget constraint:

$$\sum_{n \in \{H, L\}} (\alpha_n^U + \alpha_n^R)(y_n - z_n) + \alpha^M \left\{ G(\delta^T) \beta (y_H - z_H) + G(\delta^T) (1 - \beta) (y_L - z_L) + (1 - G(\delta^T)) (y_L - z_L) \right\} + \tau_q (Q^U + Q^R) + \tau_c (C^U + C^R) = 0. \quad (6)$$

The first term represents income tax revenue from immobile individuals. The second term with curly brackets accounts for income tax revenue from mobile individuals. As in the social welfare expression, the three terms within curly brackets respectively capture the income tax revenue from mobile individuals who migrate and become H -type, those who migrate but remain L -type, and those who do not migrate despite being mobile and therefore remain L -type. The final two terms in the budget constraint collect commodity tax revenue from aggregate urban and rural q and c consumption as defined in (4).⁶

There is asymmetric information between the government and individuals: each individual knows their own wage type, but the government does not. Therefore, the optimal allocation must satisfy the standard incentive compatibility constraint:

$$\mathcal{U}^H \left(p_c, p_q, p_x, z_H, \frac{y_H}{w_H}, E \right) \geq \hat{\mathcal{U}} \left(p_c, p_q, p_x, z_L, \frac{y_L}{w_H}, E \right) \quad (7)$$

where the right-hand side denotes the utility a high-type individual would obtain by mimicking a low-type. Note that the introduction of fixed migration costs does not modify the classical incentive compatibility condition. A mobile individual who migrates and becomes H -type receives utility $\mathcal{U}^H - \delta$ when behaving truthfully and $\hat{\mathcal{U}} - \delta$ when mimicking, so the fixed cost δ cancels from the inequality in (7).

The government can use the commodity tax rate τ_q to exercise perfect control over aggregate q consumption and therefore over the total external effect E . Hence, the expression for E in (4) can be treated as an artificial constraint, allowing E to be taken as a choice variable in the government's problem.

⁶ C^U and C^R are defined analogously to Q^U and Q^R in (4).

Notice also that the government can use the commodity tax instruments τ_q and τ_c to fully control the prices p_q and p_c (the numeraire good x has a fixed price p_x). As standard in the Mirrleesian tradition, the government solves for optimal nonlinear taxes by choosing allocations z_n and y_n for each wage type. Furthermore, as discussed above, the government can also perfectly determine the total external effect E . Consequently, it can fully control the indirect utilities of both types and, in turn, the migration threshold in (3). I therefore treat δ^T as a choice variable and impose the migration-threshold expression (3) as another artificial constraint in the government's program.

3 Optimal Policy Rules

Before presenting the results, it is useful to clarify notation. Subscripts on indirect utilities denote partial derivatives; for instance, $\mathcal{U}_x^n$ is the partial derivative of the indirect utility of type n with respect to variable x . As mentioned earlier, marginal contributions to externality are written as $\frac{\partial f}{\partial Q^U} = f_u$ for urban residents and $\frac{\partial f}{\partial Q^R} = f_r$ for rural residents. For ease of exposition, we assume the externality is negative, so $f_u > 0$, $f_r > 0$ with $u_E^n < 0$. The results can be reinterpreted straightforwardly for positive externalities (i.e., for $f_u < 0$ and $f_r < 0$), as will be done in the illustrative simulations.

The Lagrangian of the government's optimization problem is constructed from the objective function (5) together with constraints (3), (4), (6), and (7). The full Lagrangian and the corresponding first-order conditions are provided in Appendix. The Lagrange multipliers associated with each constraint are denoted as follows: (i) λ : budget constraint (6), (ii) φ : incentive compatibility constraint (7), (iii) κ : migration-threshold condition (3), (iv) μ : externality condition (4).

Assumption 1. *The utility function in (1) is assumed to be additively separable with log subutility in consumption goods. That is:*

$$u^n(c_n, q_n, x_n, l_n, E) = \gamma_c \log(c_n) + \gamma_q \log(q_n) + \gamma_x \log(x_n) - \frac{l_n^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} - E \quad (8)$$

with $\gamma_c + \gamma_q + \gamma_x = 1$. The term l denotes labor, and E is the externality defined in (4).

It is in principle possible to derive analytical expressions for optimal policy instruments without imposing a specific functional form for preferences. Indeed, general optimal commodity and marginal income taxation formulas under non-atmospheric externalities

and without particular utility restrictions are derived in, among others, [Micheletto \(2008\)](#), [Eckerstorfer and Wendner \(2013\)](#), and [Aronsson and Sjögren \(2018\)](#).

The purpose of the present study, however, is not to provide general policy rules, but rather to highlight the specific mechanism through which optimal income taxation is influenced by region-specific externality generation and wage-increasing migration. The utility function introduced in Assumption 1 yields compensated (Hicksian) demand responses to price changes that are proportional to the level of demand. This property considerably simplifies the optimal policy formulas, allowing the focus to remain on the main mechanisms of interest. Nonetheless, the mechanism identified here would carry over to a more general framework that does not impose any specific functional form.

Proposition 1. *The marginal social value of a reduction in the externality, expressed in terms of public funds (or the shadow value of marginally relaxing the externality constraint), reads:*

$$\frac{\mu}{\lambda} = \sum_{n \in \{H, L\}} a_n MW P_{E,z}^n,$$

where a_n denotes the aggregate population share of type- n individuals, and $MW P_{E,z}^n = -\mathcal{U}_E^n / \mathcal{U}_z^n$ represents the marginal willingness to pay of a type- n individual for a reduction in externality. ■

See Appendix for the proof.

The result presented in Proposition 1 is intuitive. The value society assigns to a reduction in the externality, μ/λ , equals the aggregate willingness to pay for such a reduction. When the externality is negative, individuals' willingness to pay for its reduction is positive, implying $\mu/\lambda > 0$.

One can examine the optimal policy rules for the presence of μ/λ to determine whether a particular instrument is influenced by externality considerations. If a policy rule includes the term μ/λ , it indicates that the instrument is affected by externality considerations; conversely, the absence of this term implies that externalities do not play a role in that instrument's optimal rule.

Proposition 2.

- i. *The optimal commodity tax rate on the externality-generating good equals a weighted average of the marginal externality contributions in the urban and rural areas,*

multiplied by the marginal social value of externality reduction:

$$\tau_q = \frac{\mu}{\lambda}(f_u\theta_u + f_r\theta_r)$$

with $\theta_u = Q^U/(Q^U + Q^R)$, $\theta_r = Q^R/(Q^U + Q^R)$ implying $\theta_u + \theta_r = 1$. Here, the weights θ_u and θ_r represent the shares of the urban and rural regions in aggregate q consumption.

- ii. *The optimal commodity tax rate on the clean good is unaffected by externality considerations, i.e., $\tau_c = 0$.* ■

See Appendix for the proof.

Intuitively, the efficient tax rate that internalizes the externality equals the shadow price of the externality (as defined in Proposition 1) multiplied by the marginal contribution of consumption to the externality. Hence, the “fair” tax rate on urban q consumption is $\frac{\mu}{\lambda}f_u$, while the “fair” tax rate on rural q consumption is $\frac{\mu}{\lambda}f_r$.

The government, however, is restricted to imposing a uniform commodity tax rate across regions. It therefore sets a single tax rate on commodity q that reflects a weighted average of the region-specific fair tax rates, where the weights correspond to each region’s contribution to aggregate consumption of the externality-generating good, q . To illustrate, consider an extreme case in which the rural region consumes no q , i.e., $\theta_r = 0$. In this situation, the fair tax rate for the rural region is irrelevant for determining the optimal τ_q , since all externality-generating consumption occurs in the urban region.

In the more general case where both regions consume q , but the urban region accounts for a larger share of total q consumption (i.e., $\theta_u > \theta_r$), the government assigns a greater weight to the urban fair tax rate. As a result, the chosen uniform tax rate τ_q is closer to the urban region’s fair rate.

When the government is restricted to applying a uniform commodity tax rates, it cannot fully internalize differing marginal externality generations across regions. As a result, the policy generally leads to either under- or overconsumption of the externality-generating good. To illustrate, suppose the externality is negative and that the marginal contribution is higher in the urban region. In this case, $f_u > f_r$, which implies

$$\frac{\mu}{\lambda}f_u > \frac{\mu}{\lambda}(f_u\theta_u + f_r\theta_r) > \frac{\mu}{\lambda}f_r,$$

where the middle term is the optimal τ_q . Hence, the optimal commodity tax rate on q lies between the fair rates for the two regions: it is higher than the rural fair rate and lower than the urban fair rate. As a consequence, aggregate q consumption in the urban region remains above its efficient level, while rural consumption remains below its efficient level. The commodity tax alone therefore does not fully internalize the externality. This pattern mirrors the classic findings on optimal commodity taxation under non-atmospheric externalities (see, e.g., [Micheletto \(2008\)](#) or [Aronsson and Sjögren \(2018\)](#)) in a regional context. Accordingly, the government has an incentive to use its other instruments, namely, income taxes, to mitigate the overconsumption of q in urban areas and the underconsumption in rural areas.

In cases where the commodity tax on the externality-generating good cannot fully internalize the externality, as in our setting, the tax on the normal good can in principle be used to correct under- or overconsumption in rural and urban regions by exploiting the normal good's substitutability or complementarity with the externality-generating good. For example, if q and c are substitutes, the government could reduce demand for q by subsidizing c through a negative τ_c .

However, under Assumption 1, compensated demand responses to price changes are proportional, meaning that the demand for q in both regions reacts to changes in τ_c at the same rate. As a result, τ_c cannot be used to differentially affect q consumption across regions, which is necessary to address region-specific under- or overconsumption. Consequently, introducing a tax on the normal good would generate additional consumption distortions without achieving the desired reallocation. Therefore, in the optimum, the government sets $\tau_c = 0$.

Proposition 3. *The optimal marginal income tax rates, in addition to managing the equity–efficiency trade-off of redistribution, also refine aggregate consumption of the externality-generating good q across urban and rural regions by adjusting individual consumption levels:*

$$T'(y_H) = MRS_{y,z}^H \frac{\partial q_H}{\partial z_H} \left[\left(\tau_q - \frac{\mu}{\lambda} f_u \right) H_u + \left(\tau_q - \frac{\mu}{\lambda} f_r \right) H_r \right],$$

$$T'(y_L) = \frac{\varphi}{\lambda a_L} \hat{U}_z(\widehat{MRS}_{y,z} - MRS_{y,z}^L) + MRS_{y,z}^L \frac{\partial q_L}{\partial z_L} \left[\left(\tau_q - \frac{\mu}{\lambda} f_u \right) L_u + \left(\tau_q - \frac{\mu}{\lambda} f_r \right) L_r \right],$$

where $MRS_{y,z}^n = \frac{u_{l \frac{1}{w_n}}^n}{u_z^n}$ for $n \in \{H, L\}$ and $\widehat{MRS}_{y,z} = \frac{\hat{u}_{l \frac{1}{w_H}}}{\hat{u}_z}$ denote the marginal rates of

substitution between gross and net income for truthful type- n and mimicking individuals, respectively. The term a_L represents the aggregate share of low-type individuals in the population, while H_i and L_i for $i \in \{U, R\}$ denote the shares of high- and low-type individuals residing in region i , respectively, with $H_u + H_r = 1$ and $L_u + L_r = 1$. ■

See Appendix for the proof.

In a standard two-type Mirrleesian optimal income tax problem without externalities, we would have $T'(y_H) = 0$, while $T'(y_L)$ would coincide with the first term on the right-hand side of the expression for $T'(y_L)$ in Proposition 3. This is the well-known result of [Stiglitz \(1982\)](#) concerning the equity–efficiency trade-off in redistribution and is not the focus of this paper. The additional terms in the optimal tax formulas in Proposition 3 arise entirely from externality considerations.

In our setting, marginal income tax rates serve an additional purpose: adjusting the q consumption of *individuals who have already chosen their region of residence*. Consider the expression for $T'(y_H)$. Marginal income taxes directly affect gross income y_H , and indirectly net income z_H . The term $MRS_{y,z}^H(\partial q_H / \partial z_H)$ captures how a change in $T'(y_H)$ feeds into high-type individuals' q_H consumption via its effect on their net income z_H .

Maintaining the assumption that $f_u > f_r$ as above, it follows that $\frac{\mu}{\lambda} f_u > \tau_q > \frac{\mu}{\lambda} f_r$. The bracketed term in Proposition 3 then adjusts $T'(y_H)$ according to the distribution of high-type individuals across regions. If most high types reside in the urban region (so $H_u \gg H_r$), the bracketed term tends to raise $T'(y_H)$ in order to curb their q consumption and offset the urban overconsumption. Conversely, if high types are predominantly rural, $T'(y_H)$ is reduced to increase rural q consumption toward its efficient level.

The interpretation of the additional term in the expression for $T'(y_L)$ is analogous.

Proposition 4. *The marginal social value of an increase in the migration threshold, expressed in terms of public funds (or the shadow value of tightening the migration threshold constraint), is influenced by externality considerations:*

$$-\frac{\kappa}{\lambda} = \Pi + \frac{\partial Q^U}{\partial \delta^T} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) + \frac{\partial Q^R}{\partial \delta^T} \left(\tau_q - \frac{\mu}{\lambda} f_r \right),$$

where Π denotes the income tax revenue effect of a marginal increase in the migration threshold, and $\partial Q^U / \partial \delta^T > 0$ and $\partial Q^R / \partial \delta^T < 0$. ■

See Appendix for the proof.

Next, we turn to net and average taxes, for which, unfortunately, obtaining analytical expressions is not possible. Nevertheless, the shadow value of tightening the migration threshold constraint in Proposition 4 provides economic intuition regarding their roles. Beyond its effect on income tax revenue, a marginal change in δ^T alters urban and rural q consumption by shifting the mass of mobile individuals who choose to migrate. An increase in δ^T raises urban q consumption by inducing more mobile individuals to locate in the urban region, and decreases rural q consumption. Hence, $\partial Q^U / \partial \delta^T > 0$ and $\partial Q^R / \partial \delta^T < 0$.

Maintaining the assumption of a negative externality and the ordering $\frac{\mu}{\lambda} f_u > \tau_q > \frac{\mu}{\lambda} f_r$ once again, the last two terms in the expression for $-\kappa/\lambda$ are negative. They therefore negatively contribute to the shadow value of tightening the constraint (the welfare effect, in terms of public funds, of marginally increasing the migration threshold).

Our main result follows directly from Proposition 4.

Corollary 1. (Main Result) *The optimal migration threshold defined in (3) depends on the indirect utility difference ($\mathcal{U}^H - \mathcal{U}^L$), which in turn is determined by the optimal allocations $\{y_n, z_n\}_{n \in \{H, L\}}$, that is, by net taxes $T(y_n)$ and average taxes $T(y_n)/y_n$. Because the shadow value of a marginal change in the migration threshold depends on externality considerations, as illustrated in Proposition 4, it follows that the optimal policy rules for net and average taxes are themselves influenced by externalities: they exploit the migration margin to adjust aggregate q consumption across the urban and rural regions. ■*

In order to understand the specific policy implications of Proposition 4 and Corollary 1, consider the following thought experiment. Suppose first that there is no externality, i.e., $f_u = f_r = 0$, which then straightforwardly implies $\tau_q = 0$. In this case, the last two terms in the expression for $-\kappa/\lambda$ (in Proposition 4) are zero.

Now suppose there is a non-atmospheric externality. That is, under the assumptions of a negative externality and $f_u > f_r$, we have $\frac{\mu}{\lambda} f_u > \tau_q > \frac{\mu}{\lambda} f_r$. Compared to the case of no externality, this negative externality reduces the shadow value of a marginal increase in the migration threshold, ceteris paribus, as mentioned above. Consequently, net and average taxes should adjust $(\mathcal{U}^H - \mathcal{U}^L)$ in a way that lowers δ^T , reducing the share of mobile individuals migrating to the urban region. This requires stronger redistribution to decrease $\delta^T = \beta(\mathcal{U}^H - \mathcal{U}^L)$. As fewer individuals migrate, aggregate urban q consumption, where

the marginal contribution to the externality is higher, falls, while rural q consumption rises. If, instead, the marginal contribution to the negative externality were higher in the rural region, the direction of these adjustments would reverse.

The logic is symmetric for positive externalities. Suppose $f_u < f_r \leq 0$, so that the marginal contribution to the positive externality is larger in the urban region. Then the government subsidy to q is below its fair level for urban residents but above the fair level for rural residents: $-\frac{\mu}{\lambda}f_u > -\tau_q > -\frac{\mu}{\lambda}f_r$. In this case, the last two terms in $-\kappa/\lambda$ increase the shadow value of raising the migration threshold. Accordingly, optimal policy calls for weaker redistribution in order to raise $(\mathcal{U}^H - \mathcal{U}^L)$ and δ^T . More mobile individuals would migrate to the urban region, helping correct the underconsumption of q there and amplify the positive externalities, while rural q consumption would decrease.

Finally, it is important to emphasize that net and average taxes directly determine individual indirect utilities and therefore the migration threshold. While one might expect marginal income taxes to serve a similar purpose, note that they primarily affect labor supply responses conditional on the individual's chosen location. Thus, when the government can freely differentiate between net (or average) taxes and marginal income taxes, the latter are not effective instruments for shifting utility levels. Instead, the government can directly manipulate the utilities of different wage types by adjusting the net or average taxes they face.

The next section employs simple simulation exercises to illustrate how regional non-atmospheric externalities influence optimal net and average taxes, i.e., optimal redistribution and migration.

4 Illustrative Simulations

As is well known, obtaining comparative statics results in Mirrleesian models such as the one studied here is generally not possible. Thus, it is useful to reinforce the analytical findings through illustrative numerical simulations. The goal of this section is therefore not to determine realistic tax rates that can be directly recommended for policy implementation, but rather to numerically demonstrate the mechanism highlighted in and around Corollary 1. Accordingly, we adopt a reasonable parametrization and examine the direction of the changes in policy instruments across various externality specifications.

4.1 Simulation Specifications

To ensure consistency between the analytical derivations and numerical simulations, individuals' preferences are assumed to be those specified under Assumption 1. Following [Jacquet et al. \(2013\)](#), the fixed utility cost of migration is distributed according to a logistic distribution,

$$G(\delta) = \frac{e^{-\omega + \rho\delta}}{1 + e^{-\omega + \rho\delta}}. \quad (9)$$

The externality is generated as

$$E = f(Q^U, Q^R) = \psi_u Q^U + \psi_r Q^R, \quad (10)$$

which implies marginal external effects $\partial f / \partial Q^U = f_u = \psi_u$ and $\partial f / \partial Q^R = f_r = \psi_r$.

4.2 Parametrization

High- and low-type individuals earn wage rates of $w_L = 3$ and $w_H = 8$, respectively. We assume the following expenditure shares: $\gamma_q = 0.5$ for the externality-generating good (the bundle of region-specific goods), $\gamma_c = 0.49$ for the non-numéraire normal good, and $\gamma_x = 0.01$ for the numéraire normal good. That is, if prices were identical, individuals would allocate half of their expenditure to the externality-generating good and the remaining half to normal goods. The Frisch elasticity of labor supply is set to a standard value of $\epsilon = 0.33$ ([Chetty, 2012](#)). All baseline prices are normalized to unity, such that $P_q = P_c = P_x = 1$.

Half of the population is immobile. This group is equally divided among urban high, urban low, rural high, and rural low types, such that $\alpha_H^U = \alpha_L^U = \alpha_H^R = \alpha_L^R = 0.125$. The remaining half of the population is mobile, i.e., $\alpha^M = 0.5$. Among individuals who migrate from the rural to the urban region, half become high-type, implying $\beta = 0.5$.

Table 1: Parametrization

Parameter	Value	Description
w_H, w_L	8, 3	Imposed
α_H^U, α_L^U	0.125, 0.125	Imposed
α_H^R, α_L^R	0.125, 0.125	Imposed
α^M	0.5	Imposed
β	0.5	Imposed
$\gamma_q, \gamma_c, \gamma_x$	0.50, 0.49, 0.01	Imposed
ϵ	0.33	Imposed
P_q, P_c, P_x	1, 1, 1	Imposed
ω, ρ	1.062, 2.79	Calibrated

Note: Parameters (ω, ρ) are calibrated in the benchmark economy with no externalities ($\psi_u = \psi_r = 0$), whose results are presented in in panel A of Table 2.

The parameters of the fixed migration cost CDF in equation (9), namely (ω, ρ) , are calibrated in a benchmark Mirrleesian optimal income tax economy. We feed in all parameters described above and assume that there are no externalities, i.e., $\psi_u = \psi_r = 0$, since this forms our benchmark environment. We choose (ω, ρ) such that (i) 30% of mobile individuals choose to migrate to the urban region, i.e. $G(\delta^T) = 0.30$, and (ii) the migration elasticity with respect to net income differences between high and low types equals 0.6, i.e., $\frac{dG(\delta^T)}{dz_H} \frac{z_H - z_L}{G(\delta^T)} = 0.6$. This calibration yields $\omega = 1.062$ and $\rho = 2.79$.

It may be helpful to emphasize once again that the economy with $\psi_u = \psi_r = 0$, the results of which are provided in panel A of Table 2, serves both as (i) the environment in which (ω, ρ) are calibrated and (ii) the benchmark economy against which optimal policies in the presence of externalities are compared.

Table 1 summarizes the full parametrization.

4.3 Results

Using the baseline parameters in Table 1, I numerically simulate the optimal policies under four externality scenarios. The results are reported in Table 2.

Panel A considers a benchmark case without externalities ($\psi_u = 0, \psi_r = 0$). The remaining panels examine non-atmospheric externalities. Panel B ($\psi_u = 0.5, \psi_r = 0$)

considers a scenario in which *urban* residents' consumption of good q generates a negative externality for everyone, while rural q consumption has no external effects. Panel C ($\psi_u = 0$, $\psi_r = 0.384$) mirrors this case, but with *rural* residents' consumption generating a negative externality and urban consumption remaining externality-free. Panel D ($\psi_u = -0.1$, $\psi_r = 0$) examines a setting in which urban residents' consumption of good q generates a positive externality, whereas rural consumption produces no external effects.

Table 2: Numerical results

Panel A [$\psi_u = 0$, $\psi_r = 0$]		Panel B [$\psi_u = 0.5$, $\psi_r = 0$]	
τ_q, τ_c	0.00, 0.00	τ_q, τ_c	1.18, 0.00
$T(y_L), T(y_H)$	-1.41, 2.93	$T(y_L), T(y_H)$	-2.90, 1.27
$T(y_L)/y_L, T(y_H)/y_H$	-0.53, 0.33	$T(y_L)/y_L, T(y_H)/y_H$	-1.22, 0.16
$T'(y_L), T'(y_H)$	0.03, 0.00	$T'(y_L), T'(y_H)$	0.14, 0.12
Q^U, Q^R	1.00, 1.34	Q^U, Q^R	0.54, 0.78
$G(\delta^T)$	0.300	$G(\delta^T)$	0.283
Panel C [$\psi_u = 0$, $\psi_r = 0.384$]		Panel D [$\psi_u = -0.1$, $\psi_r = 0$]	
τ_q, τ_c	1.18, 0.00	τ_q, τ_c	-0.20, 0.00
$T(y_L), T(y_H)$	-2.21, -0.62	$T(y_L), T(y_H)$	-0.67, 2.94
$T(y_L)/y_L, T(y_H)/y_H$	-0.88, -0.08	$T(y_L)/y_L, T(y_H)/y_H$	-0.24, 0.33
$T'(y_L), T'(y_H)$	0.07, -0.09	$T'(y_L), T'(y_H)$	0.04, -0.05
Q^U, Q^R	0.69, 0.71	Q^U, Q^R	1.28, 1.45
$G(\delta^T)$	0.401	$G(\delta^T)$	0.359

Note: All parameters are identical across panels (and given in Table 1), except for the marginal external effects in each region, (ψ_u, ψ_r) . Positive values of ψ_u and ψ_r indicate negative externalities, while negative values indicate positive externalities.

First, recall the well-known result of [Atkinson and Stiglitz \(1976\)](#), which shows that when nonlinear income taxes are optimally chosen, preferences are identical, and consumption is separable from labor supply, uniform commodity taxation is optimal. Consequently, in the presence of an untaxed numéraire good and no externalities, as in Panel A, the optimal commodity tax rates satisfy $\tau_q = \tau_c = 0$.

When consumption of good q generates negative externalities (Panels B and C), the optimal policy features a positive tax on q , i.e. $\tau_q > 0$. Recall from the discussion around Proposition 2 that, under non-atmospheric externalities, a uniform commodity tax on the

externality-generating good cannot fully internalize the externality. In particular, when urban residents contribute more to the marginal external damage (Panel B), the optimal uniform τ_q induces over-consumption of q in the urban region and under-consumption of q in the rural region. The opposite pattern arises when rural residents have the higher marginal external contribution (Panel C).

Conversely, when good q generates positive externalities (Panel D), the optimal τ_q becomes negative, implying a subsidy on q -consumption. In this case, as also discussed around Proposition 2, a uniform τ_q causes under-consumption of q in the urban region and over-consumption of q in the rural region.

Recall once again from the discussions around Proposition 2 that, when preferences exhibit proportional compensated demand responses, as imposed in Assumption 1, the optimal τ_c remains zero even in the presence of non-atmospheric externalities.

Equipped with the intuition regarding regional over- and under-consumption of q , we now turn to the issue of optimal redistribution, which constitutes our main result. It is helpful to start by comparing the net and average taxes in panels A and B. As is evident, the government implements higher net and average transfers to L -type individuals when urban residents generate a negative externality compared to the case with no externality. The intuition follows directly from Corollary 1. Since individuals' motivation for migrating to the urban area is the prospect of becoming H -type, the government attempts to reduce the relative attractiveness of becoming H -type by making L -types better off. The purpose of this policy is to correct the over-consumption of q in the urban region (i.e., to reduce external costs) and under-consumption of q in the rural region. Put differently, the higher transfers to L -types function as a "bribe" discouraging mobile workers from migrating to the urban region in the hope of becoming H -type.

Note, however, that the net and average taxes on H -types decline. Although this may seem at odds with the preceding intuition, it reflects the so-called double dividend of externality taxation. Because the government raises additional revenue by taxing the externality-generating good, it can finance higher transfers to L -types while imposing lower taxes on H -types in panel B relative to panel A. In the end, the fraction of mobile workers migrating to the urban region, $G(\delta^T)$ falls in panel B relative to panel A.

Comparing panels B and C reinforces the main result of this study more clearly. In panel B, $\psi_u = 0.5$ is set arbitrarily. In panel C, on the other hand, $\psi_r = 0.384$ is chosen

deliberately so that τ_q in panel C matches that of panel B. The advantage of holding τ_q constant across panels B and C is that the levels of net and average taxes, and thus the degree of redistribution, are directly comparable, since the tax revenue from commodity taxation is approximately the same.

The government redistributes substantially less in panel C relative to panel B. Not only are transfers to L -types much lower in panel C, but the government also provides transfers (rather than taxes) to H -types in panel C, something that does not occur in panel B. The intuition once again follows from Corollary 1. Unlike panel B, rural residents are the only contributors to the negative externality in panel C. Hence, the government seeks to increase the attractiveness of becoming an H -type by reducing redistribution, strengthening incentives to migrate to the urban region so as to correct the over- (under-) consumption of q in the rural (urban) region. Put differently, in panel C the government effectively “bribes” individuals to migrate to the urban region in order to mitigate the external costs arising from rural q -consumption.

Consequently, the fraction of mobile workers migrating to the urban region is higher in panel C than in panel B. Notice also that Q^U is higher in panel C relative to panel B, whereas Q^R is lower. This reflects the government’s attempt to reduce q -consumption in the urban region in panel B, and to reduce q -consumption in the rural region in panel C.

In panel D, urban q -consumption generates a positive externality. Thus, the government seeks to encourage greater q -consumption in the urban region in order to correct its under-consumption and exploit its positive external effects. Thus, transfers to L -types are lower than the benchmark, panel A. At the same time, taxes on H -types are almost exactly the same as in panel A. This implies that the government, relative to benchmark, finances the nation-wide subsidies on good q entirely by a reduction in the transfers to L -types. Consequently, the fraction of mobile workers migrating to the urban region is higher in panel D relative to panels A.

Marginal income taxes follow the intuition developed in and around Proposition 3. With the parametrization reported in Table 1, a larger share of high-types resides in the urban region: immobile high-types are evenly split across regions, while some mobile individuals migrate to the urban region and become H -type. By contrast, most L -type workers reside in the rural region: although immobile low-types are also evenly split, the majority of mobile L -types in the rural region choose not to migrate.

When urban residents' q -consumption generates negative marginal external effects (Panel B), the government seeks to discourage their consumption in order to correct for over-consumption. If the majority of the H -types reside in the urban region, this can partly be achieved through a higher $T'(H)$, which is exactly what we observe in Panel B relative to Panel A. By the same logic, one would expect a reduction in $T'(L)$ in Panel B relative to Panel A. However, note that transfers to L -types are substantially larger in Panel B than in Panel A. To deter mimicking, the traditional Stiglitz term (the first term in the expression for $T'(L)$ in Proposition 3) rises, and this effect dominates, resulting in an increase in $T'(L)$ in Panel B relative to Panel A.

The changes in optimal marginal tax rates in Panels C and D relative to Panel A can be interpreted similarly. In both cases, the government aims to encourage q -consumption in the urban region and discourage it in the rural region. When the majority of H -types reside in the urban region, this can partly be achieved through a lower $T'(H)$. Analogously, a higher $T'(L)$ can be used to reduce rural q -consumption if the majority of the L -types reside in the rural region.

5 Conclusion

Urbanization generates both positive and negative externalities: it brings agglomeration benefits that raise aggregate productivity, yet it also entails social costs such as increased exposure to air pollution, crime, and congestion. Regardless of which force dominates, this environment gives rise to a non-atmospheric externality, meaning that the marginal contribution of the urban region to the net externality, whether positive or negative, differs than that of the rural region (*non-atmospheric externalities*). This paper examines the design of tax policy that internalizes these regional externalities while simultaneously achieving optimal redistribution.

For this purpose, I develop a model with two types (high and low) and two regions (urban and rural). Individuals supply labor and choose their consumption of a normal good and an externality-generating good. A mass of immobile individuals are fixed in their regions of residence. In addition, a mass of mobile individuals, initially low-type and located in the rural region, decide whether to migrate to the urban region by incurring a fixed utility cost. Conditional on migration, they become high-type with some probability.

A utilitarian government maximizes social welfare by optimally choosing nonlinear income taxes and commodity taxes that are constrained to be uniform across regions.

This paper shows that non-atmospheric externalities affect optimal redistribution when commodity taxes are uniform and migration is motivated by higher urban wages. As an example, consider a hypothetical scenario where urban consumption contributes to a net negative externality while the consumption of rural residents does not have external effects. A uniform commodity tax across regions cannot account for the marginal external costs generated by urban residents, and thus, it fails to prevent excessive migration into the urban area. The government therefore relies on the income tax system. By strengthening redistribution, the government reduces potential gains from migrating and limits excessive urban inflows.

Differentiated commodity taxes are the first-best response to non-atmospheric externalities ([Micheletto, 2008](#); [Aronsson and Sjögren, 2018](#)). However, implementing region-specific commodity taxes can be politically and administratively costly. This paper shows that progressive income taxation provides a viable alternative. By changing the returns to migration, it influences migration decisions and helps the government manage mobility across regions. Overall, the results highlight that standard redistributive tools, which are easier to implement and consistent with horizontal and vertical equity, can play an important role in internalizing regional externalities.

Appendix

Lagrangian and the first-order-conditions. The Lagrangian of the government's program combining the objective (5) and the constraints (6), (7), (3), (4) reads:

$$\begin{aligned}
\mathcal{L} = & \sum_{n \in \{H, L\}} (\alpha_n^U + \alpha_n^R) \mathcal{U}^n + \alpha^M \left\{ \beta \int_{\underline{\delta}}^{\delta^T} (\mathcal{U}^H - \delta) g(\delta) d\delta \right. \\
& \left. + (1 - \beta) \int_{\underline{\delta}}^{\delta^T} (\mathcal{U}^L - \delta) g(\delta) d\delta + (1 - G(\delta^T)) \mathcal{U}^L \right\} \\
& + \lambda \left[\sum_{n \in \{H, L\}} (\alpha_n^U + \alpha_n^R) (y_n - z_n) + \alpha^M \left\{ G(\delta^T) \beta (y_H - z_H) + G(\delta^T) (1 - \beta) (y_L - z_L) \right. \right. \\
& \left. \left. + (1 - G(\delta^T)) (y_L - z_L) \right\} + \tau_q (Q^U + Q^R) + \tau_c (C^U + C^R) \right] \\
& + \varphi \left[\mathcal{U}^H(p_c, p_q, p_x, z_H, \frac{y_H}{w_H}, E) - \hat{\mathcal{U}}(p_c, p_q, p_x, z_L, \frac{y_L}{w_H}, E) \right] \\
& + \mu \left[E - f(Q^U, Q^R) \right] \\
& + \kappa \left[\delta^T - \beta(\mathcal{U}^H - \mathcal{U}^L) \right].
\end{aligned}$$

The first order conditions can be derived as:

$$\begin{aligned}
z_H : \quad & a_H \mathcal{U}_z^H + \lambda \left\{ -a_H + \tau_q \left[\frac{\partial Q^U}{\partial z_H} + \frac{\partial Q^R}{\partial z_H} \right] + \tau_c \left[\frac{\partial C^U}{\partial z_H} + \frac{\partial C^R}{\partial z_H} \right] \right\} + \varphi \mathcal{U}_z^H \\
& - \mu \left\{ f_u \frac{\partial Q^U}{\partial z_H} + f_r \frac{\partial Q^R}{\partial z_H} \right\} - \kappa \beta \mathcal{U}_z^H = 0,
\end{aligned} \tag{11}$$

$$\begin{aligned}
z_L : \quad & a_L \mathcal{U}_z^L + \lambda \left\{ -a_L + \tau_q \left[\frac{\partial Q^U}{\partial z_L} + \frac{\partial Q^R}{\partial z_L} \right] + \tau_c \left[\frac{\partial C^U}{\partial z_L} + \frac{\partial C^R}{\partial z_L} \right] \right\} - \varphi \hat{\mathcal{U}}_z \\
& - \mu \left\{ f_u \frac{\partial Q^U}{\partial z_L} + f_r \frac{\partial Q^R}{\partial z_L} \right\} + \kappa \beta \mathcal{U}_z^L = 0,
\end{aligned} \tag{12}$$

$$\begin{aligned}
y_H : \quad & a_H \mathcal{U}_l^H \frac{1}{w_H} + \lambda \left\{ a_H + \tau_q \left[\frac{\partial Q^U}{\partial y_H} + \frac{\partial Q^R}{\partial y_H} \right] + \tau_c \left[\frac{\partial C^U}{\partial y_H} + \frac{\partial C^R}{\partial y_H} \right] \right\} + \varphi \mathcal{U}_l^H \frac{1}{w_H} \\
& - \mu \left\{ f_u \frac{\partial Q^U}{\partial y_H} + f_r \frac{\partial Q^R}{\partial y_H} \right\} - \kappa \beta \mathcal{U}_l^H \frac{1}{w_H} = 0,
\end{aligned} \tag{13}$$

$$\begin{aligned}
y_L : \quad & a_L \mathcal{U}_l^L \frac{1}{w_L} + \lambda \left\{ a_L + \tau_q \left[\frac{\partial Q^U}{\partial y_L} + \frac{\partial Q^R}{\partial y_L} \right] + \tau_c \left[\frac{\partial C^U}{\partial y_L} + \frac{\partial C^R}{\partial y_L} \right] \right\} - \varphi \hat{\mathcal{U}}_l \frac{1}{w_H} \\
& - \mu \left\{ f_u \frac{\partial Q^U}{\partial y_L} + f_r \frac{\partial Q^R}{\partial y_L} \right\} + \kappa \beta \mathcal{U}_l^L \frac{1}{w_L} = 0,
\end{aligned} \tag{14}$$

$$\begin{aligned}
\tau_q : \quad & a_H \mathcal{U}_{p_q}^H + a_L \mathcal{U}_{p_q}^L + \lambda \left\{ Q^U + Q^R + \tau_q \left[\frac{\partial Q^U}{\partial \tau_q} + \frac{\partial Q^R}{\partial \tau_q} \right] + \tau_c \left[\frac{\partial C^U}{\partial \tau_q} + \frac{\partial C^R}{\partial \tau_q} \right] \right\} \\
& + \varphi \left(\mathcal{U}_{p_q}^H - \hat{\mathcal{U}}_{p_q} \right) - \mu \left\{ f_u \frac{\partial Q^U}{\partial \tau_q} + f_r \frac{\partial Q^R}{\partial \tau_q} \right\} - \kappa \beta \left(\mathcal{U}_{p_q}^H - \mathcal{U}_{p_q}^L \right) = 0,
\end{aligned} \tag{15}$$

$$\begin{aligned}
\tau_c : \quad & a_H \mathcal{U}_{p_c}^H + a_L \mathcal{U}_{p_c}^L + \lambda \left\{ C^U + C^R + \tau_q \left[\frac{\partial Q^U}{\partial \tau_c} + \frac{\partial Q^R}{\partial \tau_c} \right] + \tau_c \left[\frac{\partial C^U}{\partial \tau_c} + \frac{\partial C^R}{\partial \tau_c} \right] \right\} \\
& + \varphi \left(\mathcal{U}_{p_c}^H - \hat{\mathcal{U}}_{p_c} \right) - \mu \left\{ f_u \frac{\partial Q^U}{\partial \tau_c} + f_r \frac{\partial Q^R}{\partial \tau_c} \right\} - \kappa \beta \left(\mathcal{U}_{p_c}^H - \mathcal{U}_{p_c}^L \right) = 0,
\end{aligned} \tag{16}$$

$$\begin{aligned}
E : \quad & a_H \mathcal{U}_E^H + a_L \mathcal{U}_E^L + \lambda \left\{ \tau_q \left[\frac{\partial Q^U}{\partial E} + \frac{\partial Q^R}{\partial E} \right] + \tau_c \left[\frac{\partial C^U}{\partial E} + \frac{\partial C^R}{\partial E} \right] \right\} \\
& + \varphi \left(\mathcal{U}_E^H - \hat{\mathcal{U}}_E \right) + \mu \left\{ 1 - f_u \frac{\partial Q^U}{\partial E} - f_r \frac{\partial Q^R}{\partial E} \right\} - \kappa \beta \left(\mathcal{U}_E^H - \mathcal{U}_E^L \right) = 0,
\end{aligned} \tag{17}$$

$$\delta^T : \quad \lambda \Pi + \lambda \tau_q \left[\frac{\partial Q^U}{\partial \delta^T} + \frac{\partial Q^R}{\partial \delta^T} \right] + \lambda \tau_c \left[\frac{\partial C^U}{\partial \delta^T} + \frac{\partial C^R}{\partial \delta^T} \right] - \mu \left\{ f_u \frac{\partial Q^U}{\partial \delta^T} + f_r \frac{\partial Q^R}{\partial \delta^T} \right\} + \kappa = 0. \tag{18}$$

The terms denoting aggregate q consumption across regions, Q^i for $i \in \{U, R\}$, are defined in (4). The terms for aggregate c consumption across regions, C^i , are defined analogously. The aggregate population share of high-type individuals is defined as $a_H = \alpha_H^U + \alpha_H^R + \alpha^M \beta G(\delta^T)$, i.e., the sum of high-type individuals in urban and rural areas and migrating individuals who become high-type. The aggregate population share of low-type individuals is defined as $a_L = \alpha_L^U + \alpha_L^R + \alpha^M (1 - \beta) G(\delta^T) + (1 - G(\delta^T)) \alpha^M$, i.e., the sum of low-type individuals in urban and rural areas, migrating individuals who do not become high-type, and individuals who do not migrate despite being low-type. Finally, Π denotes the effect of a marginal increase in the migration threshold, δ^T , on income tax revenues.

Compensated demand responses. For the proofs that follow, it is useful to define the compensated demand responses associated with price changes. The compensated own- and cross-price effects, for $i \in \{U, R\}$, are defined as:

$$\begin{aligned}
\frac{\partial \tilde{Q}^i}{\partial p_q} &= q_H \frac{\partial Q^i}{\partial z_H} + q_L \frac{\partial Q^i}{\partial z_L} + \frac{\partial Q^i}{\partial \tau_q}, \\
\frac{\partial \tilde{C}^i}{\partial p_c} &= c_H \frac{\partial C^i}{\partial z_H} + c_L \frac{\partial C^i}{\partial z_L} + \frac{\partial C^i}{\partial \tau_c}, \\
\frac{\partial \tilde{Q}^i}{\partial p_c} &= c_H \frac{\partial Q^i}{\partial z_H} + c_L \frac{\partial Q^i}{\partial z_L} + \frac{\partial Q^i}{\partial \tau_c}, \\
\frac{\partial \tilde{C}^i}{\partial p_q} &= q_H \frac{\partial C^i}{\partial z_H} + q_L \frac{\partial C^i}{\partial z_L} + \frac{\partial C^i}{\partial \tau_q}.
\end{aligned} \tag{19}$$

To illustrate more clearly how these expressions correspond to Slutsky-style compensated demand effects, consider, for example, $\frac{\partial \tilde{Q}^U}{\partial p_q}$, the compensated effect of a change in p_q on aggregate q consumption in the urban area. Using the expression for Q^U provided in (4), we can write:

$$\frac{\partial \tilde{Q}^U}{\partial p_q} = \left(\frac{\partial q_H}{\partial z_H} q_H + \frac{\partial q_H}{\partial \tau_q} \right) (\alpha_H^U + \alpha^M \beta G(\delta^T)) + \left(\frac{\partial q_L}{\partial z_L} q_L + \frac{\partial q_L}{\partial \tau_q} \right) (\alpha_L^U + \alpha^M (1 - \beta) G(\delta^T)). \tag{20}$$

The first component of the expression above represents the compensated demand (substitution) effect of a change in p_q through urban high-types, while the second component represents the corresponding effect through urban low-types. The remaining compensated effects can be derived analogously.

The compensated effects of a change in the total level of the externality E can also be defined in a Slutsky sense in a similar way:

$$\begin{aligned}
\frac{\partial \tilde{Q}^i}{\partial E} &= MW P_{E,z}^H \frac{\partial Q^i}{\partial z_H} + MW P_{E,z}^L \frac{\partial Q^i}{\partial z_L} + \frac{\partial Q^i}{\partial E}, \\
\frac{\partial \tilde{C}^i}{\partial E} &= MW P_{E,z}^H \frac{\partial C^i}{\partial z_H} + MW P_{E,z}^L \frac{\partial C^i}{\partial z_L} + \frac{\partial C^i}{\partial E},
\end{aligned} \tag{21}$$

where $MW P_{E,z}^n = -\mathcal{U}_E^n / \mathcal{U}_z^n$, for $n \in \{H, L\}$, represents the marginal willingness to pay of an n -type individual for a marginal reduction in the externality.

To clarify the relationship of these expressions to Slutsky-style compensated demand effects, consider the compensated effect of a change in E on urban q consumption, $\frac{\partial \tilde{Q}^U}{\partial E}$, as an illustrative example. Using (4), we can write:

$$\frac{\partial \tilde{Q}^U}{\partial E} = \left(\frac{\partial q_H}{\partial z_H} MW P_{E,z}^H + \frac{\partial q_H}{\partial E} \right) (\alpha_H^U + \alpha^M \beta G(\delta^T)) + \left(\frac{\partial q_L}{\partial z_L} MW P_{E,z}^L + \frac{\partial q_L}{\partial E} \right) (\alpha_L^U + \alpha^M (1 - \beta) G(\delta^T)). \tag{22}$$

The first term captures the compensated demand effect with respect to a marginal change in E via urban high-types, while the second term captures the corresponding effect

via urban low-types. The remaining compensated effects with respect to E can be derived analogously.

Proof of Proposition 1. Rewrite (17) by using $MWP_{E,z}^n = -\mathcal{U}_E^n/\mathcal{U}_z^n$ for $n \in \{H, L\}$, $M\hat{W}P_{E,z} = -\hat{\mathcal{U}}_E/\hat{\mathcal{U}}_z$ and the definitions of $a_H\mathcal{U}_z^H$ and $a_L\mathcal{U}_z^L$ arising from (11) and (12):

$$\begin{aligned}
& -MWP_{E,z}^H \left\{ \lambda a_H - \lambda \tau_q \left[\frac{\partial Q^U}{\partial z_H} + \frac{\partial Q^R}{\partial z_H} \right] - \lambda \tau_c \left[\frac{\partial C^U}{\partial z_H} + \frac{\partial C^R}{\partial z_H} \right] - \varphi \mathcal{U}_z^H \right. \\
& \left. + \mu \left\{ f_u \frac{\partial Q^U}{\partial z_H} + f_r \frac{\partial Q^R}{\partial z_H} \right\} + \kappa \beta \mathcal{U}_z^H \right\} \\
& -MWP_{E,z}^L \left\{ \lambda a_L - \lambda \tau_q \left[\frac{\partial Q^U}{\partial z_L} + \frac{\partial Q^R}{\partial z_L} \right] - \lambda \tau_c \left[\frac{\partial C^U}{\partial z_L} + \frac{\partial C^R}{\partial z_L} \right] - \varphi \hat{\mathcal{U}}_z \right. \\
& \left. + \mu \left\{ f_u \frac{\partial Q^U}{\partial z_L} + f_r \frac{\partial Q^R}{\partial z_L} \right\} - \kappa \beta \mathcal{U}_z^L \right\} \\
& + \lambda \tau_q \left[\frac{\partial Q^U}{\partial E} + \frac{\partial Q^R}{\partial E} \right] + \lambda \tau_c \left[\frac{\partial C^U}{\partial E} + \frac{\partial C^R}{\partial E} \right] - \varphi MWP_{E,z}^H \mathcal{U}_z^H + \varphi M\hat{W}P_{E,z} \hat{\mathcal{U}}_z \\
& + \mu \left(1 - f_u \frac{\partial Q^U}{\partial E} - f_r \frac{\partial Q^R}{\partial E} \right) + \kappa \beta MWP_{E,z}^H \mathcal{U}_z^H - \kappa \beta MWP_{E,z}^L \mathcal{U}_z^L = 0.
\end{aligned} \tag{23}$$

Rearrange to get:

$$\begin{aligned}
& -\lambda \left(a_H MWP_{E,z}^H + a_L MWP_{E,z}^L \right) \\
& - \lambda \tau_q \left\{ -MWP_{E,z}^H \frac{\partial Q^U}{\partial z_H} - MWP_{E,z}^H \frac{\partial Q^R}{\partial z_H} - MWP_{E,z}^L \frac{\partial Q^U}{\partial z_L} - MWP_{E,z}^L \frac{\partial Q^R}{\partial z_L} - \frac{\partial Q^U}{\partial E} - \frac{\partial Q^R}{\partial E} \right\} \\
& - \lambda \tau_c \left\{ -MWP_{E,z}^H \frac{\partial C^U}{\partial z_H} - MWP_{E,z}^H \frac{\partial C^R}{\partial z_H} - MWP_{E,z}^L \frac{\partial C^U}{\partial z_L} - MWP_{E,z}^L \frac{\partial C^R}{\partial z_L} - \frac{\partial C^U}{\partial E} - \frac{\partial C^R}{\partial E} \right\} \\
& - \mu f_u \left\{ MWP_{E,z}^H \frac{\partial Q^U}{\partial z_H} + MWP_{E,z}^L \frac{\partial Q^U}{\partial z_L} + \frac{\partial Q^U}{\partial E} \right\} - \mu f_r \left\{ MWP_{E,z}^H \frac{\partial Q^R}{\partial z_H} + MWP_{E,z}^L \frac{\partial Q^R}{\partial z_L} + \frac{\partial Q^R}{\partial E} \right\} \\
& \varphi \hat{\mathcal{U}}_z \left(M\hat{W}P_{E,z} - MWP_{E,z}^L \right) = -\mu.
\end{aligned} \tag{24}$$

Notice that under a separable utility function, such as the one imposed in (8) of Assumption 1, we have $M\hat{W}P_{E,z} - MWP_{E,z}^L = 0$. Using this and substituting (21) into the expression above yields:

$$-\lambda \sum_{n \in \{H, L\}} a_n MWP_{E,z}^n + \lambda \tau_q \left(\frac{\partial \tilde{Q}^U}{\partial E} + \frac{\partial \tilde{Q}^R}{\partial E} \right) + \lambda \tau_c \left(\frac{\partial \tilde{C}^U}{\partial E} + \frac{\partial \tilde{C}^R}{\partial E} \right) = -\mu + \mu f_u \frac{\partial \tilde{Q}^U}{\partial E} + \mu f_r \frac{\partial \tilde{Q}^R}{\partial E}. \tag{25}$$

Dividing all the terms by λ and rearranging yields:

$$\frac{\mu}{\lambda} = \sigma \left(\sum_{n \in \{H, L\}} a_n MWP_{E,z}^n - \tau_q \sum_{i \in \{U, R\}} \frac{\partial \tilde{Q}^i}{\partial E} - \tau_c \sum_{i \in \{U, R\}} \frac{\partial \tilde{C}^i}{\partial E} \right), \tag{26}$$

where $\frac{1}{\sigma} = 1 - f_u \frac{\partial \tilde{Q}^U}{\partial E} - f_r \frac{\partial \tilde{Q}^R}{\partial E}$.

Substitute the expressions for optimal τ_q and τ_c from Proposition 2 (proofs below) into the expression above and rearrange to get:

$$\frac{\mu}{\lambda} = \left(1 - f_u \frac{\partial \tilde{Q}^U}{\partial E} - f_r \frac{\partial \tilde{Q}^R}{\partial E} + (f_u \theta_u + f_r \theta_r) \left(\frac{\partial \tilde{Q}^U}{\partial E} + \frac{\partial \tilde{Q}^R}{\partial E} \right) \right). \quad (27)$$

Let the preferences of individuals be represented by equation (8) in Assumption 1. Then, the demand function for good q is given by $q_n = \gamma_q \frac{z_n}{p_q}$ for $n \in \{H, L\}$. Furthermore, we have $MWP_{E,z}^n = -\mathcal{U}_E^n / \mathcal{U}_z^n = -1/(-1/z_n) = z_n$ under the same utility specification. Using these results together with (4) and (19), it follows that $\frac{\partial \tilde{Q}^i}{\partial E} = Q^i$ for $i \in \{U, R\}$. Substituting this result, along with the definitions of θ_u and θ_r from Proposition 2, into (27) yields the expression in Proposition 1. $\blacksquare$

Proof of Proposition 2. Multiply (11) by q_H to obtain:

$$\begin{aligned} a_H \mathcal{U}_z^H q_H - \lambda a_H q_H + \lambda \tau_q q_H \left[\frac{\partial Q^U}{\partial z_H} + \frac{\partial Q^R}{\partial z_H} \right] + \lambda \tau_c q_H \left[\frac{\partial C^U}{\partial z_H} + \frac{\partial C^R}{\partial z_H} \right] \\ + \varphi q_H \mathcal{U}_z^H - \mu q_H \left(f_u \frac{\partial Q^U}{\partial z_H} + f_r \frac{\partial Q^R}{\partial z_H} \right) - q_H \kappa \beta \mathcal{U}_z^H = 0. \end{aligned} \quad (28)$$

Multiply (12) by q_L to obtain:

$$\begin{aligned} a_L \mathcal{U}_z^L q_L - \lambda a_L q_L + \lambda \tau_q q_L \left[\frac{\partial Q^U}{\partial z_L} + \frac{\partial Q^R}{\partial z_L} \right] + \lambda \tau_c q_L \left[\frac{\partial C^U}{\partial z_L} + \frac{\partial C^R}{\partial z_L} \right] \\ + \varphi q_L \mathcal{U}_z^L - \mu q_L \left(f_u \frac{\partial Q^U}{\partial z_L} + f_r \frac{\partial Q^R}{\partial z_L} \right) - q_L \kappa \beta \mathcal{U}_z^L = 0. \end{aligned} \quad (29)$$

Use Roy's identity, i.e., $\mathcal{U}_{p_q}^n = -q_n \mathcal{U}_z^n$ and $\hat{\mathcal{U}}_{p_q} = -\hat{q} \hat{\mathcal{U}}_z$, to rewrite (15) as:

$$\begin{aligned} -a_H \mathcal{U}_z^H q_H - a_L \mathcal{U}_z^L q_L + \lambda (Q^U + Q^R) + \lambda \tau_q \left[\frac{\partial Q^U}{\partial \tau_q} + \frac{\partial Q^R}{\partial \tau_q} \right] + \lambda \tau_c \left[\frac{\partial C^U}{\partial \tau_q} + \frac{\partial C^R}{\partial \tau_q} \right] \\ - \varphi q_H \mathcal{U}_z^H + \varphi \hat{q} \hat{\mathcal{U}}_z - \mu \left(f_u \frac{\partial Q^U}{\partial \tau_q} + f_r \frac{\partial Q^R}{\partial \tau_q} \right) + \kappa \beta q_H \mathcal{U}_z^H - \kappa \beta q_L \mathcal{U}_z^L = 0. \end{aligned} \quad (30)$$

Summing (28), (29), and (30), using the fact that $\hat{q} = q_L$ due to separability in preferences, and rearranging yields:

$$\begin{aligned} \tau_q \left\{ q_H \left(\frac{\partial Q^U}{\partial z_H} + \frac{\partial Q^R}{\partial z_H} \right) + q_L \left(\frac{\partial Q^U}{\partial z_L} + \frac{\partial Q^R}{\partial z_L} \right) + \frac{\partial Q^U}{\partial \tau_q} + \frac{\partial Q^R}{\partial \tau_q} \right\} \\ + \tau_c \left\{ q_H \left(\frac{\partial C^U}{\partial z_H} + \frac{\partial C^R}{\partial z_H} \right) + q_L \left(\frac{\partial C^U}{\partial z_L} + \frac{\partial C^R}{\partial z_L} \right) + \frac{\partial C^U}{\partial \tau_q} + \frac{\partial C^R}{\partial \tau_q} \right\} \\ = \frac{\mu}{\lambda} \left\{ f_u \left(\frac{\partial Q^U}{\partial z_H} q_H + \frac{\partial Q^U}{\partial z_L} q_L + \frac{\partial Q^U}{\partial \tau_q} \right) + f_r \left(\frac{\partial Q^R}{\partial z_H} q_H + \frac{\partial Q^R}{\partial z_L} q_L + \frac{\partial Q^R}{\partial \tau_q} \right) \right\}. \end{aligned} \quad (31)$$

Next, following a procedure similar to the calculations above, I multiply (11) by c_H and (12) by c_L , rewrite (16) using Roy's identity, sum the resulting expressions, and rearrange to obtain the following expression. The intermediate calculations are omitted for brevity.

$$\begin{aligned} & \tau_q \left\{ c_H \left(\frac{\partial Q^U}{\partial z_H} + \frac{\partial Q^R}{\partial z_H} \right) + c_L \left(\frac{\partial Q^U}{\partial z_L} + \frac{\partial Q^R}{\partial z_L} \right) + \frac{\partial Q^U}{\partial \tau_c} + \frac{\partial Q^R}{\partial \tau_c} \right\} \\ & + \tau_c \left\{ c_H \left(\frac{\partial C^U}{\partial z_H} + \frac{\partial C^R}{\partial z_H} \right) + c_L \left(\frac{\partial C^U}{\partial z_L} + \frac{\partial C^R}{\partial z_L} \right) + \frac{\partial C^U}{\partial \tau_c} + \frac{\partial C^R}{\partial \tau_c} \right\} \\ & = \frac{\mu}{\lambda} \left\{ f_u \left(\frac{\partial Q^U}{\partial z_H} c_H + \frac{\partial Q^U}{\partial z_L} c_L + \frac{\partial Q^U}{\partial \tau_c} \right) + f_r \left(\frac{\partial Q^R}{\partial z_H} c_H + \frac{\partial Q^R}{\partial z_L} c_L + \frac{\partial Q^R}{\partial \tau_c} \right) \right\} \end{aligned} \quad (32)$$

The compensated demand responses introduced in (19) can be used to rewrite (31) and (32), respectively, as:

$$\tau_q \sum_{i \in \{U, R\}} \frac{\partial \tilde{Q}^i}{\partial p_q} + \tau_c \sum_{i \in \{U, R\}} \frac{\partial \tilde{C}^i}{\partial p_q} = \frac{\mu}{\lambda} \left\{ f_u \frac{\partial \tilde{Q}^U}{\partial p_q} + f_r \frac{\partial \tilde{Q}^R}{\partial p_q} \right\}, \quad (33)$$

$$\tau_q \sum_{i \in \{U, R\}} \frac{\partial \tilde{Q}^i}{\partial p_c} + \tau_c \sum_{i \in \{U, R\}} \frac{\partial \tilde{C}^i}{\partial p_c} = \frac{\mu}{\lambda} \left\{ f_u \frac{\partial \tilde{Q}^U}{\partial p_c} + f_r \frac{\partial \tilde{Q}^R}{\partial p_c} \right\}. \quad (34)$$

The utility specification in (8) of Assumption 1 yields the demand functions $q_n = \gamma_q \frac{z_n}{p_q}$ and $c_n = \gamma_c \frac{z_n}{p_c}$ for $n \in \{H, L\}$. Using these expressions, together with the definitions of compensated demand responses in (19), the expressions for Q^i for $i \in \{U, R\}$ in (4), and the analogous expressions for C^i , it follows that the compensated cross-price responses are zero: $\frac{\partial \tilde{Q}^i}{\partial p_c} = \frac{\partial \tilde{C}^i}{\partial p_q} = 0$. Substituting this result into (34) yields $\tau_c = 0$.

Substituting $\tau_c = 0$ into (33) gives:

$$\tau_q = \frac{\mu}{\lambda} (f_u \theta_u + f_r \theta_r) \quad \text{where} \quad \theta_u = \frac{\frac{\partial \tilde{Q}^U}{\partial p_q}}{\frac{\partial \tilde{Q}^U}{\partial p_q} + \frac{\partial \tilde{Q}^R}{\partial p_q}}, \quad \theta_r = \frac{\frac{\partial \tilde{Q}^R}{\partial p_q}}{\frac{\partial \tilde{Q}^U}{\partial p_q} + \frac{\partial \tilde{Q}^R}{\partial p_q}}. \quad (35)$$

Under the demand functions $q_n = \gamma_q \frac{z_n}{p_q}$ and $c_n = \gamma_c \frac{z_n}{p_c}$ for $n \in \{H, L\}$, implied by (8), together with the compensated demand response definitions in (19) and the definitions of Q^i for $i \in \{U, R\}$, it follows that $\frac{\partial \tilde{Q}^U}{\partial p_q} = \frac{-1+\gamma_q}{p_q} Q^U$ and $\frac{\partial \tilde{Q}^R}{\partial p_q} = \frac{-1+\gamma_q}{p_q} Q^R$, i.e., compensated demand responses are proportional to the corresponding demand levels. Substituting these expressions into (35) yields the optimal τ_q reported in Proposition 2. $\blacksquare$

Proof of Proposition 3. I start by the derivation of the optimal marginal income tax rate on the high-type, $T'(y_H)$. Rearrange the first-order-conditions (11) and (13)

respectively to write:

$$\mathcal{U}_z^H(a_H + \varphi - \kappa\beta) = \lambda \left\{ a_H - \tau_q \left[\frac{\partial Q^U}{\partial z_H} + \frac{\partial Q^R}{\partial z_H} \right] - \tau_c \left[\frac{\partial C^U}{\partial z_H} + \frac{\partial C^R}{\partial z_H} \right] \right\} + \mu \left(f_u \frac{\partial Q^U}{\partial z_H} + f_r \frac{\partial Q^R}{\partial z_H} \right), \quad (36)$$

$$\mathcal{U}_l^H \frac{1}{w_H} (a_H + \varphi - \kappa\beta) = -\lambda \left\{ a_H + \tau_q \left[\frac{\partial Q^U}{\partial y_H} + \frac{\partial Q^R}{\partial y_H} \right] + \tau_c \left[\frac{\partial C^U}{\partial y_H} + \frac{\partial C^R}{\partial y_H} \right] \right\} + \mu \left(f_u \frac{\partial Q^U}{\partial y_H} + f_r \frac{\partial Q^R}{\partial y_H} \right). \quad (37)$$

Dividing (37) by (36) side-by-side, using that $\tau_c = 0$ at the optimum and rearranging yields:

$$\frac{\mathcal{U}_l^H \frac{1}{w_H}}{\mathcal{U}_z^H} = \frac{-\lambda a_H - \lambda \frac{\partial Q^U}{\partial y_H} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) - \lambda \frac{\partial Q^R}{\partial y_H} \left(\tau_q - \frac{\mu}{\lambda} f_r \right)}{\lambda a_H - \lambda \frac{\partial Q^U}{\partial z_H} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) - \lambda \frac{\partial Q^R}{\partial z_H} \left(\tau_q - \frac{\mu}{\lambda} f_r \right)}. \quad (38)$$

Substitute in that $MRS_{y,z}^H = \frac{\mathcal{U}_l^H \frac{1}{w_H}}{\mathcal{U}_z^H}$ and rearrange to get:

$$\begin{aligned} MRS_{y,z}^H \lambda a_H - MRS_{y,z}^H \lambda \frac{\partial Q^U}{\partial z_H} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) - MRS_{y,z}^H \lambda \frac{\partial Q^R}{\partial z_H} \left(\tau_q - \frac{\mu}{\lambda} f_r \right) \\ = -\lambda a_H - \lambda \frac{\partial Q^U}{\partial y_H} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) - \lambda \frac{\partial Q^R}{\partial y_H} \left(\tau_q - \frac{\mu}{\lambda} f_r \right). \end{aligned} \quad (39)$$

Dividing both sides by λa_H and using that (2), which implies $T'(y_H) = 1 + MRS_{y,z}^H$ leads to:

$$T'(y_H) = -\frac{1}{a_H} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) \left(-\frac{\partial Q^U}{\partial z_H} MRS_{y,z}^H + \frac{\partial Q^U}{\partial y_H} \right) - \frac{1}{a_H} \left(\tau_q - \frac{\mu}{\lambda} f_r \right) \left(-\frac{\partial Q^R}{\partial z_H} MRS_{y,z}^H + \frac{\partial Q^R}{\partial y_H} \right). \quad (40)$$

Notice first that this expression can be grouped over $-\frac{1}{a_H} \left(-\frac{\partial Q^U}{\partial z_H} MRS_{y,z}^H + \frac{\partial Q^U}{\partial y_H} \right)$. Moreover, recall from (4) that $Q^U = \sum_{n \in \{H,L\}} \alpha_n^U q_n + \alpha^M G(\delta^T) [\beta q_H + (1 - \beta) q_L]$ and $Q^R = \sum_{n \in \{H,L\}} \alpha_n^R q_n + \alpha^M (1 - G(\delta^T)) q_L$. At the same time, $\frac{\partial q_L}{\partial y_H} = \frac{\partial q_L}{\partial z_H} = 0$ under the demand functions $q_n = \gamma_q \frac{z_n}{p_q}$ for $n \in \{H, L\}$ implied by (8) of Assumption 1. Using these expressions and rearranging yields:

$$T'(y_H) = - \left(-MRS_{y,z}^H \frac{\partial q_H}{\partial z_H} + \frac{\partial q_H}{\partial y_H} \right) \left\{ \left(\tau_q - \frac{\mu}{\lambda} f_u \right) \frac{\alpha_H^U + \alpha^M G(\delta^T) \beta}{a_H} + \left(\tau_q - \frac{\mu}{\lambda} f_r \right) \frac{\alpha_H^R}{a_H} \right\} \quad (41)$$

Notice that $\alpha_H^U + \alpha^M G(\delta^T) \beta$ corresponds to the mass of high-type individuals in the urban region, while α_H^R corresponds to the mass of high-type individuals in the rural region. Accordingly, define the shares of high-type individuals residing in the urban and rural regions as $H_u := \frac{\alpha_H^U + \alpha^M G(\delta^T) \beta}{a_H}$ and $H_r := \frac{\alpha_H^R}{a_H}$ (within the total mass of high-type, a_H), respectively. The demand function $q_H = \gamma_q \frac{z_H}{p_q}$ further implies that $\frac{\partial q_H}{\partial y_H} = 0$. Substituting

these expressions into the previous equation yields the formula for the optimal $T'(y_H)$ presented in Proposition 3.

The derivation of the optimal marginal income tax rate on the low-type, $T'(y_L)$, follows analogous steps. Rearranging (12) and (14), and dividing the latter by the former in a manner similar to (36), (37), and (38), while also taking into account that the optimal $\tau_c = 0$, yields:

$$\frac{\mathcal{U}_l^L \frac{1}{w_L}}{\mathcal{U}_z^L} = \frac{-\lambda a_L - \lambda \frac{\partial Q^U}{\partial y_L} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) - \lambda \frac{\partial Q^R}{\partial y_L} \left(\tau_q - \frac{\mu}{\lambda} f_r \right) + \varphi \hat{\mathcal{U}}_l \frac{1}{w_H}}{\lambda a_L - \lambda \frac{\partial Q^U}{\partial z_L} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) - \lambda \frac{\partial Q^R}{\partial z_L} \left(\tau_q - \frac{\mu}{\lambda} f_r \right) + \varphi \hat{\mathcal{U}}_z}. \quad (42)$$

Substitute in $MRS_{y,z}^L = \frac{\mathcal{U}_l^L \frac{1}{w_L}}{\mathcal{U}_z^L}$ and rearrange to get:

$$\begin{aligned} & MRS_{y,z}^L \lambda a_L - MRS_{y,z}^L \lambda \frac{\partial Q^U}{\partial z_L} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) - MRS_{y,z}^L \lambda \frac{\partial Q^R}{\partial z_L} \left(\tau_q - \frac{\mu}{\lambda} f_r \right) + MRS_{y,z}^L \varphi \hat{\mathcal{U}}_z \\ &= -\lambda a_L - \lambda \frac{\partial Q^U}{\partial y_L} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) - \lambda \frac{\partial Q^R}{\partial y_L} \left(\tau_q - \frac{\mu}{\lambda} f_r \right) + \varphi \hat{\mathcal{U}}_l \frac{1}{w_H}. \end{aligned} \quad (43)$$

Dividing both sides by λa_L and using $T'(y_L) = 1 + MRS_{y,z}^L$ implied by (2) yields:

$$\begin{aligned} T'(y_L) &= \frac{\varphi}{\lambda a_L} \hat{\mathcal{U}}_l \frac{1}{w_H} - \frac{\varphi}{\lambda a_L} \hat{\mathcal{U}}_z MRS_{y,z}^L - \frac{1}{a_L} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) \left(-\frac{\partial Q^U}{\partial z_L} MRS_{y,z}^L + \frac{\partial Q^U}{\partial y_L} \right) \\ &\quad - \frac{1}{a_L} \left(\tau_q - \frac{\mu}{\lambda} f_r \right) \left(-\frac{\partial Q^R}{\partial z_L} MRS_{y,z}^L + \frac{\partial Q^R}{\partial y_L} \right). \end{aligned} \quad (44)$$

Multiplying the first term by $\frac{\hat{\mathcal{U}}_z}{\hat{\mathcal{U}}_z^n}$ and using the definitions of the marginal rates of substitutions, $MRS_{y,z}^n = \frac{\mathcal{U}_l^n \frac{1}{w_n}}{\mathcal{U}_z^n}$ and $\hat{M}RS_{y,z} = \frac{\hat{\mathcal{U}}_l \frac{1}{w_H}}{\hat{\mathcal{U}}_z}$, leads to:

$$\begin{aligned} T'(y_L) &= \frac{\varphi}{\lambda a_L} \hat{\mathcal{U}}_z \left(\hat{M}RS_{y,z} - MRS_{y,z}^L \right) - \frac{1}{a_L} \left(\tau_q - \frac{\mu}{\lambda} f_u \right) \left(-\frac{\partial Q^U}{\partial z_L} MRS_{y,z}^L + \frac{\partial Q^U}{\partial y_L} \right) \\ &\quad - \frac{1}{a_L} \left(\tau_q - \frac{\mu}{\lambda} f_r \right) \left(-\frac{\partial Q^R}{\partial z_L} MRS_{y,z}^L + \frac{\partial Q^R}{\partial y_L} \right). \end{aligned} \quad (45)$$

Following analogous steps to derive (41) from (40), we obtain from (45) the expression for the optimal $T'(y_L)$ reported in Proposition 3, where $L_u := \frac{\alpha_L^U + \alpha^M G(\delta^T)(1-\beta)}{a_L}$ and $L_r := \frac{\alpha_L^R + \alpha^M (1-G(\delta^T))}{a_L}$ denote the shares of low-type individuals residing in urban and rural regions, respectively. ■

Proof of Proposition 4. Dividing all terms in (18) by λ , imposing the optimality condition $\tau_c = 0$, and rearranging yields the expression reported in Proposition 4. ■

References

- Ahlfeldt, G. M. and Pietrostefani, E. (2019). The economic effects of density: A synthesis. *Journal of Urban Economics*, 111:93–107.
- Aigner, R. (2014). Environmental taxation and redistribution concerns. *FinanzArchiv/Public Finance Analysis*, 70(2):249–277.
- Aronsson, T. and Johansson-Stenman, O. (2010). Positional concerns in an olg model: Optimal labor and capital income taxation*. *International Economic Review*, 51(4):1071–1095.
- Aronsson, T. and Sjögren, T. (2018). Optimal taxation, redistribution, and environmental externalities. *International Review of Environmental and Resource Economics*, 11(3):233–308.
- Atkinson, A. and Stiglitz, J. (1976). The design of tax structure: Direct versus indirect taxation. *Journal of Public Economics*, 6(1–2):55–75.
- Borck, R. and Schrauth, P. (2021). Population density and urban air quality. *Regional Science and Urban Economics*, 86:103596.
- Borck, R. and Schrauth, P. (2024). Urban pollution: A global perspective. *Journal of Environmental Economics and Management*, 126:103013.
- Bovenberg, A. and van der Ploeg, F. (1994). Environmental policy, public finance and the labour market in a second-best world. *Journal of Public Economics*, 55(3):349–390.
- Bovenberg, A. L. and de Mooij, R. A. (1994). Environmental levies and distortionary taxation. *American Economic Review*, 84(4):1085–1089.
- Bovenberg, A. L. and Van Der Ploeg, F. (1998). Tax reform, structural unemployment and the environment. *The Scandinavian Journal of Economics*, 100(3):593–610.
- Carozzi, F. and Roth, S. (2023). Dirty density: Air quality and the density of american cities. *Journal of Environmental Economics and Management*, 118:102767.
- Chang, Y. S., Lee, Y. J., and Choi, S. S. B. (2017). Is there more traffic congestion in larger cities? -scaling analysis of the 101 largest u.s. urban centers-. *Transport Policy*, 59:54–63.

- Chetty, R. (2012). Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply. *Econometrica*, 80(3):969–1018.
- Ciccone, A. and Hall, R. E. (1996). Productivity and density of economic activity. *American Economic Review*, 86(1):54–70.
- Combes, P.-P. and Gobillon, L. (2015). *Handbook of Regional and Urban Economics*, chapter The Empirics of Agglomeration Economies, pages 247–348. Elsevier.
- Cremer, H., Gahvari, F., and Ladoux, N. (1998). Externalities and optimal taxation. *Journal of Public Economics*, 70(3):343–364.
- Eckerstorfer, P. (2014). Relative consumption concerns and the optimal tax mix. *Journal of Public Economic Theory*, 16(6):936–958.
- Eckerstorfer, P. and Wendner, R. (2013). Asymmetric and non-atmospheric consumption externalities, and efficient consumption taxation. *Journal of Public Economics*, 106:42–56.
- Glaeser, E. and Sacerdote, B. (1999). Why is there more crime in cities? *Journal of Political Economy*, 107(S6):S225–S258.
- Jacobs, B. and de Mooij, R. A. (2015). Pigou meets mirrlees: On the irrelevance of tax distortions for the second-best pigouvian tax. *Journal of Environmental Economics and Management*, 71:90–108.
- Jacobs, B. and van der Ploeg, F. (2019). Redistribution and pollution taxes with non-linear engel curves. *Journal of Environmental Economics and Management*, 95:198–226.
- Jacquet, L., Lehmann, E., and Van der Linden, B. (2013). Optimal redistributive taxation with both extensive and intensive responses. *Journal of Economic Theory*, 148(5):1770–1805.
- Kessing, S. G., Lipatov, V., and Zoubek, J. M. (2020). Optimal taxation under regional inequality. *European Economic Review*, 126:103439.
- Micheletto, L. (2008). Redistribution and optimal mixed taxation in the presence of consumption externalities. *Journal of Public Economics*, 92(10–11):2262–2274.

- Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *The Review of Economic Studies*, 38(2):175.
- Mirrlees, J. A. (2011). *Tax By Design: The Mirrlees Review*. Institute for Fiscal Studies, Oxford, Oxford University Press.
- Musgrave, R. A. and Musgrave, P. B. (1989). *Public Finance in Theory and Practice*. McGraw-Hill, 5 edition.
- Paul, C. J. M. and Siegel, D. S. (1999). Scale economies and industry agglomeration externalities: A dynamic cost function approach. *American Economic Review*, 89(1):272–290.
- Pirttilä, J. and Tuomala, M. (1997). Income tax, commodity tax and environmental policy. *International Tax and Public Finance*, 4:379–393.
- Sandmo, A. (1975). Optimal taxation in the presence of externalities. *The Swedish Journal of Economics*, 77(1):86.
- Schneider, K. (1997). Involuntary unemployment and environmental policy: The double dividend hypothesis. *The Scandinavian Journal of Economics*, 99(1):45–59.
- Stiglitz, J. E. (1982). Self-selection and pareto efficient taxation. *Journal of Public Economics*, 17(2):213–240.
- Wendner, R. (2014). Ramsey, pigou, heterogeneous agents, and nonatmospheric consumption externalities. *Journal of Public Economic Theory*, 16(3):491–521.
- Zivin, J. G. and Neidell, M. (2012). The impact of pollution on worker productivity. *American Economic Review*, 102(7):3652–3673.